A Supplement to "Unified M-Estimation of Fixed-Effects Spatial Dynamic Models with Short Panels"

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This supplementary material provides (*i*) theoretical details on the four submodels, i.e., the FE-SDPD model with only SE effect, the FE-SDPD model with only SL effect, the FE-SDPD model with both SE and SL effects, and the FE-SDPD model with both SL and STL effects, (*ii*) a much more comprehensive set of Monte Carlo results, (*iii*) an empirical illustration with matlab codes, and (*iv*) more detailed proofs of the theoretical results.

1 Unified Estimation and Inference for Some Submodels

The general theories and methods introduced in this section can easily be simplified to suit various submodels discussed in Section 2, from which important insights can be gained on the properties of the CQMLE and the proposed M-estimator. The simplified methods are also helpful for practical applications, and allow easy comparisons of our approach with the standard small T or large T approaches if available. We concentrate on the submodels that contain spatial dependence, namely, the FE-SDPD model with only SE dependence, the FE-SDPD model with only SL dependence, the FE-SDPD model with both SL and STL dependence, and the FE-SDPD model with both SL and SE dependence.

The FE-SDPD model with SE effect. Setting $\lambda_1 = \lambda_2 = 0$, Model (3.1) reduces to an FE-SDPD with only SE dependence of a SAR form, which has been rigorously treated in Su and Yang (2015) based on a full QML approach where the initial differences are modeled. It would be certainly interesting to see how the proposed approach compares with this full QML approach. The conditional quasi Gaussian loglikelihood (3.3) simplifies to:

$$\ell_{\texttt{SE}}(\psi) = -\tfrac{n(T-1)}{2}\log(\sigma_v^2) - \tfrac{1}{2}\log|\Omega| - \tfrac{1}{2\sigma_v^2}\Delta u(\theta)'\Omega^{-1}\Delta u(\theta), \tag{1.1}$$

where $\psi = \{\beta', \sigma_v^2, \rho, \lambda_3\}'$ and $\theta = (\beta', \rho)'$ and $u(\theta) = \Delta Y - \rho\Delta Y_{-1} - \Delta X\beta$. Given λ_3, $\ell_{\texttt{SE}}(\psi)$ is maximized at $\tilde\theta(\lambda_3) = (\Delta\mathbb{X}'\Omega^{-1}\Delta\mathbb{X})^{-1}\Delta\mathbb{X}'\Omega\Delta Y$ and $\tilde\sigma_v^2(\lambda_3) = \frac{1}{n(T-1)}\Delta\tilde u'(\lambda_3)\Omega^{-1}\Delta\tilde u(\lambda_3)$, where $\Delta\tilde u(\lambda_3) = \Delta Y - \Delta\mathbb{X}\,\tilde\theta(\lambda_3)$, and $\Delta\mathbb{X} = (\Delta X, \Delta Y_{-1})$. Substituting $\tilde\theta(\lambda_3)$ and $\tilde\sigma_v^2(\lambda_3)$ back into $\ell_{\texttt{SE}}(\psi)$ gives the concentrated quasi loglikelihood function of λ_3,

$$\ell^c_{\texttt{SE}}(\lambda_3) = -\tfrac{n(T-1)}{2}\log(\tilde\sigma_v^2(\lambda_3)) - \tfrac{1}{2}\log|\Omega|. \tag{1.2}$$

Maximizing $\ell^c_{\texttt{SE}}(\lambda_3)$ gives the CQMLE $\tilde\lambda_3$ of λ_3, and thus the CQMLEs $\tilde\theta \equiv \tilde\theta(\tilde\lambda_3)$ and $\tilde\sigma_v^2 \equiv \tilde\sigma_v^2(\tilde\lambda_3)$ of β and σ_v^2, respectively.

Now, $S_{\text{SE}}(\psi) = \frac{\partial}{\partial\psi}\ell_{\text{SE}}(\psi)$ has elements: $\frac{1}{\sigma_v^2}\Delta X'\Omega^{-1}\Delta u(\theta)$, $\frac{1}{2\sigma_v^4}\Delta u(\theta)'\Omega^{-1}\Delta u(\theta) - \frac{n(T-1)}{2\sigma_v^2}$, $\frac{1}{\sigma_v^2}\Delta u(\theta)'\Omega^{-1}\Delta Y_{-1}$, $\frac{1}{2\sigma_v^2}\Delta u(\theta)'(C^{-1}\otimes A_3)\Delta u(\theta) - (T-1)\text{tr}(G_3)$. Only the ρ-element of $\text{E}[S_{\text{SE}}(\psi_0)]$ is non-zero, noting that $\mathbf{D}_{-1}$ in Lemma 3.1 reduces to $D(\rho)\otimes I_n$,

