

Estimating Firm-Level Production Functions with Spatial Dependence*

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Abstract

This paper generalizes firm-level production function estimations to allow for firm-to-firm spatial linkages. In particular, a firm’s productivity is allowed to depend on related firms’ lagged outputs (via local supplier-customer linkages), on related firms’ lagged labor inputs (via sharing local labor pools), and on related firms’ current productivity shocks. We propose a three-stage estimation strategy and apply it to a Japanese firm-to-firm dataset comprising 14,178 firms over the period 2009–2018. We find significant and positive spatial coefficients in the Japanese firm-level productivity process via all three proposed channels. We formulate the asymptotic properties of the proposed estimator, including a set of sufficient conditions, and conduct Monte Carlo simulations to validate these properties.

Key Words: productivity estimation; spatial dependence; supplier-customer network; factor market pooling; knowledge spillover

JEL Classification: C31; D24

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1 Introduction

Firm-level productivity (production function) estimation is critical to both positive and normative research, in inferring the characteristics of firm-level production activities and identifying the effect of policy/exogenous shocks (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Akerberg, Caves and Frazer, 2015; Wooldridge, 2009). Equally important, a large literature has documented/analyzed how firms interact with each other via the supplier-customer linkages, factor markets, and knowledge spillovers (e.g., Demir, Fieger, Xu and Yang, 2024; Miyauchi, 2023; Alfaro-Ureña, Manelici and Vasquez, 2022; Ellison, Glaeser and Kerr, 2010; Helsley and Strange, 1990; Diamond and Simon, 1990; Helsley and Strange, 2002; Jaffe, Trajtenberg and Henderson, 1993; Audretsch and Feldman, 1996; Matray, 2021).

Previous literature on estimating firm-level production functions typically ignores potential spatial dependence across firms. The firm-level production activities are often taken to be independent in estimations, before the estimated productivities are used to analyze potential determinants of firm productivity (e.g., a firm’s exposure to foreign direct investment or imports at the sector level, or supplying to multinational corporations at the firm level). The recent work by Iyoha (2023) highlights the need to estimate firm productivities simultaneously taking into account the presence of productivity spillovers. Her work, however, models the interdependence across firms “in reduced form” in terms of their productivities. This leads to a rather difficult setup for estimations, and prevents her from formulating the asymptotic properties (and variance-covariance) of the estimator. As a result, inferences have to be made based on bootstrap methods.

In this paper, we propose methodologies that model productivity dependence across firms structurally, motivated by the mechanisms highlighted by the spatial and firm-to-firm production network literatures. In particular, a firm’s productivity is allowed to depend on related firms’ lagged outputs (e.g., via local supplier-customer linkages), on related firms’ lagged labor inputs (e.g., via shared local labor pools), and on related firms’ current productivity shocks (e.g., via knowledge spillovers with the boundary defined by the geographical area and/or by the production network). We propose a three-stage GMM-type estimation strategy, which draws on the approaches proposed in the productivity estimation literature (e.g., Wooldridge, 2009; Akerberg, Caves and Frazer, 2015), and the spatial econometrics literature (e.g., Kelejian and Prucha, 1998, 1999; Kapoor, Kelejian and Prucha, 2007; Lee and Yu, 2014; Elhorst, 2014). The resulting three-stage efficient GMM estimator has standard asymptotic properties, with variance-covariance estimators that take into account the spatial interactions across firms in each of the three dimensions proposed. The estimation procedure provides the estimates of the production function parameters (the labor and cap-

ital elasticities in value-added), the degree of autoregressive correlation in the productivity process, and the spatial parameters (characterizing the dependence of productivity on related firms' lagged outputs and lagged inputs respectively, and the strength of spatial error correlation of the productivity shocks). Firms' productivities are unobservable to researchers and are correlated to firms' characteristics. Our estimation strategy has both the flavor of control function approach (see, e.g., [Qu, Lee and Yang, 2021](#), and the productivity estimation literature cited above) and the flavor of correlated random effects method (see, e.g., [Li and Yang, 2021](#)).

We apply the developed methodology and estimation algorithm to the Japanese BSJBSA-TSR linked dataset for the period 2009–2018. The dataset combines the firm-level financial statement information from the Basic Survey of Japanese Business Structure and Activities (BSJBSA), and the firm-to-firm supplier-customer relationship from Tokyo Shoko Research (TSR). The estimation sample covers 14,178 firms per year (both publicly listed and unlisted firms in Japan of medium/large sizes, across 200 commuting zones and 18 industries), and in particular, provides information on each firm's most important domestic suppliers and customers (up to 24 connections, respectively). We find significant and positive spatial coefficients in the Japanese firm-level productivity process via all three proposed channels. In particular, a 1% increase in the average sales of a firm's suppliers/customers in the previous period in the same commuting zone helps improve the firm's current productivity by 0.005%. A larger local labor market also enhances a firm's productivity: specifically, a 1% increase in the average labor inputs in the previous period by firms located in the same commuting zone raises a firm's current productivity by 0.055%. There is also evidence of contemporary knowledge spillovers among firms located in the same commuting zone (with a positive and significant spatial error correlation coefficient of 0.68) and/or with supplier-customer relationships. In sum, the proposed estimator suggests that spatial interactions across firms play a significant role in determining the Japanese firm-level productivity both statistically and economically.

In addition to formulating the asymptotic properties of the proposed estimator, we conduct Monte Carlo simulations to evaluate the finite sample performance of the estimator. The Monte Carlo simulations demonstrate that the proposed estimator yields point estimates that are consistent for the true parameters both in the absence and in the presence of spatial effects. In other words, it returns statistically insignificant coefficient estimates of the spatial dependence parameters, when the underlying DGPs are free of such structures, and consistent estimates of the spatial dependence parameters when the underlying DGPs are characterized with such structures (via the lagged output, the lagged labor input, or the productivity shock channel). This finding holds for DGPs with varying strengths of spatial

dependence, and DGPs with positive or negative spatial dependence. The proposed estimation algorithm also generates standard error estimates of the parameters that are consistent with the Monte Carlo simulated standard deviations, and with a convergence rate (when the sample size changes) in line with the theory. In contrast, the conventional productivity estimators (which ignore potential spatial interactions across firms) lead to biased estimates of the production function parameters when the underlying DGPs exhibit spatial dependence structure (and thus, for which the conventional estimator is misspecified). The extents of the bias worsen when the underlying DGPs' spatial dependence strengthens. These findings imply that analyzing spatial interactions across firms based on the productivities estimated by the conventional estimators will lead to biased inferences. Instead, the proposed estimator in this paper offers a framework to simultaneously estimate firm production functions and spatial interactions across firms in one unified setup.

The rest of the paper is organized as follows. We set up the model in Section 2. In Section 3, we develop the estimation algorithms and formulate the asymptotic properties of the proposed estimator. Section 4 introduces the Japanese firm-level and firm-to-firm datasets. Section 5 conducts Monte Carlo simulations to evaluate the performance of the proposed estimator in comparison with the conventional estimator. In Section 6, we apply the proposed methodology empirically to the Japanese dataset, and Section 7 concludes. All figures referred to in the text are provided in the Online Appendix.

2 Model

Consider the following production function, where a firm's value-added depends on its primary factor inputs and productivity:

$$va_{it} = \alpha_0 + \alpha_l l_{it} + \alpha_k k_{it} + \omega_{it} + \xi_{it}, \quad (1)$$

where va_{it} and l_{it} denote the log of value-added and labor input of firm i in period t , k_{it} the log of capital stock of firm i at the beginning of period t , ω_{it} the log of productivity of firm i in period t , and ξ_{it} the value-added shock to firm i in period t . In the conventional setup, a firm's productivity ω_{it} is assumed to be dependent on its lagged productivity $\omega_{i,t-1}$ via an unknown function $f(\cdot)$ as in [Olley and Pakes \(1996\)](#), [Levinsohn and Petrin \(2003\)](#), [Akerberg, Caves and Frazer \(2015\)](#), and [Wooldridge \(2009\)](#). One might also consider a firm's current productivity to depend on the lagged characteristics $\mathbf{x}_{i,t-1}$ of the firm (such as its lagged exporting status, R&D expenditure, and changes in ownership) à la [De Loecker \(2013\)](#), [Doraszelski and Jaumandreu \(2013\)](#), and [Braguinsky, Ohyama, Okazaki and Syverson \(2015\)](#).

We generalize the conventional setup of the production function to allow spatial dependence across firms. In particular, a firm i 's current productivity is allowed to depend on the lagged output $y_{j,t-1}$ of its related firms j in the set $\mathcal{N}_{i,t-1}^y$, and the lagged inputs $\mathbf{\Omega}_{j,t-1} \equiv \{l_{j,t-1}, k_{j,t-1}, \mathbf{m}_{j,t-1}\}$ of a possibly different set $\mathcal{N}_{i,t-1}^\Omega$ of related firms, where $\mathbf{m}_{j,t-1}$ denotes the log of: $1 \times M$ vector of intermediate inputs of firm j in period $t - 1$:

$$\omega_{it} = f(\omega_{i,t-1}) + \lambda \sum_{j \in \mathcal{N}_{i,t-1}^y} \mathbf{w}_{ij,t-1}^y y_{j,t-1} + \sum_{j \in \mathcal{N}_{i,t-1}^\Omega} \mathbf{w}_{ij,t-1}^\Omega \mathbf{\Omega}_{j,t-1} \boldsymbol{\beta}_\Omega + \mathbf{x}_{i,t-1} \boldsymbol{\beta}_x + u_{it}, \quad (2)$$

where $y_{j,t-1}$ denotes the log of gross output of firm j in period $t - 1$. Furthermore, the innovation u_{it} to the productivity of firm i in period t is allowed to be spatially correlated with those of related firms in the set $\mathcal{N}_{it}^u$ contemporarily:

$$u_{it} = \mu \sum_{j \in \mathcal{N}_{it}^u} \mathbf{w}_{ij,t}^u u_{jt} + v_{it}, \quad i, j = 1, 2, 3, \dots, N \text{ and } t = 2, 3, \dots, T. \quad (3)$$

The weight assigned to each of the related firms in the set $\mathcal{N}_{i,t-1}^y$ is specified by $\mathbf{w}_{ij,t-1}^y$, and correspondingly those for firms in $\mathcal{N}_{i,t-1}^\Omega$ and $\mathcal{N}_{it}^u$ are specified by $\mathbf{w}_{ij,t-1}^\Omega$ and $\mathbf{w}_{ij,t}^u$, respectively. Note that the set of related firms that a firm's productivity depends upon can be defined by supplier-customer relationship, by ownership structure, by physical location, by industry of sales, or by combinations of them, and can differ across the three channels of spatial dependence, as the context of the study may deem appropriate.

As in the literature, it is assumed that firms observe their productivities before making production decisions, but the productivity ω_{it} is unknown to econometricians and is the target of estimation (along with the production function parameters). This makes the estimation and inference of the extended production function specified in (1)–(3) challenging. Note that the modelling and estimation of the productivity process ω_{it} in (1)–(3) will adopt the control function approach from the productivity estimation literature, and also incorporate elements of the correlated random effects method (e.g., [Li and Yang, 2021](#)), thus bypassing the need for inclusion of fixed effects.

2.1 Assumptions

We adopt a set of standard assumptions in the productivity estimation literature, and an additional set of assumptions to accommodate the setup with spatial dependence across firms as introduced in Equations (1)–(3).

Assumption 1. $E(\xi_{it} | l_{it}, k_{it}, \mathbf{m}_{it}) = 0$.

Assumption 2. $E(\xi_{it}|l_{jt}, k_{jt}, \mathbf{m}_{jt}, l_{j,t-1}, k_{j,t-1}, \mathbf{m}_{j,t-1}, \dots, l_{j1}, k_{j1}, \mathbf{m}_{j1}) = 0$.

Assumption 3. $E(u_{it}|k_{it}, l_{j,t-1}, k_{j,t-1}, \mathbf{m}_{j,t-1}, \mathbf{x}_{j,t-1}, \dots, l_{j1}, k_{j1}, \mathbf{m}_{j1}, \mathbf{x}_{j1}) = 0$.

Assumption 4. The residuals, ξ_{it} , are assumed to be i.i.d. across both i and t , and have finite fourth moments: $\xi_{it} \stackrel{iid}{\sim} (0, \sigma_\xi^2)$, $E|\xi_{it}^{4+\eta}| < \infty$, for some $\eta > 0$.

Assumption 5. The productivity innovations, u_{it} , are spatially correlated as specified in Equation (3), with residual v_{it} assumed to be i.i.d. across both i and t , and have finite fourth moments: $v_{it} \stackrel{iid}{\sim} (0, \sigma_v^2)$, $E|v_{it}^{4+\eta}| < \infty$, for some $\eta > 0$.

Assumption 6. The residuals, ξ_{it} and v_{it} , are uncorrelated.

Assumption 7. $(\mathbf{I}_N - \mu \mathbf{W}_t^u)$ are non-singular for $t = 1, 2, \dots, T$, with $\mu \in (\max_t \frac{1}{\lambda_{\min,t}}, \min_t \frac{1}{\lambda_{\max,t}})$, where $\mathbf{I}_N$ is the identity matrix of size N , $\mathbf{W}_t^u \equiv \{w_{ij,t}^u\}$, and $\lambda_{\min,t}$ and $\lambda_{\max,t}$ are the smallest and largest eigenvalues of $\mathbf{W}_t^u$.

Assumption 8. The row and column sums of the matrices, $\mathbf{W}_{t-1}^y$, $\mathbf{W}_{t-1}^\Omega$, $\mathbf{W}_t^u$ and $(\mathbf{I}_N - \mu \mathbf{W}_t^u)$ are uniformly bounded in absolute value for $t = 2, 3, \dots, T$, where $\mathbf{W}_{t-1}^y \equiv \{w_{ij,t-1}^y\}$ and $\mathbf{W}_{t-1}^\Omega \equiv \{w_{ij,t-1}^\Omega\}$. The elements of the three spatial weight matrices are at most of order $\bar{h}_n^{-1}$ such that $\bar{h}_n/N \rightarrow 0$ as $N \rightarrow \infty$.

Assumption 9. The regressor matrix $\{\Omega_t, y_{t-1}, \Omega_{t-1}, \mathbf{x}_{t-1}\}$ has a full column rank, and the elements are uniformly bounded for $t = 2, 3, \dots, T$.

Assumption 1 is the standard assumption made in the literature for firm-level productivity estimations. Assumption 2 requires that the residuals ξ_{it} in the value-added equation (1) are conditionally mean independent of the current and past input usages of the firm itself, and also those of the other firms. This is not as stringent an assumption as it might appear, because the productivity term ω_{it} in Equation (1) has absorbed potential spatial dependence across firms to the extent modelled by Equation (2). Note that $\mathbf{m}_{it}$ and $\mathbf{m}_{jt}$ appear in the conditioning set of the first two assumptions, although only $\mathbf{m}_{j,t-1}$ appears in the model, because the inference of ω_{it} will utilize the information of $\mathbf{m}_{it}$ (as will become clear in Section 3.2). Assumption 3 basically states that the innovation u_{it} to productivity is conditionally mean independent of the state variable (capital), as well as the past input choices and characteristics of the firm itself and the other firms. Together, the first three assumptions will help identify the set of moment conditions and instruments for estimating the parameters in Equations (1) and (2). Assumptions 4–6 are made to develop the variance-covariance estimator of the parameters. In particular, the finite fourth moment condition for v_{it} is required for the estimation of the spatial parameter μ in Equation (3). Assumptions

7–9 are adopted from [Kelejian and Prucha \(1999\)](#), [Kapoor, Kelejian and Prucha \(2007\)](#), and [Elhorst \(2014\)](#) to ensure that the spatial parameter estimates exist. Note that we will construct the connectivity matrices such that they are row-normalized (with zeros in the diagonal by construction) and satisfy Assumption 8.

3 Estimation Algorithms

In this section, we propose a three-stage estimation procedure based on Generalized Method of Moments (GMM) to obtain consistent estimates of the parameters in Equations (1)–(3).

3.1 Moment Conditions

Given Assumptions 1–5, the following conditions hold with respect to the error terms in Equation (1) and Equation (2):

$$E(\xi_t | l_t, k_t, \mathbf{m}_t, \boldsymbol{\Omega}_{t-1}) = \mathbf{0}, \quad (4)$$

$$E(\xi_t + u_t | k_t, \boldsymbol{\Omega}_{t-1}, \mathbf{W}_{t-1}^y y_{t-1}, \mathbf{W}_{t-1}^\Omega \boldsymbol{\Omega}_{t-1}, \mathbf{x}_{t-1}) = \mathbf{0}, \quad (5)$$

where $\xi_t \equiv (\xi_{1t}, \dots, \xi_{Nt})'$ and $u_t \equiv (u_{1t}, \dots, u_{Nt})'$ denote the $N \times 1$ vector of the residual terms from Equation (1) and Equation (2), respectively, across firms in period t ; $l_t \equiv (l_{1t}, \dots, l_{Nt})'$ denotes the $N \times 1$ vector of labor inputs across firms in period t ; k_t and $\mathbf{m}_t$ are similarly defined; $\boldsymbol{\Omega}_{t-1} \equiv [l_{t-1} \ k_{t-1} \ \mathbf{m}_{t-1}]$; $y_{t-1} \equiv (y_{1,t-1}, \dots, y_{N,t-1})'$ denotes the $N \times 1$ vector of gross outputs across firms in period $t-1$; $\mathbf{x}_{t-1}$ is similarly defined. The matrices $\mathbf{W}_{t-1}^y \equiv \{\mathbf{w}_{ij,t-1}^y\}$ and $\mathbf{W}_{t-1}^\Omega \equiv \{\mathbf{w}_{ij,t-1}^\Omega\}$ are $N \times N$ connectivity matrices in period $t-1$ that specify the dependence of firm i 's productivity in period t on related firms j 's lagged outputs and lagged inputs, respectively. Note that the conditional mean is defined element (firm) wise in each period t . This set of conditionally mean independent conditions will lead to the moment conditions specified below in Equation (16).

Furthermore, by [Kelejian and Prucha \(1999\)](#) and [Kapoor, Kelejian and Prucha \(2007\)](#), the following three moment conditions hold with respect to the error term in Equation (3):

$$E \begin{bmatrix} \frac{1}{N} v_t' v_t \\ \frac{1}{N} v_t' \mathbf{W}_t^{u'} \mathbf{W}_t^u v_t \\ \frac{1}{N} v_t' \mathbf{W}_t^u v_t \end{bmatrix} = \begin{bmatrix} \sigma_v^2 \\ \frac{\sigma_v^2}{N} \text{tr}(\mathbf{W}_t^{u'} \mathbf{W}_t^u) \\ 0 \end{bmatrix}. \quad (6)$$

where $v_t \equiv (v_{1t}, \dots, v_{Nt})'$ denotes the $N \times 1$ vector of the residual term from Equation (3) across firms in period $t = 2, 3, \dots, T$.

3.2 Estimation Strategy

3.2.1 Stage 1

This stage basically follows the productivity estimation literature (e.g., [Levinsohn and Petrin, 2003](#); [Wooldridge, 2009](#)). The productivity ω_{it} is assumed to be observable to the firm (or managers of the firm), but not to the econometrician. However, since a firm would in theory choose the optimal level of intermediate input $\mathbf{m}_{it}$ to maximize profits, given its initial capital stock k_{it} , labor force l_{it} , and realized productivity level ω_{it} , the econometrician could invert the relationship to infer a firm's productivity given its initial capital stock and observed input choices:

$$\omega_{it} = g(l_{it}, k_{it}, \mathbf{m}_{it}), \quad (7)$$

where $g(\cdot, \cdot, \cdot)$ is some unknown general function in the observed input levels. Equation (7), together with Equation (1), imply the following reduced-form value-added function:

$$\begin{aligned} va_t &= \alpha_0 \iota_N + \alpha_l l_t + \alpha_k k_t + \omega_t + \xi_t \\ &= \alpha_0 \iota_N + \alpha_l l_t + \alpha_k k_t + \mathbf{g}(l_t, k_t, \mathbf{m}_t) + \xi_t \\ &\equiv \mathbf{h}(l_t, k_t, \mathbf{m}_t) + \xi_t, \end{aligned} \quad (8)$$

where $va_t \equiv (va_{1t}, \dots, va_{Nt})'$ denotes the $N \times 1$ vector of value-added across firms in period t ; and ι_N is a $N \times 1$ vector of one's. The shorthand $\mathbf{g}(l_t, k_t, \mathbf{m}_t)$ is a $N \times 1$ column vector with $g(l_{it}, k_{it}, \mathbf{m}_{it})$ as its i -th entry; similarly, $\mathbf{h}(l_t, k_t, \mathbf{m}_t)$ is a $N \times 1$ column vector with $h(l_{it}, k_{it}, \mathbf{m}_{it})$ as its i -th entry, where $h(l_{it}, k_{it}, \mathbf{m}_{it}) \equiv \alpha_0 + \alpha_l l_{it} + \alpha_k k_{it} + g(l_{it}, k_{it}, \mathbf{m}_{it})$. As in [Akerberg, Caves and Frazer \(2015\)](#) and [Wooldridge \(2009\)](#), one could approximate $h(\cdot, \cdot, \cdot)$ in Equation (8) by a n -degree polynomial that contain at least l_{it} , k_{it} and $\mathbf{m}_{it}$. For example, in the case where $\mathbf{m}_{it}$ contains only one type of intermediate input and is hence a scalar, $h(l_{it}, k_{it}, m_{it})$ can be approximated by $\sum_{p,q,r} (l_{it}^p k_{it}^q m_{it}^r) \delta_{p,q,r}$, with nonnegative integers p , q and r such that $p + q + r \leq n$. That is:

$$h(l_{it}, k_{it}, \mathbf{m}_{it}) = \alpha_0 + c(l_{it}, k_{it}, \mathbf{m}_{it}) \boldsymbol{\delta}, \quad (9)$$

where $c(l_{it}, k_{it}, \mathbf{m}_{it})$ is a $1 \times Q$ vector of functions in $(l_{it}, k_{it}, \mathbf{m}_{it})$ and $\boldsymbol{\delta}$ a $Q \times 1$ vector of parameters. For example, for a 2nd-order polynomial h function ($n = 2$), $c(l_{it}, k_{it}, \mathbf{m}_{it}) = [l_{it}, k_{it}, m_{it}, l_{it}^2, l_{it}k_{it}, l_{it}m_{it}, k_{it}^2, k_{it}m_{it}, m_{it}^2]$.

Given the condition (4), Equation (8) given Equation (9) can be estimated using the

following set of instrument variables (IVs) for period t :

$$\mathcal{Z}_{t,I} = (\iota_N, \mathbf{c}_t, \mathbf{c}_{t-1}), \quad (10)$$

where the shorthand $\mathbf{c}_t$ is a $N \times Q$ matrix with $c(l_{it}, k_{it}, \mathbf{m}_{it})$ as its i -th row entry. Note that since $g(\cdot, \cdot, \cdot)$ is allowed to be a general function (including linearity in the arguments as a special case), the slope coefficients (α_l, α_k) on the inputs are not identified from Equation (8), as highlighted by [Akerberg, Caves and Frazer \(2015\)](#). However, it enables an estimate $\hat{h}(l_{it}, k_{it}, \mathbf{m}_{it})$ of $h(l_{it}, k_{it}, \mathbf{m}_{it})$. In turn, the slope coefficients of the production function can be identified in a later stage, along with the other parameters, as laid out in the next section.

The set of IVs listed in (10)—and in the condition (4)—includes the input variables only up to one lag, and hence corresponds to weaker conditions than stated in Assumption 2. One could potentially enlarge the set and include longer lags of the input variables in the conditioning set, given Assumption 2.

3.2.2 Stage 2

Next, given the productivity process's dynamic and spatial dependence structure specified in Equation (2), and Equation (7), we can also write the value-added function in the following alternative reduced form:

$$\begin{aligned} va_t &= \alpha_0 \iota_N + \alpha_l l_t + \alpha_k k_t + \omega_t + \xi_t \\ &= \alpha_0 \iota_N + \alpha_l l_t + \alpha_k k_t + \mathbf{f}[\mathbf{g}(l_{t-1}, k_{t-1}, \mathbf{m}_{t-1})] \\ &\quad + \lambda \mathbf{W}_{t-1}^y y_{t-1} + \mathbf{W}_{t-1}^\Omega \Omega_{t-1} \beta_\Omega + \mathbf{x}_{t-1} \beta_x + u_t + \xi_t, \end{aligned} \quad (11)$$

where the shorthand $\mathbf{f}[\mathbf{g}(l_{t-1}, k_{t-1}, \mathbf{m}_{t-1})]$ is a $N \times 1$ column vector with $f[g(l_{i,t-1}, k_{i,t-1}, \mathbf{m}_{i,t-1})]$ as its i -th entry. Recall that the matrices $\mathbf{W}_{t-1}^y \equiv \{\mathbf{w}_{ij,t-1}^y\}$ and $\mathbf{W}_{t-1}^\Omega \equiv \{\mathbf{w}_{ij,t-1}^\Omega\}$ are $N \times N$ connectivity matrices in period $t-1$ that specify the dependence of firm i 's productivity in period t on related firms j 's lagged outputs and lagged inputs, respectively. In deriving Equation (11), we have used Equation (2) to replace ω_t and Equation (7) to replace $\omega_{i,t-1}$ in the $f(\cdot)$ function such that $f(\omega_{i,t-1}) = f[g(l_{i,t-1}, k_{i,t-1}, \mathbf{m}_{i,t-1})]$. As suggested by [Wooldridge \(2009\)](#), one could use a G -th degree polynomial to approximate $f(\cdot)$ such that:

$$f(\nu) = \rho_1 \nu + \rho_2 \nu^2 + \dots + \rho_G \nu^G. \quad (12)$$

Given the condition in (5), Equation (11) can be estimated using the following set of IVs

for period t :

$$\mathcal{Z}_{t,II} = (\iota_N, k_t, \mathbf{c}_{t-1}, \mathbf{W}_{t-1}^y y_{t-1}, \mathbf{W}_{t-1}^\Omega \Omega_{t-1}, \mathbf{x}_{t-1}), \quad (13)$$

with one lag (or a longer past history) of the variables. Additional spatio-temporal lags of explanatory variables, such as $(\mathbf{W}_{t-1}^y)^2 y_{t-1}$, $(\mathbf{W}_{t-1}^y)^3 y_{t-1}$, $(\mathbf{W}_{t-1}^\Omega)^2 \Omega_{t-1}$ and $(\mathbf{W}_{t-1}^\Omega)^3 \Omega_{t-1}$, may also be added to the set of IVs to help identify the spatial coefficients.

While [Akerberg, Caves and Frazer \(2015\)](#) propose to estimate Equations (8) and (11)—without the spatial structure—sequentially, by plugging in estimates from Equation (8) into Equation (11), we adopt the approach proposed by [Wooldridge \(2009\)](#) and estimate them jointly, as it leads to more efficient estimators. In particular, denote the parameters of the system by $\boldsymbol{\theta} = (\alpha_0, \boldsymbol{\delta}', \alpha_l, \alpha_k, \lambda, \boldsymbol{\beta}'_\Omega, \boldsymbol{\beta}'_x, \rho_1, \dots, \rho_G)'$. The residuals from Equations (8) and (11) given the parameters are, respectively:

$$r_{t,I}(\boldsymbol{\theta}) = va_t - \alpha_0 \iota_N - \mathbf{c}_t \boldsymbol{\delta}, \quad (14)$$

$$\begin{aligned} r_{t,II}(\boldsymbol{\theta}) = & va_t - \alpha_0 \iota_N - \alpha_l l_t - \alpha_k k_t - \mathbf{f}[\mathbf{c}_{t-1} \boldsymbol{\delta} - \alpha_l l_{t-1} - \alpha_k k_{t-1}] \\ & - \lambda \mathbf{W}_{t-1}^y y_{t-1} - \mathbf{W}_{t-1}^\Omega \Omega_{t-1} \boldsymbol{\beta}_\Omega - \mathbf{x}_{t-1} \boldsymbol{\beta}_x, \end{aligned} \quad (15)$$

where recall that $g(l_{i,t-1}, k_{i,t-1}, \mathbf{m}_{i,t-1}) = h(l_{i,t-1}, k_{i,t-1}, \mathbf{m}_{i,t-1}) - \alpha_0 - \alpha_l l_{i,t-1} - \alpha_k k_{i,t-1} = c(l_{i,t-1}, k_{i,t-1}, \mathbf{m}_{i,t-1}) \boldsymbol{\delta} - \alpha_l l_{i,t-1} - \alpha_k k_{i,t-1}$, given Equations (8) and (9). The conditions in (4) and (5) imply that:

$$E[\mathcal{Z}'_{it} r_{it}(\boldsymbol{\theta})] \equiv E \left[\begin{pmatrix} \mathcal{Z}'_{it,I} & \mathbf{0} \\ \mathbf{0} & \mathcal{Z}'_{it,II} \end{pmatrix} \begin{pmatrix} r_{it,I}(\boldsymbol{\theta}) \\ r_{it,II}(\boldsymbol{\theta}) \end{pmatrix} \right] = \mathbf{0}, \quad (16)$$

where $\mathcal{Z}_{it,I}$, $\mathcal{Z}_{it,II}$, $r_{it,I}(\boldsymbol{\theta})$, and $r_{it,II}(\boldsymbol{\theta})$ are the i -th row entry of $\mathcal{Z}_{t,I}$, $\mathcal{Z}_{t,II}$, $r_{t,I}(\boldsymbol{\theta})$, and $r_{t,II}(\boldsymbol{\theta})$, respectively. Given Equation (16), we proceed with GMM estimation of $\boldsymbol{\theta}$.

3.2.3 Stage 3

We estimate the spatial error structure in Equation (3) based on the GMM approach of [Kelejian and Prucha \(1999\)](#) and [Kapoor, Kelejian and Prucha \(2007\)](#). Specifically, given the parameter estimates $\hat{\boldsymbol{\theta}}$ from the previous stages, we impute estimates of the productivity innovation term, $\hat{u}_t$, by taking the difference between (15) and (14), since the residuals from the second stage is $\widehat{\xi_t + u_t}$ and the residuals from the first stage is $\hat{\xi}_t$:

$$\begin{aligned} \hat{u}_t \equiv & \mathbf{c}_t \hat{\boldsymbol{\delta}} - \hat{\alpha}_l l_t - \hat{\alpha}_k k_t \\ & - \mathbf{f}[\mathbf{c}_{t-1} \hat{\boldsymbol{\delta}} - \hat{\alpha}_l l_{t-1} - \hat{\alpha}_k k_{t-1}] \\ & - \hat{\lambda} \mathbf{W}_{t-1}^y y_{t-1} - \mathbf{W}_{t-1}^\Omega \Omega_{t-1} \hat{\boldsymbol{\beta}}_\Omega - \mathbf{x}_{t-1} \hat{\boldsymbol{\beta}}_x, \end{aligned} \quad (17)$$

and use the moment conditions implied by Equation (6) to estimate μ and σ_v^2 jointly by GMM. Note that if we define $\bar{u}_t \equiv \mathbf{W}_t^u u_t$, $\bar{v}_t \equiv \mathbf{W}_t^u v_t$, and $\bar{\bar{u}}_t = (\mathbf{W}_t^u)^2 u_t$, it follows that $v_t = u_t - \mu \bar{u}_t$ and $\bar{v}_t = \bar{u}_t - \mu \bar{\bar{u}}_t$. By replacing v_t in the moment condition (6) with $u_t - \mu \bar{u}_t$ rewrites the three moment conditions in terms of u_t and $\mu \bar{u}_t$. We can follow similar steps as in Kelejian and Prucha (1999) and Kapoor, Kelejian and Prucha (2007) to derive Equation (18) below:

$$\frac{1}{T-1} \sum_{t=2}^T \left(\gamma_t - \Gamma_t \begin{pmatrix} \mu \\ \mu^2 \\ \sigma_v^2 \end{pmatrix} \right) = \mathbf{0}, \quad (18)$$

where

$$\gamma_t = \frac{1}{N} \begin{pmatrix} E(u_t' u_t) \\ E(\bar{u}_t' \bar{u}_t) \\ E(u_t' \bar{u}_t) \end{pmatrix}, \quad (19)$$

$$\Gamma_t = \frac{1}{N} \begin{pmatrix} 2E(u_t' \bar{u}_t) & -E(\bar{u}_t' \bar{u}_t) & N \\ 2E(\bar{\bar{u}}_t' \bar{u}_t) & -E(\bar{\bar{u}}_t' \bar{\bar{u}}_t) & \text{tr}(\mathbf{W}_t^u \mathbf{W}_t^u) \\ E(u_t' \bar{\bar{u}}_t + \bar{u}_t' \bar{u}_t) & -E(\bar{u}_t' \bar{\bar{u}}_t) & 0 \end{pmatrix}. \quad (20)$$

Use the estimates of the productivity innovation term from Equation (17), $\hat{u}_t$, to construct the sample counterparts of the γ_t vector and the Γ_t matrix:¹

$$\mathbf{s}_t \equiv \frac{1}{N} \begin{pmatrix} \hat{u}_t' \hat{u}_t \\ \hat{u}_t' \mathbf{W}_t^u \mathbf{W}_t^u \hat{u}_t \\ \hat{u}_t' \mathbf{W}_t^u \hat{u}_t \end{pmatrix}, \quad (21)$$

$$\mathbf{F}_t \equiv \frac{1}{N} \begin{pmatrix} 2\hat{u}_t' \mathbf{W}_t^u \hat{u}_t & -\hat{u}_t' \mathbf{W}_t^u \mathbf{W}_t^u \hat{u}_t & N \\ 2\hat{u}_t' \mathbf{W}_t^u \mathbf{W}_t^u \mathbf{W}_t^u \hat{u}_t & -\hat{u}_t' \mathbf{W}_t^u \mathbf{W}_t^u \mathbf{W}_t^u \mathbf{W}_t^u \hat{u}_t & \text{tr}(\mathbf{W}_t^u \mathbf{W}_t^u) \\ \hat{u}_t' \mathbf{W}_t^u \mathbf{W}_t^u \hat{u}_t + \hat{u}_t' \mathbf{W}_t^u \mathbf{W}_t^u \hat{u}_t & -\hat{u}_t' \mathbf{W}_t^u \mathbf{W}_t^u \mathbf{W}_t^u \hat{u}_t & 0 \end{pmatrix}, \quad (22)$$

and form the sample counterpart of the condition in Equation (18):

$$\frac{1}{T-1} \sum_{t=2}^T \left(\mathbf{s}_t - \mathbf{F}_t \begin{pmatrix} \mu \\ \mu^2 \\ \sigma_v^2 \end{pmatrix} \right) \equiv \frac{1}{T-1} \sum_{t=2}^T \epsilon_t, \quad (23)$$

¹The derivations are provided in Section A.1 of the Theoretical Appendix.

where ϵ_t is a 3×1 vector of residuals. We can then estimate μ and σ_v^2 by the transformed moment condition $E(\epsilon_t) = \mathbf{0}$. Specifically,

$$\epsilon_t(\mu, \sigma_v^2) = \frac{1}{N} \begin{pmatrix} \hat{u}_t'(\mathbf{I}_N - 2\mu\mathbf{W}_t^u + \mu^2\mathbf{W}_t^{u'}\mathbf{W}_t^u)\hat{u}_t - \sigma_v^2 N \\ \hat{u}_t'\mathbf{W}_t^{u'}(\mathbf{I}_N - 2\mu\mathbf{W}_t^{u'} + \mu^2\mathbf{W}_t^{u'}\mathbf{W}_t^u)\mathbf{W}_t^u\hat{u}_t - \sigma_v^2 \text{tr}(\mathbf{W}_t^{u'}\mathbf{W}_t^u) \\ \hat{u}_t'(\mathbf{I}_N - \mu(\mathbf{W}_t^u + \mathbf{W}_t^{u'}) + \mu^2\mathbf{W}_t^{u'}\mathbf{W}_t^u)\mathbf{W}_t^u\hat{u}_t \end{pmatrix}. \quad (24)$$

The algorithm above provides a set of estimates consistent for $\boldsymbol{\theta}$, μ and σ_v^2 . We can improve the efficiency of the estimators by deriving the weighting matrix for the GMM estimator, and repeat the procedure until the parameter estimates converge. Section 3.3 characterizes the algorithm to obtain the efficient GMM estimator.

3.3 Efficient GMM Estimator

This section itemizes the steps to implement the proposed estimation strategy and obtain the efficient GMM estimator of $\boldsymbol{\theta}$ and $\psi \equiv \{\mu, \sigma_v^2\}$.

1. Minimize the objective function: $\left[\frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \mathcal{Z}_{it}' r_{it}(\boldsymbol{\theta}) \right]' \mathbf{W}_{\boldsymbol{\theta}} \left[\frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \mathcal{Z}_{it}' r_{it}(\boldsymbol{\theta}) \right]$ with respect to $\boldsymbol{\theta}$ by setting $\mathbf{W}_{\boldsymbol{\theta}} = \mathbf{I}_{\mathcal{M}}$ to obtain the one-step estimator $\hat{\boldsymbol{\theta}}$ of $\boldsymbol{\theta}$, where $\mathbf{W}_{\boldsymbol{\theta}}$ of dimension $\mathcal{M} \times \mathcal{M}$ refers to the weighting matrix for the moment conditions used in the estimation of $\boldsymbol{\theta}$, and $\mathcal{M}$ is the combined number of moment conditions (instruments) from Stage 1 and Stage 2.
2. Given the one-step estimate of $\boldsymbol{\theta}$, obtain the residuals $\{\hat{u}_t\}_{t=2}^T$ by Equation (17). This in turn can be used to obtain an estimator of μ and σ_v^2 based on Equations (23) and (24):

$$\arg \min_{\mu, \sigma_v^2} \frac{1}{T-1} \sum_{t=2}^T \left(\boldsymbol{\varsigma}_t - \mathbf{F}_t \begin{pmatrix} \mu \\ \mu^2 \\ \sigma^2 \end{pmatrix} \right)' \mathbf{W}_{\psi} \frac{1}{T-1} \sum_{t=2}^T \left(\boldsymbol{\varsigma}_t - \mathbf{F}_t \begin{pmatrix} \mu \\ \mu^2 \\ \sigma^2 \end{pmatrix} \right), \quad (25)$$

by setting $\mathbf{W}_{\psi} = \mathbf{I}_3$ to obtain the one-step estimator of μ and σ_v^2 , where $\mathbf{W}_{\psi}$ of dimension 3×3 refers to the weighting matrix for the moment conditions used in the estimation of ψ , and there are three moment conditions in this case.

3. Derive a variance-covariance estimator $\hat{\mathbf{V}}_{\boldsymbol{\theta}}$ of the moment conditions used in the esti-

mation of $\boldsymbol{\theta}$, $\mathbf{V}_{\boldsymbol{\theta}} = \text{Var} \left(\frac{1}{\sqrt{N(T-1)}} \sum_{i=1}^N \sum_{t=2}^T \mathbf{Z}'_{it} r_{it} \right)$, noting that:²

$$\begin{aligned} \mathbf{V}_{\boldsymbol{\theta}} &= \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \sum_{j=1}^N \sum_{s=2}^T E[(\mathbf{Z}'_{it} r_{it})(\mathbf{Z}'_{js} r_{js})'] \\ &= \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \begin{pmatrix} \mathbf{Z}'_{it,I} \mathbf{Z}_{it,I} E(\xi_{it}^2) & \mathbf{Z}'_{it,I} \mathbf{Z}_{it,II} E(\xi_{it}^2) \\ \mathbf{Z}'_{it,II} \mathbf{Z}_{it,I} E(\xi_{it}^2) & \mathbf{Z}'_{it,II} \mathbf{Z}_{it,II} E(\xi_{it}^2) \end{pmatrix} \\ &\quad + \frac{1}{N(T-1)} \sum_{t=2}^T \mathbf{Z}'_t \begin{pmatrix} \mathbf{0}_{N \times N} & \mathbf{0}_{N \times N} \\ \mathbf{0}_{N \times N} & \sigma_v^2 [(\mathbf{I}_N - \mu \mathbf{W}_t^u)^{-1} (\mathbf{I}_N - \mu \mathbf{W}_t^u)^{-1'}] \end{pmatrix} \mathbf{Z}_t, \end{aligned} \quad (26)$$

by replacing the residuals ξ_t with the sample counterpart $\hat{\xi}_t$, and the parameters (μ and σ_v^2) with their estimates obtained from Steps 1–2 above.

4. Repeat Step 1, but update the weighting matrix by $\mathbf{W}_{\boldsymbol{\theta}} = \hat{\mathbf{V}}_{\boldsymbol{\theta}}^{-1}$.
5. To obtain an estimate of the variance-covariance matrix for the moment conditions used in the estimation of ψ , we extend the framework of [Kapoor, Kelejian and Prucha \(2007\)](#) to allow for non-normal errors and time-varying connectivity matrices.³ The variance-covariance matrix for the sample counterpart of the left side of the moment conditions in Equation (6) is given by:

$$\mathbf{V}_{\psi} = \begin{pmatrix} V_{\psi,11} & V_{\psi,12} & 0 \\ V_{\psi,21} & V_{\psi,22} & V_{\psi,23} \\ 0 & V_{\psi,32} & V_{\psi,33} \end{pmatrix}, \quad (27)$$

where $V_{\psi,11} = \sigma_v^4(\kappa_v + 2)$; $V_{\psi,12} = \frac{\sigma_v^4}{N(T-1)}(\kappa_v + 2)\text{tr}(\mathbf{W}^{u'} \mathbf{W}^u)$; $V_{\psi,21} = V_{\psi,12}$; $V_{\psi,22} = \frac{\sigma_v^4}{N(T-1)}[\kappa_v \text{diagv}(\mathbf{W}^{u'} \mathbf{W}^u)' \text{diagv}(\mathbf{W}^{u'} \mathbf{W}^u) + \text{tr}(\mathbf{W}^{u'} \mathbf{W}^u (\mathbf{W}^{u'} \mathbf{W}^u + \mathbf{W}^u \mathbf{W}^{u'}))]$; $V_{\psi,23} = \frac{\sigma_v^4}{N(T-1)}\text{tr}((\mathbf{W}^{u'} \mathbf{W}^u)(\mathbf{W}^u + \mathbf{W}^{u'}))$; $V_{\psi,32} = V_{\psi,23}$; and $V_{\psi,33} = \frac{\sigma_v^4}{N(T-1)}\text{tr}(\mathbf{W}^u (\mathbf{W}^u + \mathbf{W}^{u'}))$. κ_v is the excess kurtosis of v_{it} , and $\mathbf{W}^u$ is a $N(T-1) \times N(T-1)$ block-diagonal matrix with $\mathbf{W}_2^u, \mathbf{W}_3^u, \dots, \mathbf{W}_T^u$ on the diagonal; the operator ‘diagv’ takes the diagonal elements of a matrix and converts them to a column vector.