$$\sigma_{v0}^{-2}\text{E}(\Delta u'\Omega^{-1}Y_{-1}) = -n\text{tr}[C^{-1}D(\rho_0)], \tag{1.3}$$

where the $(T-1)\times(T-1)$ matrix $D(\rho)$ has the following expression:

$$D(\rho) = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ \rho-2 & 1 & \cdots & 0 & 0 \\ (1-\rho)^2 & \rho-2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho^{T-5}(1-\rho)^2 & \rho^{T-6}(1-\rho)^2 & \cdots & 1 & 0 \\ \rho^{T-4}(1-\rho)^2 & \rho^{T-5}(1-\rho)^2 & \cdots & \rho-2 & 1 \end{pmatrix}.$$

It is easy to see that, when $|\rho|<1$, $\text{tr}[C^{-1}D(\rho)] = \frac{1}{1-\rho} - \frac{1-\rho^T)}{T(1-\rho)^2}$, a result that has appeared in the literature of non-spatial dynamic panel data models (e.g., Nickell, 1981; Lancaster, 2002; and Alvarez and Arellano, 2004), and was derived from different angles. Our derivation shows that the result holds when the errors are non-spherical in cross-sectional dimension.

The result suggests that the ρ-element of the conditional quasi score function is such that $\text{plim}_{n\to\infty}\frac{1}{nT\sigma_v^2}\Delta u'\Omega^{-1}\Delta Y_{-1} \neq 0$, unless T also approaches ∞. A necessary condition for consistency is violated, and hence the conditional QMLE of ρ is inconsistent when T is fixed. This result also suggests that even under the large n and large T set up, the conditional QMLE of ρ would incur a bias of order $O(T^{-1})$ as shown in Hahn and Kuersteiner (2002) for the regular DPD model. With (1.3) and the fact that other score elements have zero expectation, the adjusted quasi score becomes

$$S_{\text{SE}}^*(\psi) = \begin{cases} \frac{1}{\sigma_v^2}\Delta X'\Omega^{-1}\Delta u(\theta), \\ \frac{1}{2\sigma_v^4}\Delta u(\theta)'\Omega^{-1}\Delta u(\theta) - \frac{n(T-1)}{2\sigma_v^2}, \\ \frac{1}{\sigma_v^2}\Delta u(\theta)'\Omega^{-1}\Delta Y_{-1} + n\text{tr}(C^{-1}D(\rho)), \\ \frac{1}{2\sigma_v^2}\Delta u(\theta)'(C^{-1}\otimes A_3)\Delta u(\theta) - (T-1)\text{tr}(G_3). \end{cases} \tag{1.4}$$

Solving $S_{\text{SE}}^*(\psi) = 0$ leads to the M-estimator $\hat\psi_{\text{M}}$ of ψ. This root-finding process can be simplified by first solving the equations for β and σ_v^2, given $\delta = (\rho, \lambda_3)'$, resulting in the constrained M-estimators of β and σ_v^2 as $\hat\beta(\delta) = (\Delta X'\Omega^{-1}\Delta X)^{-1}\Delta X'\Omega^{-1}\Delta Y(\rho)$ and $\hat\sigma_v^2(\delta) = \frac{1}{n(T-1)}\Delta\hat u(\delta)'\Omega^{-1}\Delta\hat u(\delta)$, where $\Delta Y(\rho) = \Delta Y - \rho\Delta Y_{-1}$ and $\Delta\hat u(\delta) = \Delta u(\hat\beta(\delta), \rho)$. Substituting $\hat\beta(\delta)$ and $\hat\sigma_v^2(\delta)$ into the last two components of the AQS function in (1.4) gives the concentrated AQS functions:

$$S_{\text{SE}}^{*c}(\delta) = \begin{cases} \frac{1}{\hat\sigma_{v,\text{M}}^2(\delta)}\Delta\hat u(\delta)'\Omega^{-1}\Delta Y_{-1} + n\text{tr}(C^{-1}D(\rho)), \\ \frac{1}{2\hat\sigma_{v,\text{M}}^2(\delta)}\Delta\hat u(\delta)'(C^{-1}\otimes A_3)\Delta\hat u(\delta) - (T-1)\text{tr}(G_3). \end{cases} \tag{1.5}$$

Solving the resulted concentrated estimating equations, $S_{\text{SE}}^{*c}(\delta) = 0$, we obtain the unconstrained M-estimators $\hat\delta_{\text{M}} = (\hat\rho_{\text{M}}, \hat\lambda_{3,\text{M}})'$ of δ. The unconstrained M-estimators of β and σ_v^2 are thus $\hat{\beta}_{\mathtt{M}} \equiv \hat{\beta}(\hat{\delta}_{\mathtt{M}})$ and $\hat{\sigma}^2_{v,\mathtt{M}} \equiv \hat{\sigma}^2_v(\hat{\delta}_{\mathtt{M}})$.

Compared with the full QML estimation of Su and Yang (2015), the proposed M-estimation, though slightly less efficient, is much simpler as it is free from the specification of the initial conditions, and is thus robust against misspecifications of initial conditions. In contrast, the full QML estimation requires that the process starting time m is known a priori and that the processes evolve in the same manner before and after the data collection. Our Monte Carlo results (Sec. 2) and those in Su and Yang (2015) confirm these points.

The FE-SDPD model with SL effect. Setting $\lambda_2 = \lambda_3 = 0$, Model (3.1) reduces to a FE-SDPD model with only SL dependence. Now, $\psi = (\beta', \sigma_v^2, \rho, \lambda_1)'$. The conditional quasi Gaussian loglikelihood of ψ reduces to:

$$\ell_{\mathtt{SL}}(\psi) = -\tfrac{n(T-1)}{2}\log(\sigma_v^2) + \log|\mathbf{B}_1| - \tfrac{1}{2}\log|\mathbf{C}| - \tfrac{1}{2\sigma_v^2}\Delta v(\theta)'\mathbf{C}^{-1}\Delta v(\theta), \tag{1.6}$$

where $\theta = (\theta_1', \lambda_1)'$, $\theta_1 = (\beta', \rho)'$, and $v(\theta) = \mathbf{B}_1\Delta Y - \rho\Delta Y_{-1} - \Delta X\beta$. Given λ_1, $\ell_{\mathtt{SL}}(\psi)$ is maximized at $\tilde{\theta}_1(\lambda_1) = (\Delta\mathbb{X}'\mathbf{C}^{-1}\Delta\mathbb{X})^{-1}\Delta\mathbb{X}'\mathbf{C}^{-1}\mathbf{B}_1\Delta Y$ and $\tilde{\sigma}_v^2(\lambda_1) = \frac{1}{n(T-1)}\Delta\tilde{v}'(\lambda_1)\mathbf{C}^{-1}\Delta\tilde{v}(\lambda_1)$, where $\Delta\tilde{v}(\lambda_1) = \mathbf{B}_1\Delta Y - \Delta\mathbb{X}\,\tilde{\theta}(\lambda_1)$, and $\Delta\mathbb{X} = (\Delta X, \Delta Y_{-1})$. Substituting $\tilde{\theta}_1(\lambda_1)$ and $\tilde{\sigma}_v^2(\lambda_1)$ back into $\ell_{\mathtt{SL}}(\psi)$ gives the concentrated conditional loglikelihood function of λ_1,

$$\ell^c_{\mathtt{SL}}(\lambda_1) = \log|\mathbf{B}_1| - \tfrac{n(T-1)}{2}\log(\tilde{\sigma}_v^2(\lambda_1)) - \tfrac{1}{2}\log|\mathbf{C}|. \tag{1.7}$$

Maximizing $\ell^c_{\mathtt{SL}}(\lambda_1)$ gives the CQMLE $\tilde{\lambda}_1$ of λ_1, and thus the CQMLEs $\tilde{\theta} \equiv \tilde{\theta}(\hat{\lambda}_1)$ and $\tilde{\sigma}_v^2 \equiv \tilde{\sigma}_v^2(\tilde{\lambda}_1)$ of θ and σ_v^2, respectively.