The excess kurtosis κ_v can be estimated using the following formula given the estimates of μ and σ_v^2 :

$$\hat{\kappa}_v = \frac{\sum_{i=1}^N \sum_{t=2}^T (\hat{v}_{it} - \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \hat{v}_{it})^4}{N(T-1) \hat{\sigma}_v^4} - 3, \quad (28)$$

²The derivations of $\mathbf{V}_{\boldsymbol{\theta}}$ are provided in Section A.2 of the Theoretical Appendix.

³The derivations of $\mathbf{V}_{\psi}$ are provided in Section A.3 of the Theoretical Appendix.

where $\hat{v}_{it}$ is the i -th element of $\hat{v}_t = (\mathbf{I}_N - \hat{\mu}\mathbf{W}_t^u)\hat{u}_t$ and $\hat{\sigma}_v^4 = (\hat{\sigma}_v^2)^2$.

6. Repeat Step 2, this time setting $\mathbf{W}_\psi = \hat{\mathbf{V}}_\psi^{-1}$.
7. Repeat Steps 3–6 until convergence in the estimates: $\hat{\boldsymbol{\theta}}$, $\hat{\mu}$, $\hat{\sigma}_v^2$, $\hat{\mathbf{V}}_\theta$, and $\hat{\mathbf{V}}_\psi$.
8. Obtain a variance-covariance matrix estimator of the parameters $\boldsymbol{\theta}$ based on the asymptotic property established for the efficient GMM estimator (Lee and Yu, 2014):

$$\sqrt{N(T-1)}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \sim \mathcal{N}(0, \text{plim}_{N,T \rightarrow \infty} \boldsymbol{\Sigma}_\theta), \quad (29)$$

where $\boldsymbol{\Sigma}_\theta = (\mathbf{H}_\theta' \mathbf{V}_\theta^{-1} \mathbf{H}_\theta)^{-1}$ and $\mathbf{H}_\theta = E \left[\frac{d}{d\boldsymbol{\theta}'} \mathbf{Z}_{it}' r_{it}(\boldsymbol{\theta}) \right]$. The latter is estimated by its sample counterpart:

$$\hat{\mathbf{H}}_\theta = \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \left[\frac{d}{d\boldsymbol{\theta}'} \mathbf{Z}_{it}' r_{it}(\boldsymbol{\theta}) \right]_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}.$$

It is easy to show that $\hat{\boldsymbol{\Sigma}}_\theta - \boldsymbol{\Sigma}_\theta \xrightarrow{p} \mathbf{0}$, where $\hat{\boldsymbol{\Sigma}}_\theta \equiv (\hat{\mathbf{H}}_\theta' \hat{\mathbf{V}}_\theta^{-1} \hat{\mathbf{H}}_\theta)^{-1}$.

9. Similarly, the variance-covariance matrix $\boldsymbol{\Sigma}_\psi$ (*in the original scale*) for the parameters ψ can be estimated by:

$$\hat{\boldsymbol{\Sigma}}_\psi = \frac{1}{N(T-1)} (\hat{\mathcal{G}}_\psi' \hat{\mathbf{V}}_\psi^{-1} \hat{\mathcal{G}}_\psi)^{-1}, \quad (30)$$

where

$$\begin{aligned} \hat{\mathcal{G}}_\psi &= \frac{1}{(T-1)} \sum_{t=2}^T \frac{d\epsilon_t(\hat{\psi})}{d\psi'} \\ &= \frac{1}{N(T-1)} \sum_{t=2}^T \begin{pmatrix} 2\hat{u}_t'(\hat{\mu}\mathbf{W}_t^{u'} - \mathbf{I}_N)\mathbf{W}_t^u \hat{u}_t & -N \\ 2\hat{u}_t' \mathbf{W}_t^{u'} (\hat{\mu}\mathbf{W}_t^{u'} \mathbf{W}_t^u - \mathbf{W}_t^{u'}) \mathbf{W}_t^u \hat{u}_t & -\text{tr}(\mathbf{W}_t^{u'} \mathbf{W}_t^u) \\ \hat{u}_t' (2\hat{\mu}\mathbf{W}_t^{u'} \mathbf{W}_t^u - (\mathbf{W}_t^u + \mathbf{W}_t^{u'})) \mathbf{W}_t^u \hat{u}_t & 0 \end{pmatrix}. \end{aligned} \quad (31)$$

4 Data

Our dataset is constructed by combining two Japanese datasets. The first dataset is the Basic Survey of Japanese Business Structure and Activities (BSJBSA), provided by the Ministry of Economy, Trade and Industry (METI), Japan. The data include a firm-level annual survey of detailed business information, such as sales, employment, capital stock, industry classification (Japan Standard Industry Classification, JSIC) and intermediate purchases. The data cover both manufacturing and non-manufacturing firms that have: (1) more than 50

employees, and (2) capital stocks of more than 30 million yens (approximately 250 thousand USD in 2015).

The second dataset contains information on firm-to-firm relationship provided by Tokyo Shoko Research (TSR), a major credit reporting company in Japan. It provides a firm’s most important domestic suppliers and customers (up to 24 connections in each direction) and covers both publicly listed and unlisted firms in Japan of all sizes and industries. Because these two datasets do not use the same firm identification codes, we match them on the basis of firm name, address, phone number, and postal code. Using the BSJBSA as the denominator (since it provides the required firm-level variables for productivity estimations), the percentage of firms in BSJBSA that are matched with its counterpart in TSR is very high, typically at 93%–94%, across years during the sample period 2009–2018. [Table 1](#) provides the detailed firm counts.

We supplement the BSJBSA-TSR linked dataset with the JIP database 2021 provided by the Research Institute of Economy, Trade and Industry (RIETI), and with the information on commuting zones (CZs) constructed by [Adachi, Fukai, Kawaguchi and Saito \(2021\)](#). We impute the industry-level price deflators based on the JIP database. In particular, it contains the nominal and real values of outputs, intermediate inputs, investment, value added, and R&D expenditure for the 100 industries classified by the JIP database. We construct the deflators by the ratios of the nominal and real values for each of these variables, and merge them with the BSJBSA data (based on concordance between the BSJBSA JSIC industries and the JIP industries, provided in the JIP database). There are in total 433 JSIC 3-digit industries. A JIP industry is matched on average with 4.8 JSIC 3-digit industries. These deflators are then used to convert the BSJBSA corresponding variables into real terms. We also impute the average work hours per person in a year in an industry based on the JIP database and merge the variable with the BSJBSA data (using again the concordance between the JIP and JSIC industries).

The information on CZs is used as one criterion below in defining connectivity matrices across firms. [Adachi, Fukai, Kawaguchi and Saito \(2021\)](#) construct the CZ information for Japan using the hierarchical agglomerative clustering (HAC) method of [Tolbert and Killian \(1987\)](#) and [Tolbert and Sizer \(1996\)](#) for constructing the US CZs. By [Adachi, Fukai, Kawaguchi and Saito \(2021\)](#), there are 267 CZs in 2010 and 265 in 2015 in Japan. We use the 2010 CZ definition, and merge the CZ information with the BSJBSA observations based on the prefecture and city names of a firm’s address, and examine/adjust manually if necessary.

The variables at the firm-year level used for the production function estimation is constructed in the following manners. The number of workers is measured as the sum of regular workers and part-time workers (excluding temporary workers) in headquarter, head office,

branch office, and assignee company (available from the BSJBSA). The labor hour is constructed as the number of workers of a firm in a year (from the BSJBSA), times the average work hours per person in the corresponding year and industry (from the JIP database). The real physical capital stock is constructed by the perpetual inventory method with 2007 as the base year, using the real physical capital value in 2007 (year end) and the real investment in physical capital in each year from the BSJBSA, together with the depreciation rates at the industry level from the JIP database. The real intermediate inputs are constructed by the sum of the cost of goods sold, and general and administrative expenses, minus wages, rental costs, depreciation, and taxes reported in the BSJBSA, deflated by the input deflator (constructed using the JIP database as documented above). The real revenue is measured by sales, deflated by the output deflator imputed from the JIP database. The real value added is constructed by nominal value-added (i.e., nominal sales minus nominal intermediate inputs) deflated by the value-added deflator from the JIP database. The nominal R&D expenditure is deflated by the R&D deflator (constructed using the JIP database). The export status of a firm is directly available from the BSJBSA.

4.1 Summary Statistics

Table 2(a) provides the summary statistics of the key variables for the BSJBSA-TSR linked sample in year 2015 (based on the denominator of BSJBSA firms, not all of which have corresponding entries in TSR). The effective number of observations differs from Table 1 due to potentially missing observations on the variable of interest.

A few remarks are in order. First, the average firms tend to be large (e.g., having 490 workers, and 7.2 billion JPY physical capital, roughly equivalent to 60 million USD). This is due to the fact that the BSJBSA only covers medium and large firms. Second, the average firms report 6.8 customers and 6.7 suppliers, suggesting that the TSR’s limit of reporting the top customers and suppliers up to 24 connections in each direction is not practically binding for most of the firms.

Figures 1(a)–3(a) illustrate the number of firms, their average size in terms of employment, and their average number of customers and suppliers for each 1-digit JSIC rev12 industry. Figure 1(a) indicates that most of the firms in the sample are in the manufacturing, wholesale & retail trade, and information & communications industries. Among them, those in the service industries tend to be large in terms of employment (e.g., accommodation and food & beverage services, electricity, gas, heat supply & water, and finance & insurance; see Figure 2(a)). In terms of connectedness with other firms, those in the construction, manufacturing, and mining & quarrying industries tend to have a larger number of customers

and suppliers, while those in the service industries tend to have a smaller number of business customers but just as many suppliers as in other industries (Figure 3(a)).

Figures 4(a)–6(a) show a large heterogeneity across prefectures in terms of the number of firms, their average size, and their average number of customers and suppliers. Most of the firms in the sample are located in economically large prefectures, such as Tokyo, Osaka, Aichi, Kanagawa, and Hyogo (Figure 4(a)). Firms in these large prefectures tend also to be large in terms of employment (Figure 5(a)) and connected with a larger number of both customers and suppliers (Figure 6(a)).

The potential spatial dependence across firms through the supply chain and the input markets can be more local than the prefecture level. Bernard, Moxnes and Saito (2019) show that the median distance of any customer-supplier pair in the TSR data is 30 km and thus smaller than the typical size of prefectures. Thus, the following figures further provide the characterization at the commuting zone level. Figure 7(a) shows the number of CZs within each prefecture in 2015. Prefectures with large areas (e.g., Hokkaido and Nagano) tend to have many CZs, while those with small areas and economic sizes (e.g., Kagawa and Fukui) tend to have few CZs. Figures 8(a)–10(a) show the counterparts of Figures 4(a)–6(a) at the commuting-zone level. The commuting zone with the largest number of firms (8993) is CZ89 that covers the busiest areas around Tokyo (parts of Tokyo, Kanagawa, Chiba, and Saitama). Economically large CZs also tend to have larger firms and more connected firms. For example, the same CZ89 ranks top 5th in terms of average firm’s employment size (643.99), and top 24th in terms of average firm’s customer connections (7.44). Note also that the average firm size is much more dispersed at the right tail when we zoom in at the commuting-zone level (Figure 9(a)) compared to that at the prefecture level (Figure 5(a)). Similarly, the distributions of customer/supplier connections are much more dispersed at the commuting-zone level (Figure 10(a)) than at the prefecture level (Figure 6(a)), suggesting a large degree of heterogeneity across CZs within prefectures.

4.2 Definition of Connectivity Matrices

To model the spatial-temporal lag dependence in outputs, we define the output connectivity matrix $\mathbf{W}_{t-1}^y$ based on the set of a firm’s customers and/or suppliers located in the same commuting zone. The ij -th element of $\mathbf{W}_{t-1}^y$ takes on the value one if both firms i and j are located in the same commuting zone, and in addition, firm j is a customer or supplier of firm i , in period $t - 1$. This is in line with the research conducted by Demir, Fieler, Xu and Yang (2024), Alfaro-Ureña, Manelici and Vasquez (2022) and Ellison, Glaeser and Kerr (2010). They find that the input-output linkages across firms and the geographical proximity

of firms play an important role in productivity spillovers.

Next, for spatial dependence across firms through the input markets, we restrict our focus to the labor market channel, and define input connectivity matrix $\mathbf{W}_{t-1}^\Omega$ such that the ij -th element of $\mathbf{W}_{t-1}^\Omega$ takes on the value one if firms i and j are located in the same commuting zone in period $t - 1$. Firms located in the same commuting zone are more likely to tap into the same labor pool, considering the potential labor mobility frictions across zones. Multiple theories have been proposed about the benefits associated with a large labor pool. When firms in the same location employ more workers, the potential pool of labor in the location increases. This facilitates better worker-firm matches (e.g., [Helsley and Strange, 1990](#)); allows risk sharing and worker turnover across firms (e.g., [Diamond and Simon, 1990](#); [Krugman, 1991](#)); and induces stronger incentives for workers to invest in human capital knowing that they do not face ex post appropriation ([Rotemberg and Saloner, 2000](#)). As a result, conditional on the amount of labor input hired by a firm, the quality of labor input (and hence firm productivity) is likely higher when the total labor employed in the same location in the past period is larger. Relatedly, [Greenstone, Hornbeck and Moretti \(2010\)](#) find that estimated spillover effects resulting from the opening of Million Dollar Plants are larger for other plants that share labor pools and similar technologies with the new plant.

To model the spatial diffusion of the productivity shock u_t , we consider three variants of the connectivity matrix $\mathbf{W}_t^u$, depending on whether two firms are located in the same commuting zone, whether they have supplier/customer relationships, or both. In particular, the ij -th element of $\mathbf{W}_t^u$ takes on the value one: (i) if both firms i and j are located in the same commuting zone in period t , (ii) if both firms i and j are located in the same commuting zone, and firm j is a customer or supplier of firm i , in period t , and (iii) if firm j is a customer or supplier of firm i in period t , respectively. Supporting evidence of the first criterion used includes the work by [Jaffe, Trajtenberg and Henderson \(1993\)](#), [Audretsch and Feldman \(1996\)](#), and [Matray \(2021\)](#). These studies suggest that knowledge/innovation spillovers tend to be geographically localized. In the second variant, the spillover is further restricted specifically to firms in supplier-customer relationships. An unexpected productivity shock experienced by a firm may trickle down to its buyers via the provision of higher quality inputs, allowing its buyers to scale up their productivities. Alternatively, the technology innovation or discovery may occur simultaneously to the network of firms that belong to the same supply or value chain. The third variant instead focuses on the supply chain as the conduit of productivity spillovers, but disregards the potential distance between the customers/suppliers. Note that the second variant is a relatively sparse matrix compared to the other two variants.

The connectivity matrices defined above are then row-normalized, such that each row

has a row sum equal to one (and zero if all elements in a row are zeros).

5 Monte Carlo Simulations

In this section, we conduct Monte Carlo simulations to assess the consistency and efficiency of the estimator we proposed in Section 3 (which allows for spatial dependence across firms), and compare it with the conventional estimators (that assume no such spatial dependence). We consider five data generating processes (DGPs). The first DGP (DGP1) is favorable to the conventional estimator and assumes that the productivity ω_t follows an AR(1) process. The remaining DGPs consider spatial dependence of different structures and strengths across firms. The second DGP (DGP2) assumes the productivity ω_t to depend on own lagged productivity and the lagged outputs/inputs of connected firms as specified in Equation (2). The third DGP (DGP3) further allows the productivity shock u_t to be spatially correlated as specified in Equation (3). The fourth DGP (DGP4) is the same as DGP3 but assumes stronger spatial dependence in the lagged output/inputs of connected firms. The fifth DGP (DGP5) is the same as DGP4 but considers instead negative spatial dependence in the lagged output/inputs of connected firms.

We generate the simulation data based on the empirical sample statistics of the Japanese BSJBSA-TSR linked dataset. Appendix B provides detailed documentations of the simulation setups, which we summarize below. We follow [Akerberg, Caves and Frazer \(2015\)](#) and adopt a Leontief production function such that:

$$VA_{it} = \min \{e^{\alpha_0} L_{it}^{\alpha_l} K_{it}^{\alpha_k} e^{\omega_{it}}, e^{\alpha_m} M_{it}\} e^{\xi_{it}},$$

which implies Equation (1). In turn, gross output is linear in value-added. In particular, we set $e^{\alpha_m} = 1$ in simulating the gross output. The firm-level productivity is simulated based on Equations (2)–(3), with variations in the parameter values across the DGPs studied.

The firm-level input variables (labor and capital inputs) and the firm-to-firm connectivity matrices are simulated based on the firm-level statistics and the supplier-customer network statistics of the BSJBSA-TSR linked dataset. For example, based on the BSJBSA-TSR linked dataset, we tabulate the distribution of firms that supply to one, two, three, ..., and up to 24 other firms; and respectively, the distribution of firms that purchase from one, two, three, ..., and up to 24 other firms. We use these distribution statistics across years to simulate time-varying supplier-customer networks, which takes into account network addition, attrition, and persistency observed in the data.

Given the model structure, we assume that the error terms (ξ_{it}, v_{it}) are normally dis-

tributed with mean zeros and standard deviations of $\sigma_\xi = 0.3$ and $\sigma_v = 0.7$. We simulate a balanced panel of 500, 750 or 1000 firms for 10 or 19 time periods. For each DGP, 1000 simulated samples are drawn and estimated. We report the mean (Mean) and the standard deviation (SD) of the parameter point estimates across the 1000 Monte Carlo simulations, together with the average of the estimated standard errors (SE) derived from the variance-covariance matrices of the estimators across the 1000 Monte Carlo simulations. The parameter values used in the DGPs are listed in the first row of Tables 3–7. The parameter values that are common across DGPs are: $\alpha_0 = 0$, $\alpha_l = 0.6$, $\alpha_k = 0.4$; $\rho_1 = 0.5$ and $\rho_2 = \dots = \rho_G = 0$. To simplify the Monte Carlo exercises, we drop $\mathbf{x}_{i,t-1}$ (a firm’s lagged exporting status and/or R&D expenditure) from consideration in the simulation. In DGP2, the strength of spillovers in terms of lagged outputs and lagged labor inputs of related firms is set at: $\lambda = \beta_l = 0.01$. DGP4 considers stronger spillovers such that $\lambda = \beta_l = 0.1$, while DGP5 considers negative spillovers such that $\lambda = \beta_l = -0.1$. In DGP3–DGP5, with spatial error dependence, we set $\mu = 0.25$.

Given the simulated sample, we use the [Wooldridge \(2009\)](#) GMM estimator in combination with the ACF estimator to represent the conventional estimator (henceforth WGMM), which assumes no spatial dependence across firms.⁴ For our proposed estimator (SGMM), we use the instruments indicated in Equations (10) and (13) in estimations. In particular, the current and first lag of labor, capital and material inputs are used as the instruments for the first-stage equation (8), with a degree-1 h function in labor, capital and material inputs (à la [Akerberg, Caves and Frazer, 2015](#)). For the second-stage equation (11), the current capital along with the first lag of labor, capital and material inputs, and the lagged-one-period outputs and labor inputs of related firms ($\mathbf{W}_{t-1}^y y_{t-1}$, $(\mathbf{W}_{t-1}^y)^2 y_{t-1}$, $\mathbf{W}_{t-1}^l l_{t-1}$, $(\mathbf{W}_{t-1}^l)^2 l_{t-1}$) are used as instruments. The connectivity matrices are as defined in Section 4.2. In particular, the connectivity matrix $\mathbf{W}_t^u$ specifying the spatial correlation of productivity shocks is defined based on the customer-supplier relationships across firms. The same set of instruments are used for the WGMM estimations, but excluding the related firms’ lagged outputs and lagged labor inputs ($\mathbf{W}_{t-1}^y y_{t-1}$, $(\mathbf{W}_{t-1}^y)^2 y_{t-1}$, $\mathbf{W}_{t-1}^l l_{t-1}$, $(\mathbf{W}_{t-1}^l)^2 l_{t-1}$).

5.1 Simulation Results

[Table 3](#) reports the results for DGP1. The conventional estimator (WGMM) performs well as it should, when the DGP has no spatial dependence across firms. Importantly, our proposed

⁴As noted above, the [Wooldridge \(2009\)](#) procedure estimates Equations (8) and (11) jointly (instead of sequentially as in ACF), while the ACF estimator allows $g(\cdot, \cdot, \cdot)$ to be a general function in all the input variables (including linearity as a special case) and hence does not aim to identify the slope coefficients (α_l, α_k) in Stage 1 from Equation (8) but in Stage 2 from Equation (11).

estimator (SGMM) performs just as well. The point estimates of both estimators are close to the true parameter values, and the 95% confidence intervals (CIs) include the true parameter values for the input coefficients of the production function (α_l, α_k). While our estimator has wider confidence intervals than the conventional estimator for the input coefficients, it returns mean estimates of the spatial coefficients (λ, β_l, μ) nearly identical to zeros, consistent with the true parameter values of the underlying DGP. Both estimators obtain estimates for the autoregressive parameter (ρ_1) that are close to the true parameter value, even when the duration of the panel is relatively short. Both the conventional estimator and our proposed estimator yield standard error estimates (SE) that are on average close to their Monte Carlo standard deviations (SD). The SEs also reduce as the sample size increases at a rate consistent with the asymptotic properties laid out in Section 3.3.