The CQS function $S_{\mathtt{SL}}(\psi)$ has elements: $\frac{1}{\sigma_v^2}\Delta X'\mathbf{C}^{-1}\Delta v(\theta)$, $\frac{1}{2\sigma_v^4}\Delta v(\theta)'\mathbf{C}^{-1}\Delta v(\theta) - \frac{n(T-1)}{2\sigma_v^2}$, $\frac{1}{\sigma_v^2}\Delta v(\theta)'\mathbf{C}^{-1}\Delta Y_{-1}$, $\frac{1}{\sigma_v^2}\Delta v(\theta)'\mathbf{C}^{-1}\mathbf{W}_1\Delta Y - \text{tr}(\mathbf{B}_1^{-1}\mathbf{W}_1)$. The expectations of the first two components of $S_{\mathtt{SL}}(\psi_0)$ are zero, but these of the last two are not as by Lemma 3.1,

$$\begin{aligned} \text{E}(\Delta v'\mathbf{C}^{-1}\Delta Y_{-1}) &= -\sigma_{v0}^2\text{tr}(\mathbf{C}^{-1}\mathbf{D}_{-10}), \text{ and} && (1.8)\\ \text{E}(\Delta v'\mathbf{C}^{-1}\mathbf{W}_1\Delta Y) &= -\sigma_{v0}^2\text{tr}(\mathbf{C}^{-1}\mathbf{D}_0\mathbf{W}_1), && (1.9)\end{aligned}$$

where $\mathbf{D}_{-1}$ and $\mathbf{D}$ are given in Section 3.1 with $\mathcal{B}$ simplifies to ρB_1^{-1}. These show that the last two elements of $\text{plim}_{n\to\infty}\frac{1}{nT}S_{\mathtt{SL}}(\psi_0)$ are not zero, showing that the CQMLEs of the SL model are inconsistent. Even when T grows with n, it can be shown that the CQMLE of ρ has a bias of order $O(T^{-1})$ instead of the desired order $O((nT)^{-1})$. Some modifications are thus necessary whether T is fixed or not. The adjusted quasi score function is,

$$S^*_{\mathtt{SL}}(\psi) = \begin{cases} \frac{1}{\sigma_v^2}\Delta X'\mathbf{C}^{-1}\Delta v(\theta), \\ \frac{1}{2\sigma_v^4}\Delta v(\theta)'\mathbf{C}^{-1}\Delta v(\theta) - \frac{n(T-1)}{2\sigma_v^2}, \\ \frac{1}{\sigma_v^2}\Delta v(\theta)'\mathbf{C}^{-1}\Delta Y_{-1} + \text{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1}), \\ \frac{1}{\sigma_v^2}\Delta v(\theta)'\mathbf{C}^{-1}\mathbf{W}_1\Delta Y + \text{tr}(\mathbf{C}^{-1}\mathbf{D}\mathbf{W}_1). \end{cases} \tag{1.10}$$

The M-estimator for the FE-SDPD-SLD model is thus defined as $\hat{\psi}_{\mathtt{M}} = \arg\{S^*_{\mathtt{SL}}(\psi) = 0\}$. The root-finding process can be simplified by first solving the equations for β and σ_v^2, given $\delta = (\rho, \lambda_1)'$, leading to the constrained M-estimators $\hat{\beta}(\delta) = (\Delta X'\mathbf{C}^{-1}\Delta X)^{-1}\Delta X'\mathbf{C}^{-1}\Delta Y(\delta)$

and $\hat{\sigma}_v^2(\delta) = \frac{1}{n(T-1)}\Delta\tilde{v}(\delta)'\mathbf{C}^{-1}\Delta\tilde{v}(\delta)$ for β and σ_v^2, where $\Delta Y(\delta) = \mathbf{B}_1\Delta Y - \rho\Delta Y_{-1}$ and $\Delta\hat{v}(\delta) = \Delta v(\hat{\beta}(\delta), \delta)$. Substituting $\hat{\beta}(\delta)$ and $\hat{\sigma}_v^2(\delta)$ into the last two components of (1.10) gives the concentrated AQS function of δ:

$$S_{\mathtt{SL}}^{*c}(\delta) = \begin{cases} \frac{1}{\hat{\sigma}_{v,\mathtt{M}}^2(\delta)}\Delta\hat{v}(\delta)'\mathbf{C}^{-1}\Delta Y_{-1} + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1}), \\ \frac{1}{\hat{\sigma}_{v,\mathtt{M}}^2(\delta)}\Delta\hat{v}(\delta)'\mathbf{C}^{-1}\mathbf{W}_1\Delta Y + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}\mathbf{W}_1). \end{cases} \tag{1.11}$$

Solving the concentrated equations, $S_{\mathtt{SL}}^{*c}(\delta) = 0$, gives the unconstrained M-estimator $\hat{\delta}_{\mathtt{M}}$ of δ. The unconstrained M-estimators of β and σ_v^2 are thus $\hat{\beta}_{\mathtt{M}} \equiv \hat{\beta}(\hat{\delta}_{\mathtt{M}})$ and $\hat{\sigma}_{v,\mathtt{M}}^2 \equiv \hat{\sigma}_v^2(\hat{\delta}_{\mathtt{M}})$.