Table 4 reports the findings for the second set of simulations based on DGP2. When spatial dependence across firms via lagged outputs and lagged labor inputs are indeed present, the conventional estimator leads to biased estimates of the input coefficients. In particular, its mean estimates for α_l across variations in N and T are higher than the true parameter value. The bias does not shrink with a larger sample size, suggesting the inconsistency of the conventional estimator when spatial dependence is present in the underlying DGP. In contrast, our proposed SGMM estimator yields estimates that are close to the true parameter values for both input elasticities (α_l, α_k) and the spatial coefficients (λ, β_l), with 95% CIs that well cover the true parameter values. Finally, our proposed SGMM estimator reports statistically insignificant estimates of μ , consistent with the underlying DGP where no spatial correlation in the error terms (i.e., the productivity shocks u_t) is present.

In DGP3, the data generating process for the productivity term further allows for spatial error correlation across related firms. Table 5 shows that the conventional estimator of the labor coefficient of the production function remains to be upward biased, while our proposed estimator yields consistent estimates that are close to the true parameter values for all the coefficients of interest. In particular, we note that the SGMM estimator returns estimates of the spatial error coefficient (μ) that are close to its true parameter value when it is indeed non-zero.

Table 6 reports the simulation results for DGP4. With larger spatial coefficients ($\lambda = \beta_l = 0.1$, instead of 0.01), the conventional estimator of all coefficients ($\alpha_l, \alpha_k, \rho_1$) are upward biased, and the extents of bias are substantial (by around 23–26 percentage points for α_l , 1 percentage point for α_k , and 17–26 percentage points for ρ_1). Furthermore, the standard errors (SE) obtained by the conventional estimator deviate significantly from the Monte Carlo standard deviations (SD). In contrast, our proposed SGMM estimator continues to yield consistent estimates for the true parameters, with estimates of the standard errors

(SE) that are on average very close to the Monte Carlo standard deviations (SD).

Table 7 indicates that if the underlying DGP is characterized with large negative spatial coefficients ($\lambda = \beta_l = -0.1$, instead of 0.01), the conventional estimator of the input coefficients (α_l, α_k) are instead downward biased, and the extents of bias continue to be substantial (by around 21–23 percentage points for α_l , and 1 percentage point for α_k). The conventional estimator of ρ_1 , however, remains to be upward biased (by 12–21 percentage points). The standard errors (SE) obtained by the conventional estimator on average also deviate significantly from the Monte Carlo standard deviations (SD). In contrast, our proposed SGMM estimator yields consistent estimates for the true parameters, with estimates of the standard errors (SE) being very close on average to the Monte Carlo standard deviations (SD). In particular, it is able to capture the negative signs of the two spatial coefficients ($\lambda = \beta_l = -0.1$) and their magnitudes.

In sum, across all the DGPs, we find that the proposed SGMM estimator yields point estimates that are consistent for the true parameter values both in the absence and in the presence of spatial effects. By the SGMM estimator, the standard error estimates (SE) of the parameters are also very close to the Monte Carlo standard deviations (SD). As the sample size $N(T - 1)$ doubles (either due to doubling of N or $T - 1$), both the standard error estimates (SE) and the Monte Carlo standard deviations (SD) shrink at a rate close to $1/\sqrt{2}$, consistent with a convergence rate of $1/\sqrt{N(T - 1)}$.

6 Empirical Analysis

6.1 Estimation Sample

We apply the methodology and estimation algorithms proposed in Section 3 to the Japanese dataset introduced in Section 4. Given the BSJBSA-TSR linked data, we further restrict the sample to a balanced panel of firms with observations on the set of variables required for productivity estimations. In particular, the sample is based on firms that were surveyed for 10 consecutive years from 2009 to 2018.⁵ Second, the sample of firms used for analysis also need to have non-missing values for log of labor hours (used to measure l_{it}), log of real capital stock (k_{it}), log of real intermediate inputs (m_{it}), log of real revenues (y_{it}), and log of real value added (va_{it}), during the entire sample period 2009–2018. Recall that the real capital stock is calculated based on the perpetual inventory method with the real capital stock in 2007 (year end) as the initial value. Observations on real capital stock for a firm

⁵This excludes, for example, firms whose number of employees fell under 50 at some point during the sample period.

could be missing, for example, if the firm was not observed in 2007.

The resulting sample is a balanced panel of 14,178 firms for the period 2009–2018. Given the balanced panel of firms, the set of a firm’s customers/suppliers identified via the TSR entries is effectively restricted to those whose firm-level data also exist in BSJBSA. In particular, firm j is regarded effectively as a customer/supplier of firm i in the estimation if firm i reports firm j as a customer or supplier, *and* if firm j exists in the BSJBSA dataset (with consecutive observations on output and inputs, as required for the estimation of Equation (2)).

Table 2(b) provides the summary statistics for the estimation sample. Relative to the raw sample reported in Table 2(a), the firms in the estimation sample tends to be larger in terms of both inputs and output, and have more customers and suppliers. This is expected, as larger firms are more likely to be surveyed consecutively and have positive inputs/output throughout the years. Although larger firms tend to have more customers/suppliers, the orders of magnitude in the number of connections on average do not differ substantially between the raw and estimation samples. Despite the much smaller set of firms covered, the estimation sample accounts for 61.42% of aggregate real value added and 63.29% of real gross output of the raw sample in 2015 (and a majority of the other economic activities in terms of employment, labor hours, real capital stock, and real spending on intermediate inputs).

Figures 1(b)–10(b) repeat the characterization as in Figures 1(a)–10(a), and show that the estimation sample has similar patterns as documented for the raw sample. The rank across industries is almost identical in terms of the number of firms: manufacturing, wholesale & retail trade, and information & communications remain to be the top three industries with the largest numbers of firms (Figure 1). The set of prefectures with the largest numbers of firms is also similar to that previously documented (Figure 4). Basically, firms in the estimation sample tend to be larger in terms of employment size (Figures 2, 5, and 9), and are slightly more connected in terms of customers/suppliers (Figures 3, 6, and 10), in comparison with the raw sample.

6.2 Estimation Results

We estimate the model proposed in Equations (1)–(3) based on the estimation methodology laid out in Section 3 and the connectivity matrices defined in Section 4.2. In short, we define the output connectivity matrix $\mathbf{W}_{t-1}^y$ based on the set of a firm’s customers/suppliers located in the same commuting zone. The ij -th element of $\mathbf{W}_{t-1}^y$ takes on the value one if both firms i and j are located in the same commuting zone, and in addition, firm j is a customer or supplier of firm i , in period $t - 1$. Second, we define input connectivity matrix $\mathbf{W}_{t-1}^\Omega$ such

that the ij -th element of $\mathbf{W}_{t-1}^\Omega$ takes on the value one if both firms i and j are located in the same commuting zone in period $t - 1$. The input variable being analyzed corresponds to the lagged labor inputs of the connected firms defined by $\mathbf{W}_{t-1}^\Omega$. Third, we consider three variants of the spatial error connectivity matrix $\mathbf{W}_t^u$ and define it such that the ij -th element of $\mathbf{W}_t^u$ takes on the value one: (i) if both firms i and j are located in the same commuting zone in period t , (ii) if both firms i and j are located in the same commuting zone, and firm j is a customer or supplier of firm i , in period t , and (iii) if firm j is a customer or supplier of firm i in period t , respectively. The connectivity matrices are then row-normalized, such that each row has a row sum equal to one (and zero if all elements in a row are zeros).

In addition to the key variables (lagged productivity and connected firms' lagged output and lagged labor input) in the productivity equation (2), we also control for the firm's lagged exporting status and lagged real R&D expenditure, and the industry fixed effects (FE).

Table 8 reports the estimation results. Column 1 (based on the first definition of $\mathbf{W}_t^u$) indicates that all three spatial coefficients are significant and positive. A 1% increase in the sales of customers/suppliers in the same commuting zone in the previous period helps improve a firm's current productivity by 0.005%. A larger local labor market also enhances a firm's productivity: specifically, a 1% increase in the employment of firms located in the same commuting zone in the previous period raises a firm's current productivity by 0.055%. Finally, there is evidence of contemporary knowledge spillovers across firms located in the same commuting zone: the productivity innovations u_{it} are spatially correlated with a positive and significant slope coefficient of 0.68. The other production function parameters are also precisely estimated. Column 1 reports a labor value-added share of 0.81, a capital value-added share of 0.14, and a partial AR(1) coefficient of 0.77 for the productivity process. In addition, if a firm is an exporter in the previous period, its current productivity tends to be higher by 18.98%. A one-dollar increase in R&D expenditure in the previous period also helps to raise the current period productivity by 5.13E-07 (in log scale).

The estimates for key parameters of interest remain similar if we adopt alternative definitions of spatial error connectivity matrices $\mathbf{W}_t^u$, as reported in Column 2 and Column 3. The key difference is the strength of contemporary spatial correlation in the productivity shocks u_{it} . They tend to be quantitatively smaller by around 50% in magnitude if $\mathbf{W}_t^u$ is defined in a more restricted manner, relative to the finding in Column 1 based on common commuting zone alone.

To compare the spatial GMM findings with those if one ignores the spatial dependence across firms, Column 4 reports the estimates based on conventional estimators (restricting the spatial coefficients to be zeros but otherwise adopting the same GMM approach). We find that the labor value-added share tends to be downward biased (0.72), the capital value-added

share upward biased (0.22), and the AR(1) coefficient upward biased (0.83), in comparison with the spatial GMM estimates reported above. The effect estimates of exporter status (18.62%) and R&D expenditures (1.83E-06) are otherwise close to those obtained by the spatial GMM estimates.

6.3 Restricted Specifications

In this section, we experiment with restricted specifications of Equations (2)–(3), by setting λ , β , and/or μ equal to zeros. This helps us assess the role of each proposed spatial mechanism in affecting the outcome of production function estimations. For this set of exercises, we define $\mathbf{W}_t^u$ based on supplier-customer relationship within the same commuting zone. This specification (as shown in Column (2) of Table 8) tends to lead to the weakest estimate of μ and hence provides a conservative reference benchmark for the proposed spatial mechanisms.

Table 9 summarizes the results. Columns (1)–(3) allow for spatial error dependence ($\mu \neq 0$) but omit the dependence of productivity on the lagged outputs and/or the lagged labor inputs of related firms ($\lambda = 0$ and/or $\beta = 0$). Columns (4)–(6) further drop the spatial error dependence structure ($\mu = 0$). We note that relative to the benchmark, by ignoring the spatial error dependence structure in the productivity process, the labor elasticity (α_l) estimate is significantly downward biased and the reverse is true for the capital elasticity (α_k) estimate; meanwhile, the AR(1) coefficient estimate tends to be upward biased (as shown in Columns (4)–(6)). Thus, to a large extent, the direction of bias of the conventional estimates (in Column (4) of Table 8) relative to the SGMM estimates is driven by the omission of the spatial error dependence structure in the productivity process. Given $\mu = 0$, by omitting the dependence of productivity on the lagged outputs or the lagged labor inputs of related firms, the remaining spatial coefficient (λ or β) tends to increase, but the orders of magnitude remain very similar.

Once we allow for the spatial error dependence structure in the productivity process (as shown in Columns (1)–(3)), by omitting the dependence of productivity on the lagged outputs and/or the lagged labor inputs of related firms, the labor elasticity (α_l) estimate tends to be upward biased and the capital elasticity (α_k) estimate downward biased, the opposite of the pattern shown in Columns (4)–(6); meanwhile, the AR(1) coefficient estimate continues to be upward biased, relative to the benchmark. Thus, by omitting all spatial mechanisms, the conventional WGMM estimate tends to fall in between the two sets of labor/capital elasticity estimates (Columns (1)–(3) versus Columns (4)–(6)), and on net with the omission of spatial error structure ($\mu = 0$) exerting a greater impact on the pattern of bias for the sample under study. In Columns (1)–(3), by omitting the dependence of productivity on the

lagged outputs or the lagged labor inputs of related firms, the remaining spatial coefficient (λ or β) tends to increase but maintains similar orders of magnitude as the benchmark, an observation similar to the case with $\mu = 0$. Finally, the spatial error coefficient μ tends to weaken if the other two spatial mechanisms are omitted from the specification. Overall, the exercises in this section illustrate the sensitivity of the labor and capital elasticity estimates (and that of the AR(1) coefficient in the productivity process) to the spatial mechanisms proposed in this study.

7 Conclusion

In this paper, we develop a framework to simultaneously estimate firm production functions and spatial interactions across firms in one unified setup. We propose a three-stage efficient GMM estimation algorithm, and formulate the asymptotic properties of the proposed estimator. The Monte Carlo simulations demonstrate that the proposed estimator is consistent under DGPs with or without spatial dependence across firms. In contrast, the conventional estimators are biased when the true DGPs are indeed characterized with spatial dependence. By applying the developed methodology and estimation algorithm to the Japanese BSJBSA-TSR linked dataset for the period 2009–2018, we find that spatial interactions across firms play a significant role in determining the Japanese firm-level productivity both statistically and economically.

The paper can be extended in several directions in future research. First, the connectivity matrices in our setup are allowed to differ across different mechanisms of spatial interactions. One can potentially hypothesize alternative candidates for the connectivity matrices and conduct specification tests that select the specification that best fits the model. Second, the current framework allows for time-varying connectivity matrices. This is useful, as we can use the framework to analyze how shocks (such as transportation infrastructure developments and earthquakes) affect the connectivity matrices across time, and in turn, the firm-level performance measures (such as productivity, and production technology). Third, the current framework could also be used to analyze the centrality of firms in the sense that a firm is more central if by increasing connectivity (links) to this firm, the average productivity of all firms is improved by more than if by increasing connectivity (links) to another firm. This is useful for policy design that aims to target subsidies at the critical links of a firm network structure for the greater benefits of the economy.

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A Theoretical Appendix

A.1 Deriving the Moment Condition in Stage 3

Substituting $v_t = u_t - \mu \mathbf{W}_t^u u_t$ into the left side of Equation (6) yields:

$$\begin{aligned} \frac{1}{N} E \begin{bmatrix} (u_t - \mu \mathbf{W}_t^u u_t)'(u_t - \mu \mathbf{W}_t^u u_t) \\ (u_t - \mu \mathbf{W}_t^u u_t)' \mathbf{W}_t^{u'} \mathbf{W}_t^u (u_t - \mu \mathbf{W}_t^u u_t) \\ (u_t - \mu \mathbf{W}_t^u u_t)' \mathbf{W}_t^u (u_t - \mu \mathbf{W}_t^u u_t) \end{bmatrix} &= \frac{1}{N} E \begin{bmatrix} (u_t - \mu \bar{u}_t)'(u_t - \mu \bar{u}_t) \\ (u_t - \mu \bar{u}_t)' \mathbf{W}_t^{u'} \mathbf{W}_t^u (u_t - \mu \bar{u}_t) \\ (u_t - \mu \bar{u}_t)' \mathbf{W}_t^u (u_t - \mu \bar{u}_t) \end{bmatrix} \\ &= \frac{1}{N} E \begin{bmatrix} (u_t' u_t - 2\mu \bar{u}_t' u_t + \mu^2 \bar{u}_t' \bar{u}_t) \\ (\bar{u}_t' \bar{u}_t - 2\mu \bar{u}_t' \bar{u}_t + \mu^2 \bar{u}_t' \bar{u}_t) \\ u_t' \bar{u}_t - \mu(\bar{u}_t' \bar{u}_t + u_t' \bar{u}_t) + \mu^2 \bar{u}_t' \bar{u}_t \end{bmatrix}. \end{aligned}$$

Therefore, Equation (6) becomes:

$$\frac{1}{N} E \begin{bmatrix} (u_t' u_t - 2\mu \bar{u}_t' u_t + \mu^2 \bar{u}_t' \bar{u}_t) \\ (\bar{u}_t' \bar{u}_t - 2\mu \bar{u}_t' \bar{u}_t + \mu^2 \bar{u}_t' \bar{u}_t) \\ u_t' \bar{u}_t - \mu(\bar{u}_t' \bar{u}_t + u_t' \bar{u}_t) + \mu^2 \bar{u}_t' \bar{u}_t \end{bmatrix} = \begin{bmatrix} \sigma_v^2 \\ \frac{\sigma_v^2}{N} \text{tr}(\mathbf{W}_t^{u'} \mathbf{W}_t^u) \\ 0 \end{bmatrix}. \quad (32)$$

Rearranging terms, we have:

$$\begin{aligned} \frac{1}{N} E \begin{bmatrix} u_t' u_t \\ \bar{u}_t' \bar{u}_t \\ u_t' \bar{u}_t \end{bmatrix} &= \frac{1}{N} E \begin{bmatrix} 2\mu \bar{u}_t' u_t - \mu^2 \bar{u}_t' \bar{u}_t + N\sigma_v^2 \\ 2\mu \bar{u}_t' \bar{u}_t - \mu^2 \bar{u}_t' \bar{u}_t + \sigma_v^2 \text{tr}(\mathbf{W}_t^{u'} \mathbf{W}_t^u) \\ \mu(\bar{u}_t' \bar{u}_t + u_t' \bar{u}_t) - \mu^2 \bar{u}_t' \bar{u}_t \end{bmatrix} \\ &= \frac{1}{N} E \begin{bmatrix} 2\bar{u}_t' u_t & -\bar{u}_t' \bar{u}_t & N \\ 2\bar{u}_t' \bar{u}_t & -\bar{u}_t' \bar{u}_t & \text{tr}(\mathbf{W}_t^{u'} \mathbf{W}_t^u) \\ (\bar{u}_t' \bar{u}_t + u_t' \bar{u}_t) & -\bar{u}_t' \bar{u}_t & 0 \end{bmatrix} \begin{bmatrix} \mu \\ \mu^2 \\ \sigma_v^2 \end{bmatrix}, \quad (33) \end{aligned}$$

which yields the following relationship:

$$\gamma_t = \mathbf{\Gamma}_t \begin{bmatrix} \mu \\ \mu^2 \\ \sigma_v^2 \end{bmatrix}.$$

A.2 Deriving the Variance-Covariance Matrix for the Moment Conditions in Stages 1 and 2

Using the definition of the variance-covariance matrix of the moment conditions in the first and second stages, we have:

$$\begin{aligned}
\mathbf{V}_\theta &= \text{Var} \left(\frac{1}{\sqrt{N(T-1)}} \sum_{i=1}^N \sum_{t=2}^T \mathbf{Z}'_{it} r_{it} \right) \\
&= \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \sum_{j=1}^N \sum_{s=2}^T E[(\mathbf{Z}'_{it} r_{it})(\mathbf{Z}'_{js} r_{js})'] \\
&= \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \sum_{j=1}^N \sum_{s=2}^T \mathbf{Z}'_{it} E[r_{it} r'_{js}] \mathbf{Z}_{js} \\
&= \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \sum_{j=1}^N \sum_{s=2}^T \mathbf{Z}'_{it} E \left[\begin{pmatrix} \xi_{it} \\ \xi_{it} + u_{it} \end{pmatrix} (\xi_{js} \quad \xi_{js} + u_{js})' \right] \mathbf{Z}_{js} \\
&= \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \sum_{j=1}^N \sum_{s=2}^T \mathbf{Z}'_{it} E \begin{pmatrix} \xi_{it} \xi_{js} & \xi_{it}(\xi_{js} + u_{js}) \\ (\xi_{it} + u_{it}) \xi_{js} & (\xi_{it} + u_{it})(\xi_{js} + u_{js}) \end{pmatrix} \mathbf{Z}_{js}. \quad (34)
\end{aligned}$$

To derive $E \begin{pmatrix} \xi_{it} \xi_{js} & \xi_{it}(\xi_{js} + u_{js}) \\ (\xi_{it} + u_{it}) \xi_{js} & (\xi_{it} + u_{it})(\xi_{js} + u_{js}) \end{pmatrix}$, consider the following 4 cases:

1. $i = j, t = s$

$$\begin{aligned}
E \begin{pmatrix} \xi_{it} \xi_{js} & \xi_{it}(\xi_{js} + u_{js}) \\ (\xi_{it} + u_{it}) \xi_{js} & (\xi_{it} + u_{it})(\xi_{js} + u_{js}) \end{pmatrix} &= E \begin{pmatrix} \xi_{it} \xi_{it} & \xi_{it}(\xi_{it} + u_{it}) \\ (\xi_{it} + u_{it}) \xi_{it} & (\xi_{it} + u_{it})(\xi_{it} + u_{it}) \end{pmatrix} \\
&= \begin{pmatrix} \sigma_\xi^2 & \sigma_\xi^2 \\ \sigma_\xi^2 & \sigma_\xi^2 + E(u_{it}^2) \end{pmatrix}.
\end{aligned}$$

2. $i = j, t \neq s$

$$\begin{aligned}
E \begin{pmatrix} \xi_{it} \xi_{js} & \xi_{it}(\xi_{js} + u_{js}) \\ (\xi_{it} + u_{it}) \xi_{js} & (\xi_{it} + u_{it})(\xi_{js} + u_{js}) \end{pmatrix} &= E \begin{pmatrix} \xi_{it} \xi_{is} & \xi_{it}(\xi_{is} + u_{is}) \\ (\xi_{it} + u_{it}) \xi_{is} & (\xi_{it} + u_{it})(\xi_{is} + u_{is}) \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.
\end{aligned}$$

3. $i \neq j, t = s$

$$\begin{aligned} E \begin{pmatrix} \xi_{it}\xi_{js} & \xi_{it}(\xi_{js} + u_{js}) \\ (\xi_{it} + u_{it})\xi_{js} & (\xi_{it} + u_{it})(\xi_{js} + u_{js}) \end{pmatrix} &= E \begin{pmatrix} \xi_{it}\xi_{jt} & \xi_{it}(\xi_{jt} + u_{jt}) \\ (\xi_{it} + u_{it})\xi_{jt} & (\xi_{it} + u_{it})(\xi_{jt} + u_{jt}) \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & E(u_{it}u_{jt}) \end{pmatrix}. \end{aligned}$$

4. $i \neq j, t \neq s$

$$E \begin{pmatrix} \xi_{it}\xi_{js} & \xi_{it}(\xi_{js} + u_{js}) \\ (\xi_{it} + u_{it})\xi_{js} & (\xi_{it} + u_{it})(\xi_{js} + u_{js}) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

To estimate $E(u_{it}^2)$, note that u_{it} can be obtained by taking the product of the i -th row of $(I_N - \mu \mathbf{W}_t^u)^{-1}$ and v_t , since $u_t = (I_N - \mu \mathbf{W}_t^u)^{-1}v_t$. Then, we have:

$$\begin{aligned} E(u_{it}^2) &= E([(I_N - \mu \mathbf{W}_t^u)^{-1}]_i v_t v_t' [(I_N - \mu \mathbf{W}_t^u)^{-1}]_i') \\ &= [(I_N - \mu \mathbf{W}_t^u)^{-1}]_i E[v_t v_t'] [(I_N - \mu \mathbf{W}_t^u)^{-1}]_i' \\ &= \sigma_v^2 [(I_N - \mu \mathbf{W}_t^u)^{-1}]_i [(I_N - \mu \mathbf{W}_t^u)^{-1}]_i', \end{aligned}$$

where $[X]_i$ refers to the i -th row of matrix X and $E(v_t v_t') = \sigma_v^2 \mathbf{I}_N$. Similarly, $E(u_{it}u_{jt})$ is given by:

$$\begin{aligned} E(u_{it}u_{jt}) &= E([(I_N - \mu \mathbf{W}_t^u)^{-1}]_i v_t v_t' [(I_N - \mu \mathbf{W}_t^u)^{-1}]_j') \\ &= [(I_N - \mu \mathbf{W}_t^u)^{-1}]_i E[v_t v_t'] [(I_N - \mu \mathbf{W}_t^u)^{-1}]_j' \\ &= \sigma_v^2 [(I_N - \mu \mathbf{W}_t^u)^{-1}]_i [(I_N - \mu \mathbf{W}_t^u)^{-1}]_j'. \end{aligned}$$

Substituting all the terms back into $\mathbf{V}_\theta$, we obtain:

$$\begin{aligned}
\mathbf{V}_\theta &= \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \sum_{j=1}^N \sum_{s=2}^T \mathbf{Z}'_{it} E \begin{pmatrix} \xi_{it}\xi_{js} & \xi_{it}(\xi_{js} + u_{js}) \\ (\xi_{it} + u_{it})\xi_{js} & (\xi_{it} + u_{it})(\xi_{js} + u_{js}) \end{pmatrix} \mathbf{Z}_{js} \\
&= \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \begin{pmatrix} \mathbf{Z}'_{it,I} & 0 \\ 0 & \mathbf{Z}'_{it,II} \end{pmatrix} E \begin{pmatrix} \xi_{it}^2 & \xi_{it}^2 \\ \xi_{it}^2 & \xi_{it}^2 + u_{it}^2 \end{pmatrix} \begin{pmatrix} \mathbf{Z}_{it,I} & 0 \\ 0 & \mathbf{Z}_{it,II} \end{pmatrix} \\
&\quad + \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{j \neq i}^N \sum_{t=2}^T \begin{pmatrix} \mathbf{Z}'_{it,I} & 0 \\ 0 & \mathbf{Z}'_{it,II} \end{pmatrix} E \begin{pmatrix} 0 & 0 \\ 0 & u_{it}u_{jt} \end{pmatrix} \begin{pmatrix} \mathbf{Z}_{jt,I} & 0 \\ 0 & \mathbf{Z}_{jt,II} \end{pmatrix} \\
&= \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \begin{pmatrix} \mathbf{Z}'_{it,I} \mathbf{Z}_{it,I} E(\xi_{it}^2) & \mathbf{Z}'_{it,I} \mathbf{Z}_{it,II} E(\xi_{it}^2) \\ \mathbf{Z}'_{it,II} \mathbf{Z}_{it,I} E(\xi_{it}^2) & \mathbf{Z}'_{it,II} \mathbf{Z}_{it,II} E(\xi_{it}^2) \end{pmatrix} \\
&\quad + \frac{1}{N(T-1)} \sum_{t=2}^T \sum_{i=1}^N \sum_{j=1}^N \begin{pmatrix} \mathbf{Z}'_{it,I} & 0 \\ 0 & \mathbf{Z}'_{it,II} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & \sigma_v^2 [(I_N - \mu \mathbf{W}_t^u)^{-1}]_i [(I_N - \mu \mathbf{W}_t^u)^{-1}]_j' \end{pmatrix} \begin{pmatrix} \mathbf{Z}_{jt,I} & 0 \\ 0 & \mathbf{Z}_{jt,II} \end{pmatrix} \\
&= \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \begin{pmatrix} \mathbf{Z}'_{it,I} \mathbf{Z}_{it,I} E(\xi_{it}^2) & \mathbf{Z}'_{it,I} \mathbf{Z}_{it,II} E(\xi_{it}^2) \\ \mathbf{Z}'_{it,II} \mathbf{Z}_{it,I} E(\xi_{it}^2) & \mathbf{Z}'_{it,II} \mathbf{Z}_{it,II} E(\xi_{it}^2) \end{pmatrix} \\
&\quad + \frac{1}{N(T-1)} \sum_{t=2}^T \begin{pmatrix} \mathbf{0}_{\mathcal{M}_1 \times \mathcal{M}_1} & \mathbf{0}_{\mathcal{M}_1 \times \mathcal{M}_2} \\ \mathbf{0}_{\mathcal{M}_2 \times \mathcal{M}_1} & \sigma_v^2 \mathbf{Z}'_{t,II} [(\mathbf{I}_N - \mu \mathbf{W}_t^u)^{-1} (\mathbf{I}_N - \mu \mathbf{W}_t^u)^{-1}] \mathbf{Z}_{t,II} \end{pmatrix},
\end{aligned}$$

where $\mathcal{M}_1$ and $\mathcal{M}_2$ are the number of moment conditions (instruments) used in the first and second stages respectively (such that $\mathcal{M}_1 + \mathcal{M}_2 = \mathcal{M}$), and $\mathbf{Z}_{t,II} = [\mathbf{Z}'_{1t,II}, \mathbf{Z}'_{2t,II}, \dots, \mathbf{Z}'_{Nt,II}]'$ is a $N \times \mathcal{M}_2$ matrix.

We can estimate $\mathbf{V}_\theta$ by its sample counterpart:

$$\begin{aligned}
\widehat{\mathbf{V}}_\theta &= \frac{1}{N(T-1)} \sum_{i=1}^N \sum_{t=2}^T \begin{pmatrix} \mathbf{Z}'_{it,I} \mathbf{Z}_{it,I} \widehat{\xi}_{it}^2 & \mathbf{Z}'_{it,I} \mathbf{Z}_{it,II} \widehat{\xi}_{it}^2 \\ \mathbf{Z}'_{it,II} \mathbf{Z}_{it,I} \widehat{\xi}_{it}^2 & \mathbf{Z}'_{it,II} \mathbf{Z}_{it,II} \widehat{\xi}_{it}^2 \end{pmatrix} \\
&\quad + \frac{1}{N(T-1)} \sum_{t=2}^T \mathbf{Z}'_t \begin{pmatrix} \mathbf{0}_{N \times N} & \mathbf{0}_{N \times N} \\ \mathbf{0}_{N \times N} & \widehat{\sigma}_v^2 [(\mathbf{I}_N - \widehat{\mu} \mathbf{W}_t^u)^{-1} (\mathbf{I}_N - \widehat{\mu} \mathbf{W}_t^u)^{-1}] \end{pmatrix} \mathbf{Z}_t,
\end{aligned}$$

where $\widehat{\xi}_{it}$, $\widehat{\mu}$ and $\widehat{\sigma}_v^2$ are obtained from Steps 1–2 in Section 3.3.

A.3 Deriving the Variance-Covariance Matrix for the Moment Conditions in Stage 3

In this section, we derive the variance-covariance matrix for the moment conditions used in Stage 3. Let $\mathbf{A}$ and $\mathbf{B}$ be $n \times n$ non-stochastic matrices. For a $n \times 1$ random vector $\mathbf{e}$ with mean 0, variance σ_e^2 and finite excess kurtosis κ_e :

$$\text{cov}(\mathbf{e}'\mathbf{A}\mathbf{e}, \mathbf{e}'\mathbf{B}\mathbf{e}) = \sigma_e^4 \kappa_e a'b + \sigma_e^4 \text{tr}(\mathbf{A}(\mathbf{B}' + \mathbf{B})), \quad (35)$$

where $a = \text{diagv}(\mathbf{A})$ and $b = \text{diagv}(\mathbf{B})$. The operator ‘diagv’ takes the diagonal elements of a matrix and converts them to a column vector.

Define $\mathbf{v} = [v'_2, v'_3, \dots, v'_T]$ to be a $N(T-1) \times 1$ vector, and $\bar{\mathbf{v}} = \mathbf{W}^u \mathbf{v}$. Let κ_v be the excess kurtosis of $\mathbf{v}$. The sample counterpart of the moment conditions in Equation (6) is:

$$\frac{1}{N(T-1)} \begin{pmatrix} \mathbf{v}'\mathbf{v} \\ \mathbf{v}'\mathbf{W}^{u'}\mathbf{W}^u\mathbf{v} \\ \mathbf{v}'\mathbf{W}^u\mathbf{v} \end{pmatrix} = \begin{pmatrix} \hat{\sigma}_v^2 \\ \frac{\hat{\sigma}_v^2}{N(T-1)} \text{tr}(\mathbf{W}^{u'}\mathbf{W}^u) \\ 0 \end{pmatrix}. \quad (36)$$

The following computes each cell of the variance-covariance matrix of the vector on the left side of Equation (36):

$$\begin{aligned} V_{\psi,11} &= N(T-1) \text{cov} \left(\frac{1}{N(T-1)} \mathbf{v}'\mathbf{v}, \frac{1}{N(T-1)} \mathbf{v}'\mathbf{v} \right) \\ &= N(T-1) \frac{\sigma_v^4}{(N(T-1))^2} [\kappa_v \ell'_{N(T-1)} \ell_{N(T-1)} + \text{tr}(\mathbf{I}_{N(T-1)}(\mathbf{I}'_{N(T-1)} + \mathbf{I}_{N(T-1)}))] \\ &= \sigma_v^4 (\kappa_v + 2); \\ V_{\psi,12} &= N(T-1) \text{cov} \left(\frac{1}{N(T-1)} \mathbf{v}'\mathbf{v}, \frac{1}{N(T-1)} \bar{\mathbf{v}}'\bar{\mathbf{v}} \right) \\ &= N(T-1) \text{cov} \left(\frac{1}{N(T-1)} \mathbf{v}'\mathbf{v}, \frac{1}{N(T-1)} \mathbf{v}'\mathbf{W}^{u'}\mathbf{W}^u\mathbf{v} \right) \\ &= \frac{\sigma_v^4}{N(T-1)} [\kappa_v \text{tr}(\mathbf{W}^{u'}\mathbf{W}^u) + \text{tr}(\mathbf{I}_{N(T-1)}(\mathbf{W}^{u'}\mathbf{W}^u + \mathbf{W}^u\mathbf{W}^{u'}))] \\ &= \frac{\sigma_v^4}{N(T-1)} (\kappa_v + 2) \text{tr}(\mathbf{W}^{u'}\mathbf{W}^u); \end{aligned}$$

$$\begin{aligned}
V_{\psi,13} &= N(T-1)\text{cov}\left(\frac{1}{N(T-1)}\mathbf{v}'\mathbf{v}, \frac{1}{N(T-1)}\mathbf{v}'\bar{\mathbf{v}}\right) \\
&= N(T-1)\text{cov}\left(\frac{1}{N(T-1)}\mathbf{v}'\mathbf{v}, \frac{1}{N(T-1)}\mathbf{v}'\mathbf{W}^{u'}\mathbf{v}\right) \\
&= \frac{\sigma_v^4}{N(T-1)}[\kappa_v \text{tr}(\mathbf{W}^u) + \text{tr}(\mathbf{W}^u)] \\
&= 0; \\
V_{\psi,22} &= N(T-1)\text{cov}\left(\frac{1}{N(T-1)}\bar{\mathbf{v}}'\bar{\mathbf{v}}, \frac{1}{N(T-1)}\bar{\mathbf{v}}'\bar{\mathbf{v}}\right) \\
&= N(T-1)\text{cov}\left(\frac{1}{N(T-1)}\mathbf{v}'\mathbf{W}^{u'}\mathbf{W}^u\mathbf{v}, \frac{1}{N(T-1)}\mathbf{v}'\mathbf{W}^{u'}\mathbf{W}^u\mathbf{v}\right) \\
&= \frac{\sigma_v^4}{N(T-1)}[\kappa_v \text{diagv}(\mathbf{W}^{u'}\mathbf{W}^u)' \text{diagv}(\mathbf{W}^{u'}\mathbf{W}^u) + \text{tr}((\mathbf{W}^{u'}\mathbf{W}^u)(\mathbf{W}^{u'}\mathbf{W}^u + \mathbf{W}^u\mathbf{W}^{u'}))]; \\
V_{\psi,23} &= N(T-1)\text{cov}\left(\frac{1}{N(T-1)}\bar{\mathbf{v}}'\bar{\mathbf{v}}, \frac{1}{N(T-1)}\mathbf{v}'\bar{\mathbf{v}}\right) \\
&= N(T-1)\text{cov}\left(\frac{1}{N(T-1)}\mathbf{v}'\mathbf{W}^{u'}\mathbf{W}^u\mathbf{v}, \frac{1}{N(T-1)}\mathbf{v}'\mathbf{W}^u\mathbf{v}\right) \\
&= \frac{\sigma_v^4}{N(T-1)}[\kappa_v \text{diagv}(\mathbf{W}^{u'}\mathbf{W}^u)' \text{diagv}(\mathbf{W}^u) + \text{tr}((\mathbf{W}^{u'}\mathbf{W}^u)(\mathbf{W}^u + \mathbf{W}^{u'}))] \\
&= \frac{\sigma_v^4}{N(T-1)}\text{tr}((\mathbf{W}^{u'}\mathbf{W}^u)(\mathbf{W}^u + \mathbf{W}^{u'})); \\
V_{\psi,32} &= N(T-1)\text{cov}\left(\frac{1}{N(T-1)}\mathbf{v}'\bar{\mathbf{v}}, \frac{1}{N(T-1)}\bar{\mathbf{v}}'\bar{\mathbf{v}}\right) \\
&= N(T-1)\text{cov}\left(\frac{1}{N(T-1)}\mathbf{v}'\mathbf{W}^u\mathbf{v}, \frac{1}{N(T-1)}\mathbf{v}'\mathbf{W}^{u'}\mathbf{W}^u\mathbf{v}\right) \\
&= \frac{\sigma_v^4}{N(T-1)}[\kappa_v \text{diagv}(\mathbf{W}^u)' \text{diagv}(\mathbf{W}^{u'}\mathbf{W}^u) + \text{tr}(\mathbf{W}^u(\mathbf{W}^{u'}\mathbf{W}^u + \mathbf{W}^{u'}\mathbf{W}^u))] \\
&= 2\frac{\sigma_v^4}{N(T-1)}\text{tr}(\mathbf{W}^u\mathbf{W}^{u'}\mathbf{W}^u); \\
V_{\psi,33} &= N(T-1)\text{cov}\left(\frac{1}{N(T-1)}\mathbf{v}'\bar{\mathbf{v}}, \frac{1}{N(T-1)}\mathbf{v}'\bar{\mathbf{v}}\right) \\
&= N(T-1)\text{cov}\left(\frac{1}{N(T-1)}\mathbf{v}'\mathbf{W}^u\mathbf{v}, \frac{1}{N(T-1)}\mathbf{v}'\mathbf{W}^u\mathbf{v}\right) \\
&= \frac{\sigma_v^4}{N(T-1)}[\kappa_v \text{diagv}(\mathbf{W}^u)' \text{diagv}(\mathbf{W}^u) + \text{tr}(\mathbf{W}^u(\mathbf{W}^u + \mathbf{W}^{u'}))] \\
&= \frac{\sigma_v^4}{N(T-1)}\text{tr}(\mathbf{W}^u(\mathbf{W}^u + \mathbf{W}^{u'})).
\end{aligned}$$

B Simulation Appendix

B.1 Simulation of Connectivity Matrices

The BSJBSA-TSR linked dataset provides the distribution of the number of customers (and respectively suppliers) that a firm has, up to 24 customers (and suppliers). We assign a time-invariant random number for each firm, r_i , which is uniformly distributed in $[0, 1]$, for $i \in \{1, 2, \dots, N\}$. For the initial period, we use a weakly monotonic mapping function, $q_t^{num}(\cdot)$, to map the firm random number $r_i \in [0, 1]$ to the number of customers, given the empirical distribution. In other words, $q_t^{num}(r_i) = num_{it}$, where $q_t^{num}(\cdot)$ is the inverse of the empirical distribution function of the number of customers in period t . Given the number of customers assigned to each firm in the initial period, we randomly draw its customers from the pool of firms. Subsequently, given the mapping from the firm random number to the number of customers that firm i has at time t , $q_t^{num}(r_i) = num_{it}$, we randomly drop firms from the set of customers that a firm initially has in the previous period if $num_{it} < num_{i,t-1} * persistency_{t-1}$, where $persistency_{t-1}$ is the fraction of firm-to-firm relationships in period $t - 1$ that survive in period t as observed in the data. Alternatively, we add firms (randomly drawn from the pool of unrelated firms) to the set of customers that a firm has in the previous period after attrition (the identity of the connections dropped being randomly drawn from the pool of existing customers of a firm) if $num_{it} > num_{i,t-1} * persistency_{t-1}$. The number of suppliers that a firm has across time is simulated in similar manner.

Given data on the distribution of firms across commuting zones, we use the inverse of the empirical distribution function of commuting zones, $q_t^{cz}(\cdot)$, to map each firm $r_i \in [0, 1]$ to commuting zone in each period, such that $q_t^{cz}(r_i) = cz_{it}$. We then generate the connectivity matrix based on common commuting zone. The ij -th element of the matrix is set equal to 1, if firms i and j are located in the same commuting zone in period t and 0 otherwise. As we vary the number of firms N above 500, we limit the number of connections per firm. Specifically, we generate random numbers $r_{ij} \in [0, 1]$ from a uniform distribution for each firm-pair ij . For each ij -th element of the connectivity matrix that equals 1 in period t , it remains to be 1 if $r_{ij} < 500/N$ and reduces to 0 otherwise.

B.2 Simulation of Input and Output Variables

Based on the BSJBSA-TSR linked dataset, we obtain the mean and standard deviation of labor input (and respectively, capital) across firms in each year from 2009 to 2018. We then simulate the usage of labor input (and respectively, capital) for each firm, by drawing randomly from Normal distributions that have the same mean and standard deviation as

empirically observed specific to each input variable and year.

For the Monte Carlo simulations, we adopt a Leontief production function as in [Akerberg, Caves and Frazer \(2015\)](#) such that:

$$VA_{it} = \min \{e^{\alpha_0} L_{it}^{\alpha_l} K_{it}^{\alpha_k} e^{\omega_{it}}, e^{\alpha_m} M_{it}\} e^{\xi_{it}}, \quad (37)$$

which gives rise to the following relationship between material inputs and productivity after taking logs:

$$\alpha_m + m_{it} = \alpha_0 + \alpha_l l_{it} + \alpha_k k_{it} + \omega_{it}. \quad (38)$$

Setting $e^{\alpha_m} = 1$ as in ACF, we have: $m_{it} = \alpha_0 + \alpha_l l_{it} + \alpha_k k_{it} + \omega_{it}$. The logged output, $y_{it} = \ln Y_{it}$, is then derived using the sum of value-added and material inputs: $y_{it} = \ln(VA_{it} + M_{it})$.

B.3 Simulation Procedure

Given simulated data on $\{l_{it}\}_{i=1,t=1}^{i=N,t=T}$, $\{k_{it}\}_{i=1,t=1}^{i=N,t=T}$, $\{\mathbf{W}_t^y\}_{t=1}^{T-1}$, $\{\mathbf{W}_t^l\}_{t=1}^{T-1}$ and $\{\mathbf{W}_t^u\}_{t=1}^T$ and the parameter values for $\{\alpha_0, \alpha_l, \alpha_k, \lambda, \beta_l, \rho_1, \mu, \sigma_\xi, \sigma_v\}$, the data used for the simulations are generated as follows:

1. Set $\omega_{i,t-1} = 0$, for $t = 1$.
2. Generate $va_{i,t-1}$ based on the simulated $l_{i,t-1}$ and $k_{i,t-1}$, the parameter values for $\{\alpha_0, \alpha_l, \alpha_k\}$, the productivity $\omega_{i,t-1}$, and the random draw of $\xi_{i,t-1}$ from a Normal distribution with mean 0 and variance σ_ξ^2 .
3. Set $m_{i,t-1}$ according to Equation (38) and derive $y_{i,t-1} = \ln(VA_{i,t-1} + M_{i,t-1})$.
4. Generate ω_{it} based on Equations (2)–(3), given $y_{i,t-1}$, simulated data on $\{\mathbf{W}_t^y\}_{t=1}^{T-1}$, $\{\mathbf{W}_t^l\}_{t=1}^{T-1}$ and $\{\mathbf{W}_t^u\}_{t=1}^T$, the parameter values for $\{\lambda, \beta_l, \rho_1, \mu\}$, and the random draw of v_{it} from a Normal distribution with mean 0 and variance σ_v^2 .
5. Iterate Steps 2–4 to generate simulated data on $\{va_{it}\}_{i=1,t=1}^{i=N,t=T}$, $\{m_{it}\}_{i=1,t=1}^{i=N,t=T}$, $\{y_{it}\}_{i=1,t=1}^{i=N,t=T}$, and $\{\omega_{it}\}_{i=1,t=1}^{i=N,t=T}$.

Table 1: BSJBSA and TSR Matching Percentage

Sample	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
# firms in BSJBSA	29096	29570	30647	30584	30217	30180	30231	30151	29530	29780
# TSR firms matched	26947	27559	28486	28557	28237	28263	28196	28448	27715	27978
Percentage	92.61	93.20	92.95	93.37	93.45	93.65	93.27	94.35	93.85	93.95

Notes: This table reports the percentage of firms in BSJBSA that are matched with its counterpart in TSR. The BSJBSA set of firms is used as the denominator, since it provides the required firm-level variables for productivity estimations.

Table 2: Summary Statistics (for 2015 cross section)

(a) BSJBSA-TSR Linked Sample					
	Observations	Mean	Std	Min	Max
Labor headcounts	29044	490.48	1940.36	50	130725
Labor hours	29044	846896.70	3288626	67141.72	206818495
Real capital	18580	7205.55	75883.27	0	5394504
Real spending on intermediate inputs	29044	18826.32	127064.81	-4802.70	9208360
Real revenue	29044	21954.38	137121.55	0.93	1.06+07
Real valued added	29044	3109.33	15810.43	-51864.47	1417696
Real R&D expenditure	29044	398.37	7788.43	0	679568.72
Export status	29044	0.23	0.42	0	1
Number of customers	27788	6.77	5.54	0	24
Number of suppliers	27788	6.73	4.75	0	24
Number of customers existing in BSJBSA	27788	4.09	3.95	0	23
Number of suppliers existing in BSJBSA	27788	3.73	3.17	0	22
(b) Estimation Sample					
	Observations	Mean	Std	Min	Max
Labor headcounts	14178	567.81	2152.05	50	81740
Labor hours	14178	997316.94	3794994.36	70234.09	162990234
Real capital	14178	7767.91	80312.97	0.17	5394504
Real spending on intermediate inputs	14178	24580.23	155456.49	12.36	9208361
Real revenue	14178	28465.38	169336.04	123.84	10620083
Real valued added	14178	3912.00	19523.18	9.67	1417696
Real R&D expenditure	14178	585.11	8996.03	0	601742.00
Export status	14178	0.27	0.44	0	1
Number of customers	13757	7.32	5.71	0	24
Number of suppliers	13757	7.43	4.92	0	24
Number of customers existing in BSJBSA	13757	4.52	4.12	0	23
Number of suppliers existing in BSJBSA	13757	4.24	3.34	0	22

Notes: Refer to Section 4 for the measurement of the variables. The revenue, value added, capital, intermediate inputs, and real R&D expenditure are in million Japanese yens. The number of customers / suppliers is identified by the BSJBSA firm's corresponding entries in TSR, which provides the list of a firm's top 24 customers / suppliers. The number of customers / suppliers existing in BSJBSA refers to the subset of a firm's top 24 customers / suppliers listed in TSR that also have firm-level information in BSJBSA.

Table 3: DGP1 – No Spatial Dependence in Productivity

Estimator	N	T-1	Stat.	α_l 0.6	α_k 0.4	λ 0	β_l 0	ρ_1 0.5	μ 0	σ_v^2 0.49
WGMM	500	9	Mean	0.6000	0.4000	-	-	0.4996	-	-
			SD	(0.0051)	(0.0053)	-	-	(0.0138)	-	-
			SE	(0.0052)	(0.0054)	-	-	(0.0140)	-	-
	500	18	Mean	0.6000	0.3998	-	-	0.4996	-	-
			SD	(0.0035)	(0.0038)	-	-	(0.0095)	-	-
			SE	(0.0036)	(0.0038)	-	-	(0.0095)	-	-
	750	9	Mean	0.6002	0.4000	-	-	0.4998	-	-
			SD	(0.0042)	(0.0045)	-	-	(0.0119)	-	-
			SE	(0.0042)	(0.0044)	-	-	(0.0114)	-	-
	750	18	Mean	0.6000	0.4001	-	-	0.5000	-	-
			SD	(0.0029)	(0.0031)	-	-	(0.0077)	-	-
			SE	(0.0030)	(0.0031)	-	-	(0.0077)	-	-
	1000	9	Mean	0.6000	0.3999	-	-	0.4998	-	-
			SD	(0.0037)	(0.0039)	-	-	(0.0098)	-	-
			SE	(0.0037)	(0.0038)	-	-	(0.0099)	-	-
	1000	18	Mean	0.5999	0.4001	-	-	0.4999	-	-
			SD	(0.0025)	(0.0028)	-	-	(0.0069)	-	-
			SE	(0.0026)	(0.0027)	-	-	(0.0067)	-	-
SGMM	500	9	Mean	0.5998	0.4000	0.0000	0.0001	0.4991	-0.0009	0.4881
			SD	(0.0078)	(0.0053)	(0.0023)	(0.0031)	(0.0138)	(0.0322)	(0.0117)
			SE	(0.0076)	(0.0054)	(0.0023)	(0.0031)	(0.0140)	(0.0319)	(0.0103)
	500	18	Mean	0.6001	0.3998	0.0000	0.0000	0.4994	0.0001	0.4892
			SD	(0.0053)	(0.0038)	(0.0016)	(0.0022)	(0.0095)	(0.0232)	(0.0083)
			SE	(0.0053)	(0.0038)	(0.0016)	(0.0022)	(0.0095)	(0.0225)	(0.0073)
	750	9	Mean	0.6004	0.4000	0.0000	-0.0001	0.4996	0.0015	0.4896
			SD	(0.0060)	(0.0046)	(0.0019)	(0.0025)	(0.0119)	(0.0273)	(0.0097)
			SE	(0.0061)	(0.0045)	(0.0019)	(0.0025)	(0.0114)	(0.0274)	(0.0084)
	750	18	Mean	0.6000	0.4001	0.0000	0.0000	0.4998	0.0000	0.4896
			SD	(0.0042)	(0.0031)	(0.0014)	(0.0017)	(0.0077)	(0.0184)	(0.0067)
			SE	(0.0042)	(0.0031)	(0.0014)	(0.0017)	(0.0077)	(0.0183)	(0.0060)
	1000	9	Mean	0.5999	0.3999	0.0001	0.0000	0.4995	-0.0005	0.4892
			SD	(0.0059)	(0.0039)	(0.0019)	(0.0024)	(0.0098)	(0.0227)	(0.0084)
			SE	(0.0057)	(0.0038)	(0.0018)	(0.0024)	(0.0099)	(0.0226)	(0.0073)
	1000	18	Mean	0.6000	0.4001	0.0000	-0.0001	0.4997	0.0004	0.4898
			SD	(0.0038)	(0.0028)	(0.0013)	(0.0016)	(0.0069)	(0.0159)	(0.0058)
			SE	(0.0040)	(0.0027)	(0.0013)	(0.0017)	(0.0067)	(0.0160)	(0.0052)

Notes: For each DGP, 1000 simulated samples are drawn and estimated. We report the mean (Mean) and the standard deviation (SD) of the parameter point estimates across the 1000 Monte Carlo simulations, together with the average of the estimated standard errors (SE) derived from the variance-covariance matrices of the estimators across the 1000 Monte Carlo simulations. The parameter values used in the DGPs are listed in the first row of the table.

Table 4: DGP2 – Spatial Dependence in Productivity via Lagged Outputs and Lagged Labor Inputs of Related Firms

Estimator	N	T-1	Stat.	α_l 0.6	α_k 0.4	λ 0.01	β_l 0.01	ρ_1 0.5	μ 0	σ_v^2 0.49
WGMM	500	9	Mean	0.6229	0.4002	-	-	0.5085	-	-
			SD	(0.0052)	(0.0053)	-	-	(0.0138)	-	-
			SE	(0.0052)	(0.0054)	-	-	(0.0138)	-	-
	500	18	Mean	0.6229	0.4002	-	-	0.5094	-	-
			SD	(0.0035)	(0.0038)	-	-	(0.0095)	-	-
			SE	(0.0037)	(0.0038)	-	-	(0.0094)	-	-
	750	9	Mean	0.6224	0.4009	-	-	0.5073	-	-
			SD	(0.0042)	(0.0045)	-	-	(0.0117)	-	-
			SE	(0.0042)	(0.0045)	-	-	(0.0113)	-	-
	750	18	Mean	0.6218	0.4008	-	-	0.5096	-	-
			SD	(0.0029)	(0.0031)	-	-	(0.0078)	-	-
			SE	(0.0030)	(0.0031)	-	-	(0.0077)	-	-
	1000	9	Mean	0.6214	0.4007	-	-	0.5059	-	-
			SD	(0.0037)	(0.0039)	-	-	(0.0098)	-	-
			SE	(0.0037)	(0.0038)	-	-	(0.0098)	-	-
	1000	18	Mean	0.6214	0.4008	-	-	0.5067	-	-
			SD	(0.0025)	(0.0028)	-	-	(0.0069)	-	-
			SE	(0.0026)	(0.0027)	-	-	(0.0066)	-	-
SGMM	500	9	Mean	0.5999	0.4000	0.0100	0.0101	0.4991	-0.0009	0.4881
			SD	(0.0078)	(0.0053)	(0.0023)	(0.0031)	(0.0136)	(0.0322)	(0.0117)
			SE	(0.0076)	(0.0054)	(0.0023)	(0.0031)	(0.0139)	(0.0319)	(0.0103)
	500	18	Mean	0.6001	0.3998	0.0100	0.0100	0.4994	0.0001	0.4892
			SD	(0.0053)	(0.0038)	(0.0016)	(0.0022)	(0.0094)	(0.0232)	(0.0083)
			SE	(0.0053)	(0.0038)	(0.0016)	(0.0022)	(0.0094)	(0.0225)	(0.0073)
	750	9	Mean	0.6004	0.4000	0.0100	0.0099	0.4996	0.0015	0.4896
			SD	(0.0060)	(0.0046)	(0.0019)	(0.0025)	(0.0117)	(0.0273)	(0.0097)
			SE	(0.0061)	(0.0045)	(0.0019)	(0.0026)	(0.0113)	(0.0274)	(0.0084)
	750	18	Mean	0.6000	0.4001	0.0100	0.0100	0.4998	0.0000	0.4896
			SD	(0.0042)	(0.0031)	(0.0014)	(0.0017)	(0.0077)	(0.0184)	(0.0066)
			SE	(0.0042)	(0.0031)	(0.0014)	(0.0017)	(0.0077)	(0.0183)	(0.0060)
	1000	9	Mean	0.5999	0.3999	0.0101	0.0100	0.4995	-0.0005	0.4892
			SD	(0.0059)	(0.0039)	(0.0019)	(0.0024)	(0.0097)	(0.0228)	(0.0084)
			SE	(0.0057)	(0.0038)	(0.0018)	(0.0024)	(0.0098)	(0.0226)	(0.0073)
	1000	18	Mean	0.6000	0.4001	0.0100	0.0099	0.4998	0.0004	0.4898
			SD	(0.0038)	(0.0028)	(0.0013)	(0.0016)	(0.0069)	(0.0159)	(0.0058)
			SE	(0.0040)	(0.0027)	(0.0013)	(0.0017)	(0.0067)	(0.0160)	(0.0052)

Notes: For each DGP, 1000 simulated samples are drawn and estimated. We report the mean (Mean) and the standard deviation (SD) of the parameter point estimates across the 1000 Monte Carlo simulations, together with the average of the estimated standard errors (SE) derived from the variance-covariance matrices of the estimators across the 1000 Monte Carlo simulations. The parameter values used in the DGPs are listed in the first row of the table.

Table 5: DGP3 – Spatial Dependence in Productivity via Lagged Outputs and Lagged Labor Inputs of Related Firms, and via Productivity Shocks

Estimator	N	T-1	Stat.	α_l 0.6	α_k 0.4	λ 0.01	β_l 0.01	ρ_1 0.5	μ 0.25	σ_v^2 0.49
WGMM	500	9	Mean	0.6229	0.4002	-	-	0.5083	-	-
			SD	(0.0053)	(0.0054)	-	-	(0.0139)	-	-
			SE	(0.0052)	(0.0055)	-	-	(0.0138)	-	-
	500	18	Mean	0.6229	0.4002	-	-	0.5094	-	-
			SD	(0.0036)	(0.0038)	-	-	(0.0096)	-	-
			SE	(0.0037)	(0.0039)	-	-	(0.0094)	-	-
	750	9	Mean	0.6224	0.4009	-	-	0.5072	-	-
			SD	(0.0043)	(0.0046)	-	-	(0.0118)	-	-
			SE	(0.0042)	(0.0045)	-	-	(0.0113)	-	-
	750	18	Mean	0.6218	0.4008	-	-	0.5096	-	-
			SD	(0.0030)	(0.0031)	-	-	(0.0078)	-	-
			SE	(0.0030)	(0.0032)	-	-	(0.0077)	-	-
	1000	9	Mean	0.6214	0.4008	-	-	0.5058	-	-
			SD	(0.0038)	(0.0039)	-	-	(0.0099)	-	-
			SE	(0.0037)	(0.0039)	-	-	(0.0098)	-	-
	1000	18	Mean	0.6214	0.4008	-	-	0.5067	-	-
			SD	(0.0026)	(0.0028)	-	-	(0.0069)	-	-
			SE	(0.0026)	(0.0027)	-	-	(0.0066)	-	-
SGMM	500	9	Mean	0.5998	0.4000	0.0101	0.0101	0.4989	0.2488	0.4881
			SD	(0.0079)	(0.0054)	(0.0023)	(0.0032)	(0.0136)	(0.0322)	(0.0117)
			SE	(0.0077)	(0.0055)	(0.0023)	(0.0032)	(0.0140)	(0.0318)	(0.0103)
	500	18	Mean	0.6001	0.3998	0.0100	0.0100	0.4994	0.2499	0.4892
			SD	(0.0054)	(0.0038)	(0.0016)	(0.0023)	(0.0095)	(0.0231)	(0.0082)
			SE	(0.0054)	(0.0039)	(0.0016)	(0.0022)	(0.0095)	(0.0224)	(0.0073)
	750	9	Mean	0.6004	0.3999	0.0100	0.0099	0.4995	0.2513	0.4896
			SD	(0.0061)	(0.0046)	(0.0019)	(0.0026)	(0.0118)	(0.0273)	(0.0097)
			SE	(0.0062)	(0.0045)	(0.0019)	(0.0026)	(0.0114)	(0.0274)	(0.0084)
	750	18	Mean	0.6000	0.4001	0.0100	0.0100	0.4998	0.2499	0.4896
			SD	(0.0043)	(0.0031)	(0.0014)	(0.0017)	(0.0077)	(0.0184)	(0.0066)
			SE	(0.0043)	(0.0032)	(0.0014)	(0.0017)	(0.0077)	(0.0183)	(0.0060)
	1000	9	Mean	0.5999	0.3999	0.0101	0.0100	0.4994	0.2493	0.4892
			SD	(0.0060)	(0.0039)	(0.0019)	(0.0024)	(0.0098)	(0.0227)	(0.0084)
			SE	(0.0057)	(0.0039)	(0.0018)	(0.0024)	(0.0099)	(0.0226)	(0.0073)
	1000	18	Mean	0.6001	0.4001	0.0100	0.0100	0.4997	0.2503	0.4898
			SD	(0.0039)	(0.0028)	(0.0013)	(0.0016)	(0.0069)	(0.0158)	(0.0058)
			SE	(0.0041)	(0.0027)	(0.0013)	(0.0017)	(0.0067)	(0.0160)	(0.0052)

Notes: For each DGP, 1000 simulated samples are drawn and estimated. We report the mean (Mean) and the standard deviation (SD) of the parameter point estimates across the 1000 Monte Carlo simulations, together with the average of the estimated standard errors (SE) derived from the variance-covariance matrices of the estimators across the 1000 Monte Carlo simulations. The parameter values used in the DGPs are listed in the first row of the table.

Table 6: DGP4 – Stronger Spatial Dependence in Productivity via Lagged Outputs and Lagged Labor Inputs of Related Firms, and via Productivity Shocks

Estimator	N	T-1	Var	α_l 0.6	α_k 0.4	λ 0.1	β_l 0.1	ρ_1 0.5	μ 0.25	σ_v^2 0.49
WGMM	500	9	Mean	0.8628	0.4100	-	-	0.7129	-	-
			SD	(0.0056)	(0.0056)	-	-	(0.0063)	-	-
			SE	(0.0070)	(0.0069)	-	-	(0.0090)	-	-
	500	18	Mean	0.8497	0.4115	-	-	0.7573	-	-
			SD	(0.0039)	(0.0040)	-	-	(0.0044)	-	-
			SE	(0.0050)	(0.0046)	-	-	(0.0062)	-	-
	750	9	Mean	0.8536	0.4139	-	-	0.6937	-	-
			SD	(0.0047)	(0.0047)	-	-	(0.0053)	-	-
			SE	(0.0055)	(0.0056)	-	-	(0.0075)	-	-
	750	18	Mean	0.8382	0.4123	-	-	0.7581	-	-
			SD	(0.0032)	(0.0032)	-	-	(0.0034)	-	-
			SE	(0.0040)	(0.0037)	-	-	(0.0051)	-	-
	1000	9	Mean	0.8389	0.4116	-	-	0.6716	-	-
			SD	(0.0041)	(0.0040)	-	-	(0.0051)	-	-
			SE	(0.0047)	(0.0047)	-	-	(0.0068)	-	-
	1000	18	Mean	0.8295	0.4122	-	-	0.7184	-	-
			SD	(0.0028)	(0.0029)	-	-	(0.0034)	-	-
			SE	(0.0033)	(0.0032)	-	-	(0.0048)	-	-
SGMM	500	9	Mean	0.5998	0.4000	0.1001	0.1002	0.4994	0.2484	0.4881
			SD	(0.0079)	(0.0054)	(0.0021)	(0.0034)	(0.0080)	(0.0322)	(0.0107)
			SE	(0.0076)	(0.0055)	(0.0022)	(0.0034)	(0.0086)	(0.0319)	(0.0103)
	500	18	Mean	0.6001	0.3998	0.1000	0.1000	0.4998	0.2497	0.4892
			SD	(0.0054)	(0.0038)	(0.0016)	(0.0025)	(0.0065)	(0.0231)	(0.0076)
			SE	(0.0055)	(0.0039)	(0.0016)	(0.0025)	(0.0065)	(0.0224)	(0.0073)
	750	9	Mean	0.6004	0.3999	0.0999	0.0999	0.5002	0.2510	0.4896
			SD	(0.0060)	(0.0046)	(0.0018)	(0.0026)	(0.0067)	(0.0272)	(0.0090)
			SE	(0.0062)	(0.0045)	(0.0018)	(0.0026)	(0.0068)	(0.0274)	(0.0084)
	750	18	Mean	0.6000	0.4001	0.1000	0.1000	0.4999	0.2498	0.4897
			SD	(0.0042)	(0.0031)	(0.0013)	(0.0020)	(0.0052)	(0.0184)	(0.0061)
			SE	(0.0042)	(0.0032)	(0.0013)	(0.0020)	(0.0053)	(0.0183)	(0.0060)
	1000	9	Mean	0.5998	0.3999	0.1001	0.1001	0.4995	0.2491	0.4893
			SD	(0.0059)	(0.0039)	(0.0017)	(0.0026)	(0.0063)	(0.0227)	(0.0079)
			SE	(0.0057)	(0.0039)	(0.0017)	(0.0026)	(0.0062)	(0.0226)	(0.0073)
	1000	18	Mean	0.6000	0.4001	0.1000	0.1000	0.5000	0.2502	0.4898
			SD	(0.0039)	(0.0028)	(0.0013)	(0.0018)	(0.0047)	(0.0158)	(0.0054)
			SE	(0.0040)	(0.0027)	(0.0012)	(0.0019)	(0.0047)	(0.0160)	(0.0052)

Notes: For each DGP, 1000 simulated samples are drawn and estimated. We report the mean (Mean) and the standard deviation (SD) of the parameter point estimates across the 1000 Monte Carlo simulations, together with the average of the estimated standard errors (SE) derived from the variance-covariance matrices of the estimators across the 1000 Monte Carlo simulations. The parameter values used in the DGPs are listed in the first row of the table.

Table 7: DGP5 – Negative Spatial Dependence in Productivity via Lagged Outputs and Lagged Labor Inputs of Related Firms, and via Productivity Shocks

Estimator	N	T-1	Var	α_l 0.6	α_k 0.4	λ -0.1	β_l -0.1	ρ_1 0.5	μ 0.25	σ_v^2 0.49
WGMM	500	9	Mean	0.3656	0.3929	-	-	0.6527	-	-
			SD	(0.0054)	(0.0056)	-	-	(0.0078)	-	-
			SE	(0.0062)	(0.0066)	-	-	(0.0098)	-	-
	500	18	Mean	0.3748	0.3905	-	-	0.6969	-	-
			SD	(0.0038)	(0.0040)	-	-	(0.0058)	-	-
			SE	(0.0044)	(0.0044)	-	-	(0.0071)	-	-
	750	9	Mean	0.3728	0.3888	-	-	0.6334	-	-
			SD	(0.0045)	(0.0047)	-	-	(0.0070)	-	-
			SE	(0.0050)	(0.0053)	-	-	(0.0081)	-	-
	750	18	Mean	0.3813	0.3901	-	-	0.7058	-	-
			SD	(0.0031)	(0.0032)	-	-	(0.0043)	-	-
			SE	(0.0036)	(0.0036)	-	-	(0.0057)	-	-
	1000	9	Mean	0.3801	0.3902	-	-	0.6193	-	-
			SD	(0.0040)	(0.0041)	-	-	(0.0058)	-	-
			SE	(0.0043)	(0.0045)	-	-	(0.0071)	-	-
	1000	18	Mean	0.3863	0.3900	-	-	0.6610	-	-
			SD	(0.0027)	(0.0029)	-	-	(0.0043)	-	-
			SE	(0.0030)	(0.0031)	-	-	(0.0052)	-	-
SGMM	500	9	Mean	0.5998	0.4000	-0.0999	-0.0999	0.5000	0.2484	0.4882
			SD	(0.0078)	(0.0054)	(0.0029)	(0.0034)	(0.0089)	(0.0322)	(0.0107)
			SE	(0.0076)	(0.0055)	(0.0030)	(0.0034)	(0.0091)	(0.0319)	(0.0103)
	500	18	Mean	0.6001	0.3998	-0.1000	-0.1001	0.4996	0.2497	0.4893
			SD	(0.0054)	(0.0038)	(0.0022)	(0.0025)	(0.0070)	(0.0231)	(0.0076)
			SE	(0.0054)	(0.0039)	(0.0022)	(0.0025)	(0.0070)	(0.0224)	(0.0073)
	750	9	Mean	0.6003	0.3999	-0.1001	-0.1001	0.4995	0.2510	0.4896
			SD	(0.0061)	(0.0046)	(0.0024)	(0.0027)	(0.0074)	(0.0273)	(0.0090)
			SE	(0.0062)	(0.0045)	(0.0024)	(0.0027)	(0.0072)	(0.0274)	(0.0084)
	750	18	Mean	0.6000	0.4001	-0.1000	-0.1000	0.5000	0.2498	0.4896
			SD	(0.0042)	(0.0031)	(0.0019)	(0.0020)	(0.0057)	(0.0184)	(0.0063)
			SE	(0.0042)	(0.0032)	(0.0019)	(0.0020)	(0.0057)	(0.0183)	(0.0060)
	1000	9	Mean	0.5999	0.3999	-0.0999	-0.1000	0.5001	0.2491	0.4895
			SD	(0.0059)	(0.0039)	(0.0024)	(0.0026)	(0.0064)	(0.0227)	(0.0077)
			SE	(0.0057)	(0.0039)	(0.0023)	(0.0026)	(0.0066)	(0.0226)	(0.0073)
	1000	18	Mean	0.6000	0.4001	-0.1001	-0.1001	0.4998	0.2502	0.4899
			SD	(0.0038)	(0.0028)	(0.0018)	(0.0019)	(0.0051)	(0.0158)	(0.0055)
			SE	(0.0040)	(0.0027)	(0.0018)	(0.0019)	(0.0051)	(0.0160)	(0.0052)

Notes: For each DGP, 1000 simulated samples are drawn and estimated. We report the mean (Mean) and the standard deviation (SD) of the parameter point estimates across the 1000 Monte Carlo simulations, together with the average of the estimated standard errors (SE) derived from the variance-covariance matrices of the estimators across the 1000 Monte Carlo simulations. The parameter values used in the DGPs are listed in the first row of the table.

Table 8: Production Function Estimations (Japanese Firms 2009–2018)

	SGMM			WGMM
	(1)	(2)	(3)	(4)
$\mathbf{W}_{t-1}^y$	buyer-seller*CZ	buyer-seller*CZ	buyer-seller*CZ	
$\mathbf{W}_{t-1}^\Omega$	CZ	CZ	CZ	
$\mathbf{W}_t^u$	CZ	buyer-seller*CZ	buyer-seller	
α_0	-8.0852 (0.1115)	-7.9469 (0.1096)	-7.9957 (0.1093)	-4.4705 (0.0683)
α_l	0.8110 (0.0041)	0.7909 (0.0047)	0.7969 (0.0048)	0.7202 (0.0082)
α_k	0.1386 (0.0022)	0.1685 (0.0026)	0.1659 (0.0026)	0.2230 (0.0050)
ρ_1	0.7724 (0.0036)	0.7480 (0.0041)	0.7472 (0.0042)	0.8265 (0.0042)
$\beta_{exporter}$	0.1898 (0.0032)	0.1789 (0.0034)	0.1795 (0.0035)	0.1862 (0.0032)
$\beta_{R\&D}$	5.13E-07 (1.27E-07)	7.95E-07 (1.46E-07)	8.63E-07 (1.40E-07)	1.83E-06 (2.01E-07)
λ	0.0053 (0.0003)	0.0055 (0.0003)	0.0057 (0.0003)	
β	0.0550 (0.0020)	0.0601 (0.0022)	0.0596 (0.0022)	
μ	0.6769 (0.0062)	0.2788 (0.0042)	0.2979 (0.0034)	
σ_v^2	0.0210 (0.0002)	0.0246 (0.0002)	0.0251 (0.0002)	
no. of observations	141,780	141,780	141,780	141,780
no. of firms	14,178	14,178	14,178	14,178
industry FE	Yes	Yes	Yes	Yes

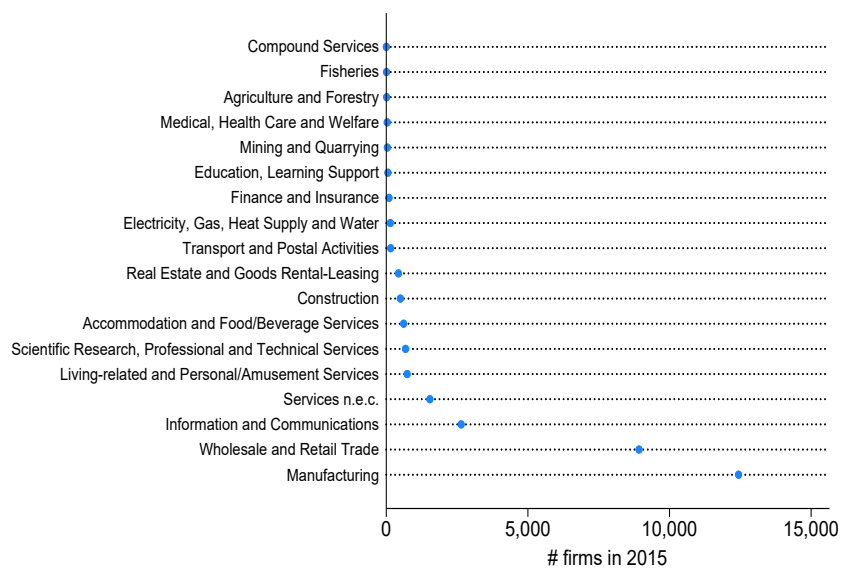
Notes: This table reports the estimations of Equations (1)–(3) based on the estimation methodology laid out in Section 3 and the connectivity matrices defined in Section 4.2. The function $h(l_{it}, k_{it}, m_{it})$ in Equation (9) is approximated by a second-order polynomial function: $l_{it}^p k_{it}^q m_{it}^r$ for $p + q + r \leq 2$, with nonnegative integers p , q and r . The slope coefficient estimates δ are omitted from the table above. The function $f(\nu)$ in Equation (12) is assumed to be of first order as in the conventional estimator. The list of instruments used for SGMM is: $\mathcal{Z}_{t,I} = (\iota_N, \mathbf{c}_t, \mathbf{c}_{t-1})$ and $\mathcal{Z}_{t,II} = (\iota_N, k_t, \mathbf{c}_{t-1}, \mathbf{W}_{t-1}^y y_{t-1}, \mathbf{W}_{t-1}^l l_{t-1}, (\mathbf{W}_{t-1}^y)^2 y_{t-1}, (\mathbf{W}_{t-1}^l)^2 l_{t-1})$. The list of instruments used for WGMM is the same as those for SGMM, but excluding the related firms' outputs and labor inputs $(\mathbf{W}_{t-1}^y y_{t-1}, (\mathbf{W}_{t-1}^y)^2 y_{t-1}, \mathbf{W}_{t-1}^l l_{t-1}, (\mathbf{W}_{t-1}^l)^2 l_{t-1})$. We iterate the efficient GMM estimation procedure until the set of parameter estimates converges.

Table 9: Production Function Estimations (Japanese Firms 2009–2018) with Restricted Specifications

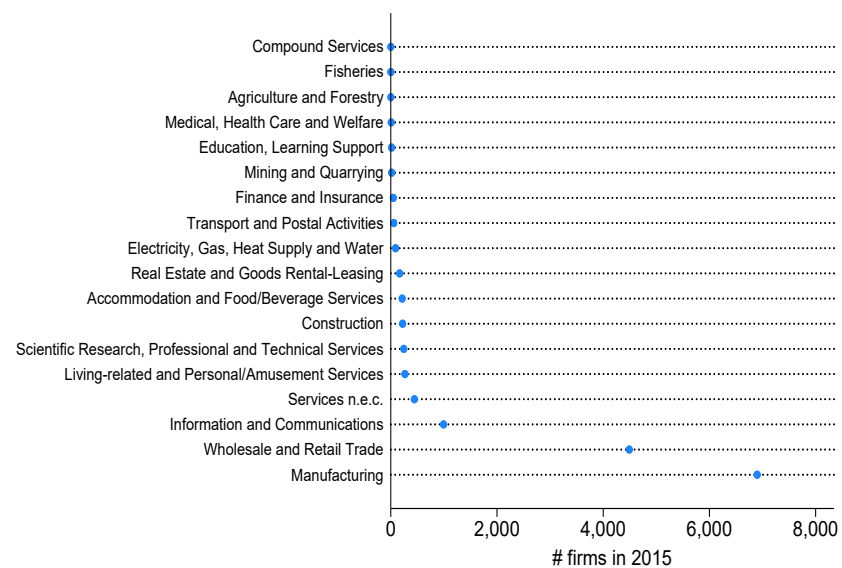
	benchmark	(1)	(2)	(3)	(4)	(5)	(6)
$\mathbf{W}_{t-1}^y$: buyer-seller*CZ	Yes	Yes	No	No	Yes	Yes	No
$\mathbf{W}_{t-1}^\Omega$: CZ	Yes	No	Yes	No	Yes	No	Yes
$\mathbf{W}_t^u$: buyer-seller*CZ	Yes	Yes	Yes	Yes	No	No	No
α_0	-7.9469 (0.1096)	-5.2952 (0.0385)	-8.5280 (0.1100)	-5.4494 (0.0406)	-8.3118 (0.1473)	-4.2629 (0.0667)	-8.6608 (0.1422)
α_l	0.7909 (0.0047)	0.8324 (0.0040)	0.8243 (0.0041)	0.8580 (0.0041)	0.5466 (0.0144)	0.6762 (0.0081)	0.6230 (0.0099)
α_k	0.1685 (0.0026)	0.1334 (0.0022)	0.1406 (0.0022)	0.1198 (0.0022)	0.2690 (0.0065)	0.2447 (0.0047)	0.2804 (0.0055)
ρ_1	0.7480 (0.0041)	0.7801 (0.0036)	0.7824 (0.0035)	0.8098 (0.0034)	0.8780 (0.0037)	0.8246 (0.0041)	0.8272 (0.0041)
$\beta_{exporter}$	0.1789 (0.0034)	0.1833 (0.0032)	0.1998 (0.0031)	0.1981 (0.0031)	0.1736 (0.0032)	0.1720 (0.0032)	0.1863 (0.0032)
$\beta_{R\&D}$	7.95E-07 (1.46E-07)	1.05E-06 (1.21E-07)	5.09E-07 (1.25E-07)	9.82E-07 (1.14E-07)	1.28E-06 (1.68E-07)	1.22E-06 (1.59E-07)	1.11E-06 (1.63E-07)
λ	0.0055 (0.0003)	0.0068 (0.0003)			0.0051 (0.0002)	0.0067 (0.0003)	
β	0.0601 (0.0022)		0.0583 (0.0019)		0.0510 (0.0017)		0.0676 (0.0022)
μ	0.2788 (0.0042)	0.2376 (0.0040)	0.2249 (0.0044)	0.1882 (0.0041)			
σ_v^2	0.0246 (0.0002)	0.0208 (0.0002)	0.0213 (0.0002)	0.0186 (0.0001)			
no. of observations	141,780	141,780	141,780	141,780	141,780	141,780	141,780
no. of firms	14,178	14,178	14,178	14,178	14,178	14,178	14,178
industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: This table reports the estimations of Equations (1)–(3), but with subsets of parameters restricted to be zeros, based on the estimation methodology laid out in Section 3 and the connectivity matrices defined in Section 4.2. See Table 8 for further details. The list of instruments used for SGMM is: $\mathcal{Z}_{t,I} = (\iota_N, \mathbf{c}_t, \mathbf{c}_{t-1})$ and $\mathcal{Z}_{t,II} = (\iota_N, k_t, \mathbf{c}_{t-1}, \mathbf{W}_{t-1}^y y_{t-1}, \mathbf{W}_{t-1}^l l_{t-1}, (\mathbf{W}_{t-1}^y)^2 y_{t-1}, (\mathbf{W}_{t-1}^l)^2 l_{t-1})$, excluding subsets of the instruments $(\mathbf{W}_{t-1}^y y_{t-1}, (\mathbf{W}_{t-1}^y)^2 y_{t-1}, \mathbf{W}_{t-1}^l l_{t-1}, (\mathbf{W}_{t-1}^l)^2 l_{t-1})$ whose corresponding spatial terms are not included in the estimations. We iterate the efficient GMM estimation procedure until the set of parameter estimates converges.

Figure 1: Number of firms in each industry in 2015

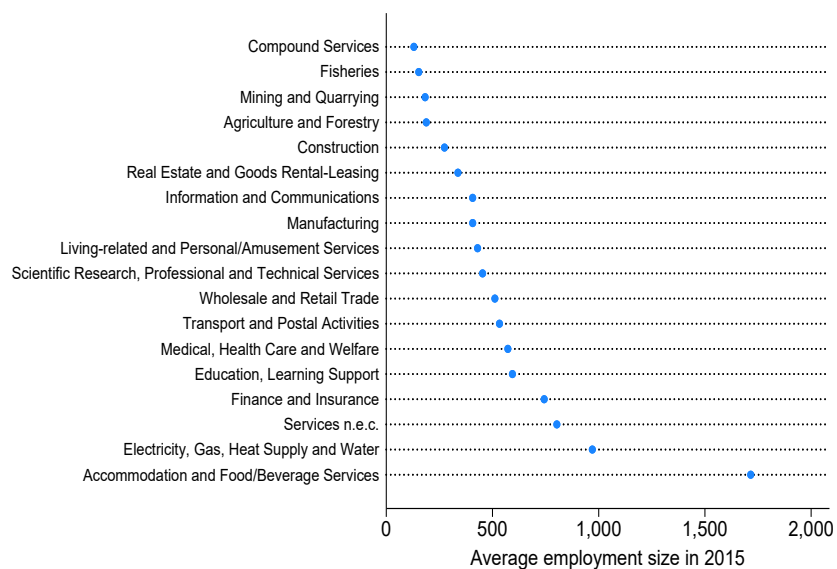


(a) BSJBSA-TSR Linked Sample

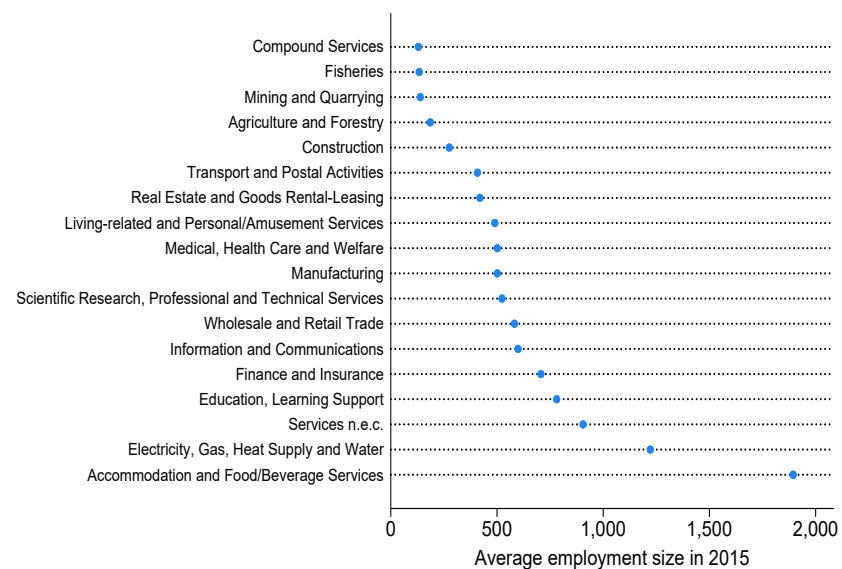


(b) Estimation Sample

Figure 2: Average firm's employment in each industry in 2015

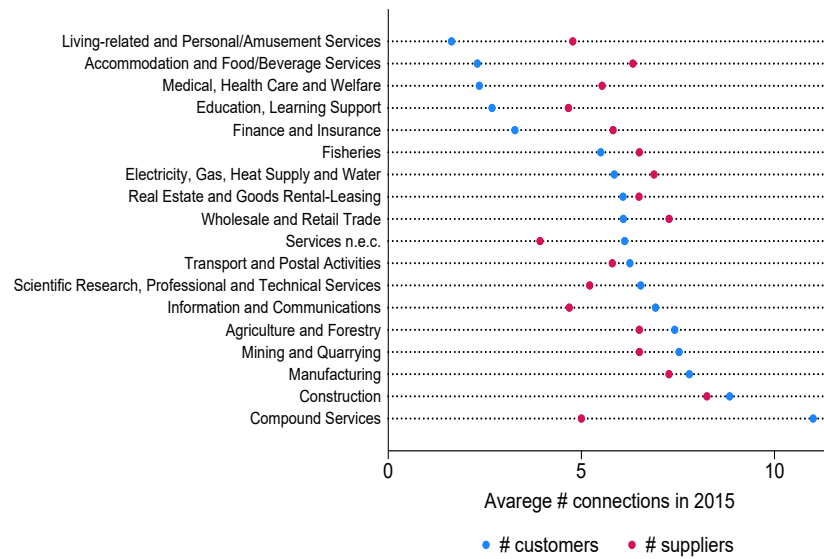


(a) BSJBSA-TSR Linked Sample

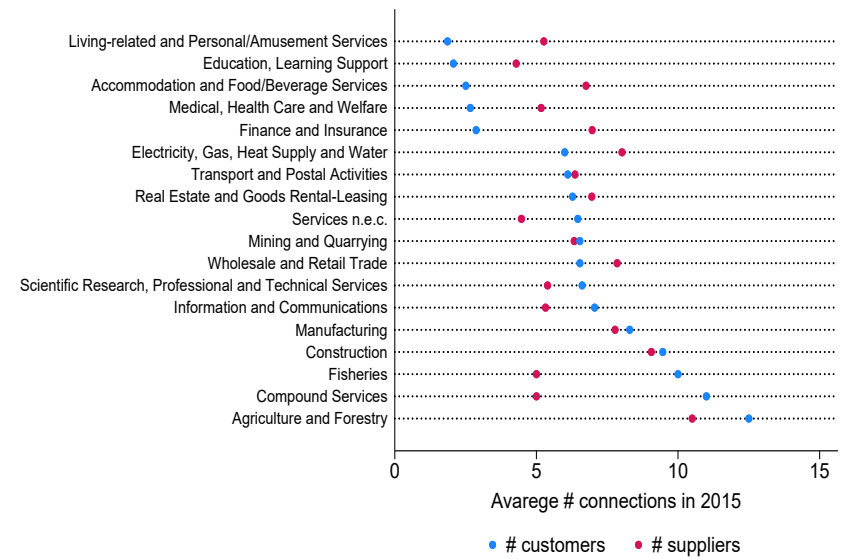


(b) Estimation Sample

Figure 3: Average number of connections in each industry in 2015

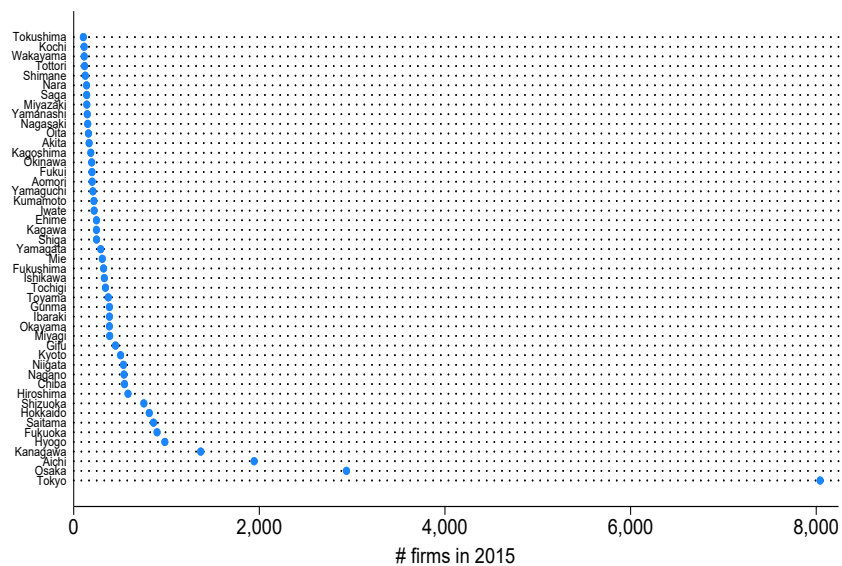


(a) BSJBSA-TSR Linked Sample

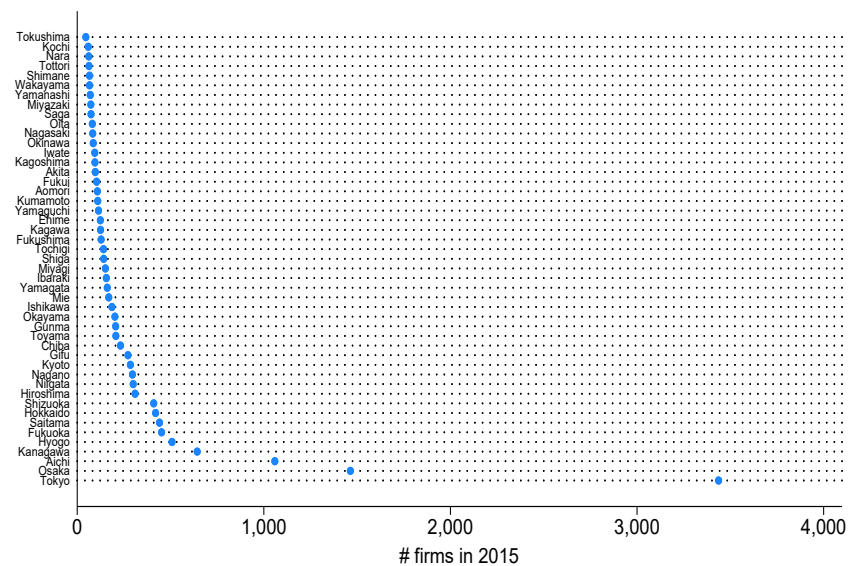


(b) Estimation Sample

Figure 4: Number of firms in each prefecture in 2015

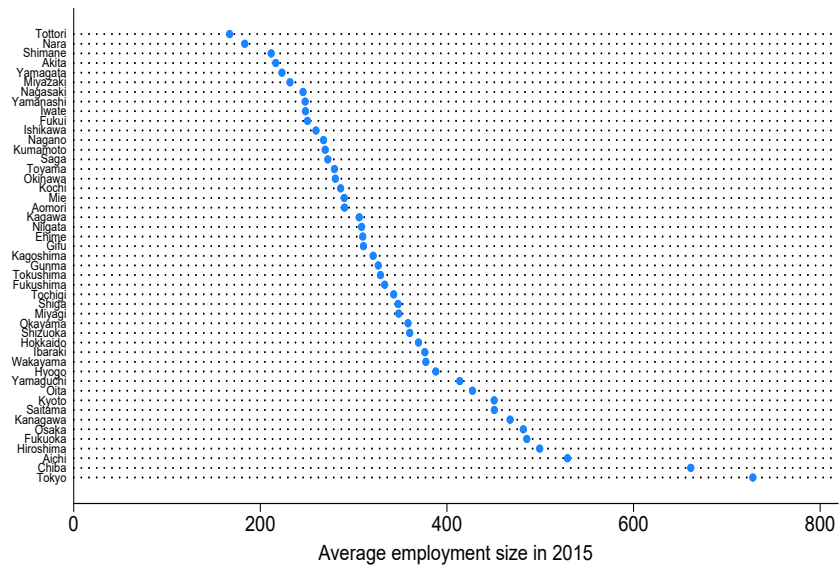


(a) BSJBSA-TSR Linked Sample

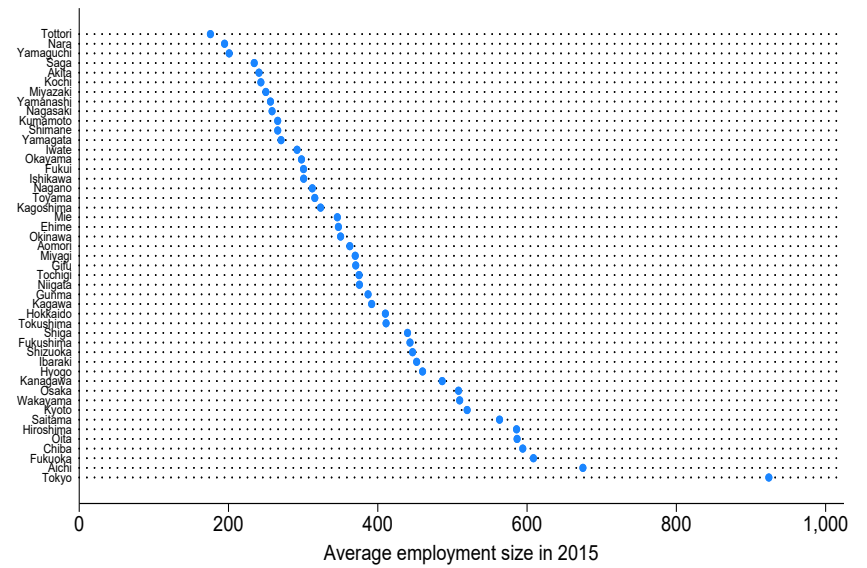


(b) Estimation Sample

Figure 5: Average firm's employment in each prefecture in 2015

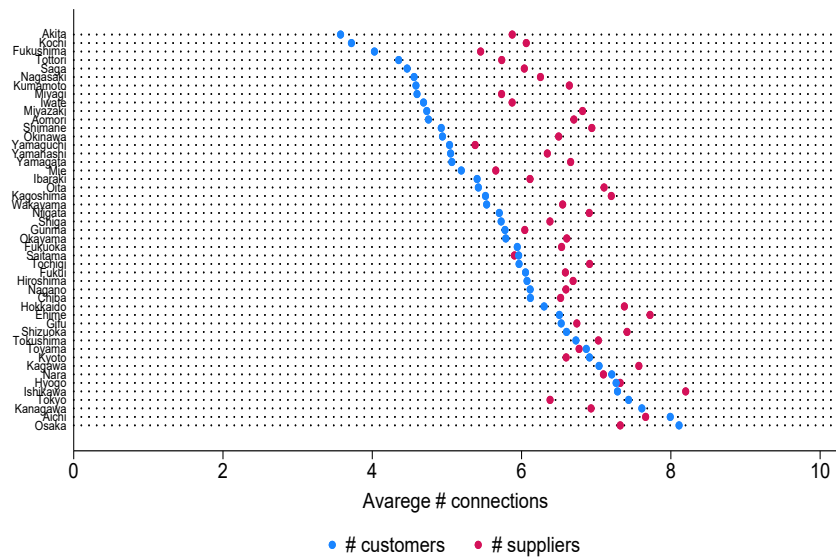


(a) BSJBSA-TSR Linked Sample

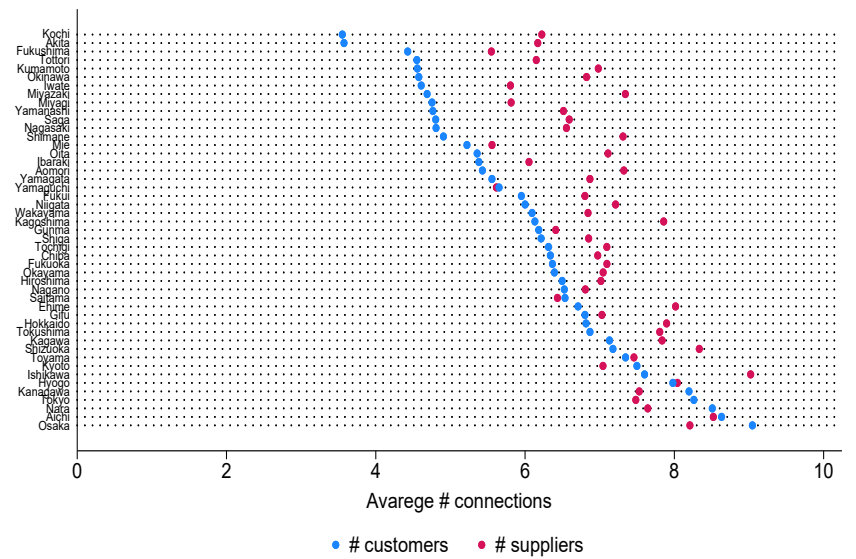


(b) Estimation Sample

Figure 6: Average number of connections in each prefecture in 2015

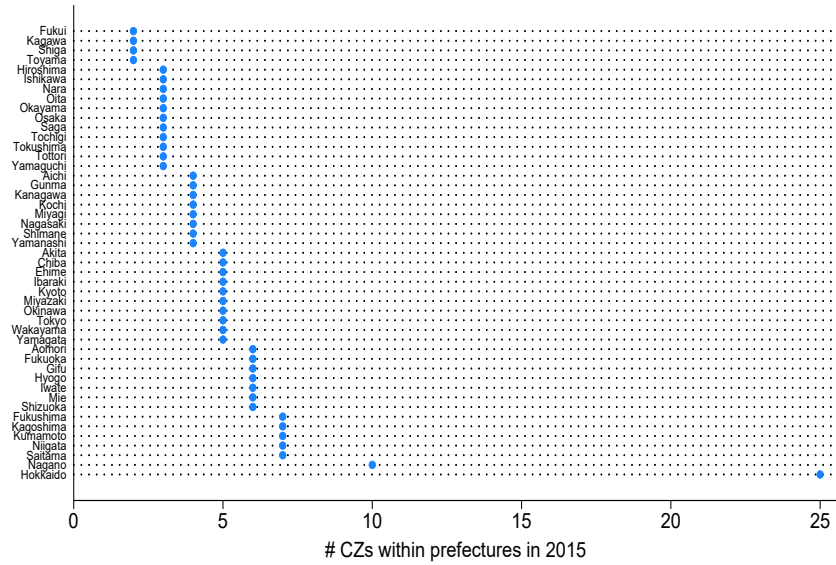


(a) BSJBSA-TSR Linked Sample

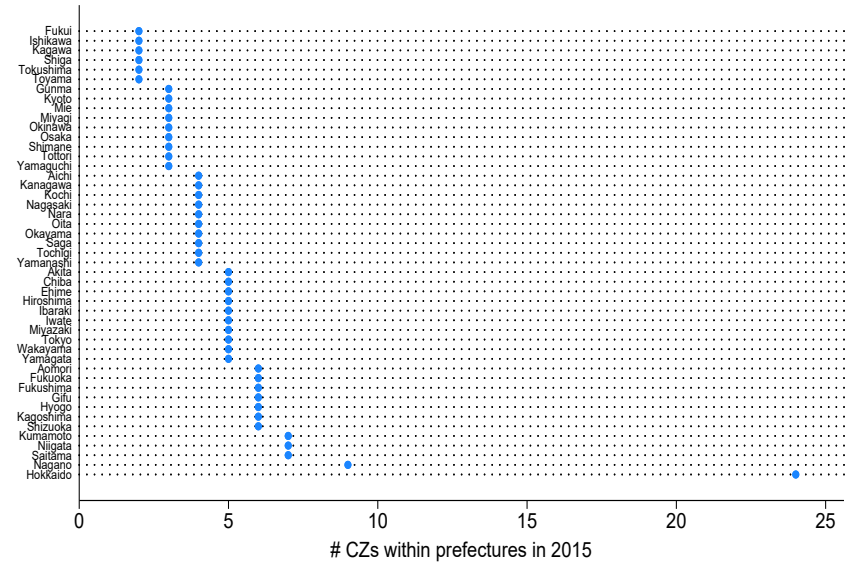


(b) Estimation Sample

Figure 7: Number of commuting zones in each prefecture in 2015



(a) BSJBSA-TSR Linked Sample



(b) Estimation Sample

Figure 8: Number of firms in each commuting zone in 2015

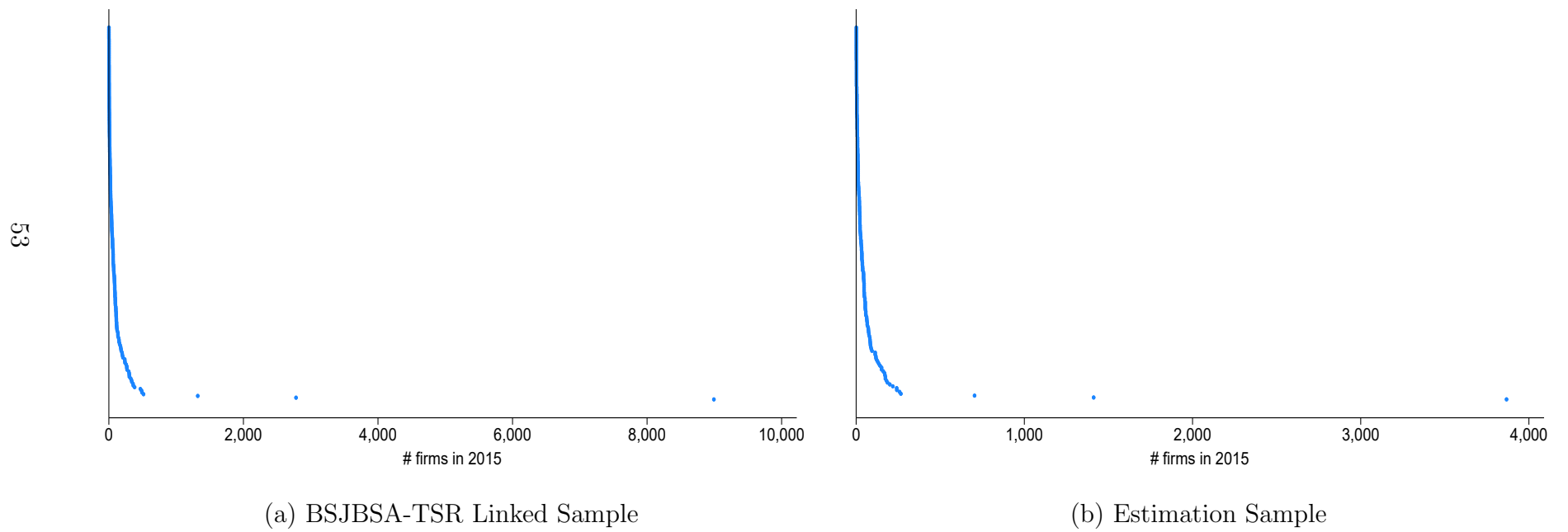


Figure 9: Average firm's employment in each commuting zone in 2015

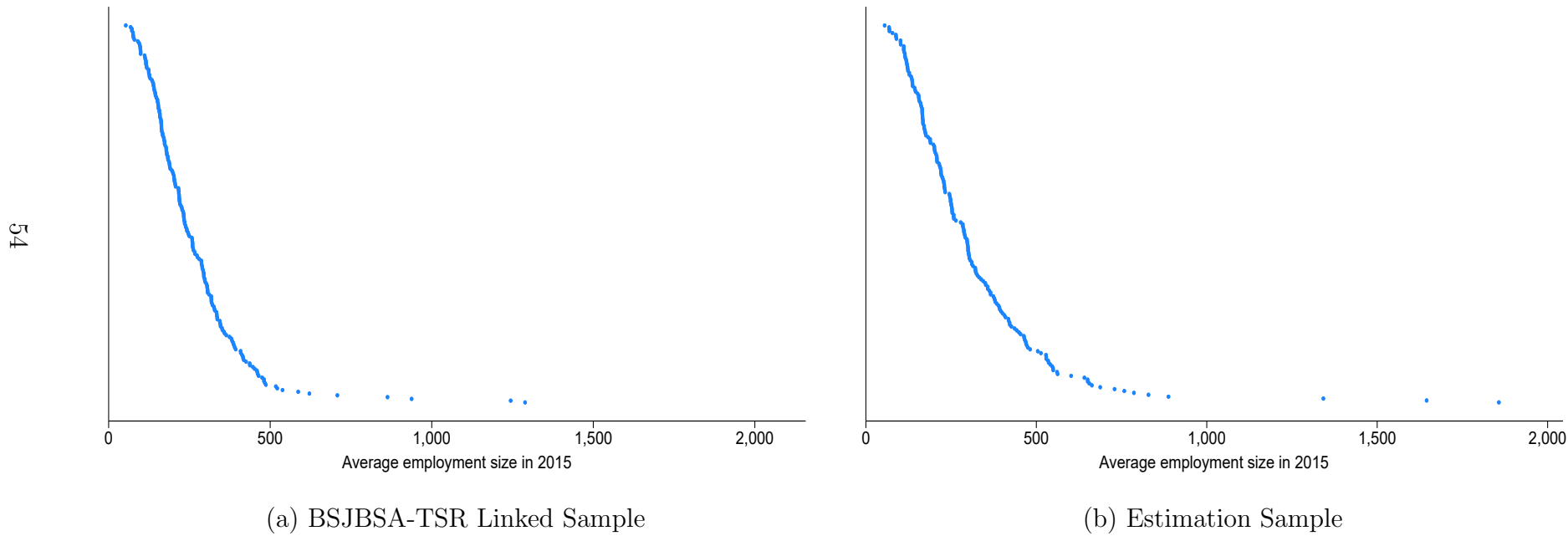
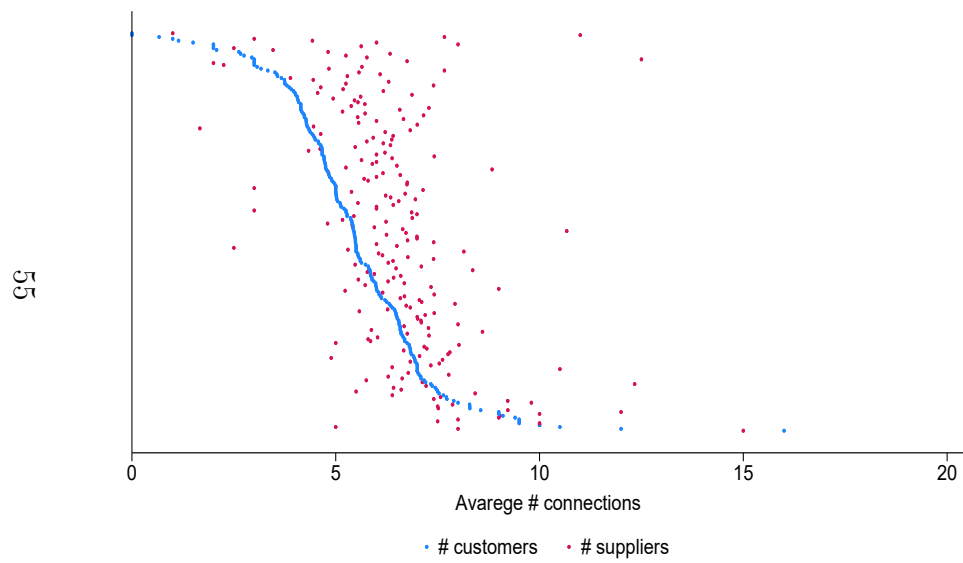
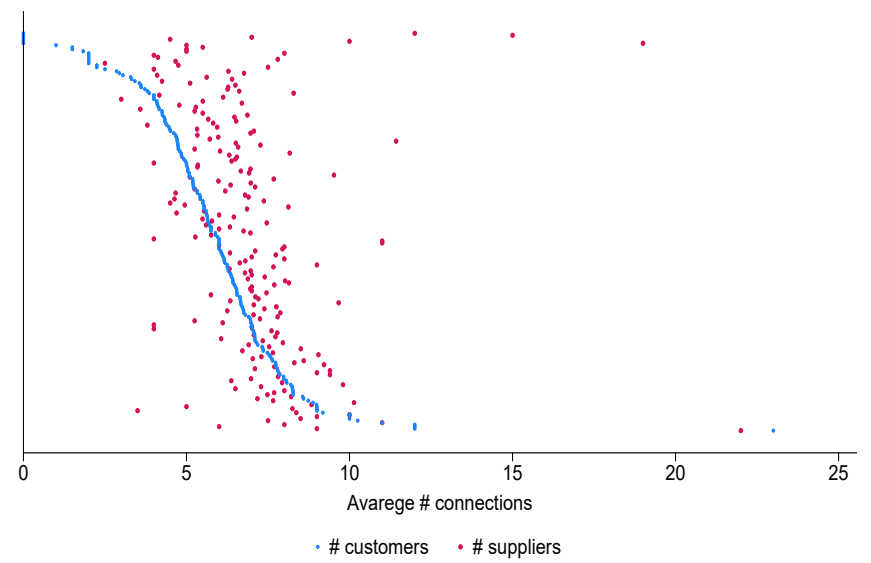


Figure 10: Average number of connections in each commuting zone in 2015



(a) BSJBSA-TSR Linked Sample



(b) Estimation Sample