The FE-SDPD model with SL and STL effects. Setting $\lambda_3 = 0$, Model (3.1) reduces to a FE-SDPD model with SL and STL dependence. Now, $\psi = (\beta', \sigma_v^2, \rho, \lambda_1, \lambda_2)'$. The conditional quasi Gaussian loglikelihood of ψ reduces to:

$$\ell_{\mathtt{STL}}(\psi) = -\tfrac{n(T-1)}{2}\log(\sigma_v^2) + \log|\mathbf{B}_1| - \tfrac{1}{2}\log|\mathbf{C}| - \tfrac{1}{2\sigma_v^2}\Delta v(\theta)'\mathbf{C}^{-1}\Delta v(\theta), \tag{1.12}$$

where $\theta = (\beta', \rho, \lambda_1, \lambda_2)'$, $\theta_1 = (\beta', \rho, \lambda_2)'$, and $v(\theta) = \mathbf{B}_1\Delta Y - (\rho I_n + \lambda_2\mathbf{W}_2)\Delta Y_{-1} - \Delta X\beta$. Given λ_1, $\ell_{\mathtt{STL}}(\psi)$ is maximized at $\tilde{\theta}_1(\lambda_1) = (\Delta\mathbb{X}'\mathbf{C}^{-1}\Delta\mathbb{X})^{-1}\Delta\mathbb{X}'\mathbf{C}^{-1}\mathbf{B}_1\Delta Y$ and $\tilde{\sigma}_v^2(\lambda_1) = \frac{1}{n(T-1)}\Delta\tilde{v}'(\lambda_1)\mathbf{C}^{-1}\Delta\tilde{v}(\lambda_1)$, where $\Delta\mathbb{X} = (\Delta X, \Delta Y_{-1}, \mathbf{W}_2\Delta Y_{-1})$ and $\Delta\tilde{v}(\lambda_1) = \mathbf{B}_1\Delta Y - \Delta\mathbb{X}\,\tilde{\theta}(\lambda_1)$. Thus, the concentrated conditional quasi loglikelihood function of λ_1 is,

$$\ell_{\mathtt{STL}}^c(\lambda_1) = +\log|\mathbf{B}_1| - \tfrac{n(T-1)}{2}\log(\tilde{\sigma}_v^2(\lambda_1)) - \tfrac{1}{2}\log|\mathbf{C}|. \tag{1.13}$$

Maximizing $\ell_{\mathtt{STL}}^c(\lambda_1)$ gives the CQMLE $\tilde{\lambda}_1$, and thus the CQMLEs $\tilde{\theta} \equiv \tilde{\theta}(\hat{\lambda}_1)$ and $\tilde{\sigma}_v^2 \equiv \tilde{\sigma}_v^2(\tilde{\lambda}_1)$.

The CQS function $S_{\mathtt{STL}}(\psi)$ contains elements: $\frac{1}{\sigma_v^2}\Delta X'\mathbf{C}^{-1}\Delta v(\theta)$, $\frac{1}{2\sigma_v^4}\Delta v(\theta)'\mathbf{C}^{-1}\Delta v(\theta) - \frac{n(T-1)}{2\sigma_v^2}$, $\frac{1}{\sigma_v^2}\Delta v(\theta)'\mathbf{C}^{-1}\Delta Y_{-1}$, $\frac{1}{\sigma_v^2}\Delta v(\theta)'\mathbf{C}^{-1}\mathbf{W}_1\Delta Y - \mathrm{tr}(\mathbf{B}_1^{-1}\mathbf{W}_1)$, $\frac{1}{\sigma_v^2}\Delta v(\theta)'\mathbf{C}^{-1}\mathbf{W}_2\Delta Y_{-1}$. It is easy to see that the first two components of $\mathrm{E}[S_{\mathtt{STL}}(\psi_0)]$ are zero, but these of the last three components are not, and are obtained from (3.11)-(3.13). Thus a necessary condition for the consistency of parameter estimators is violated, suggesting that with T fixed the conditional QMLEs of the FE-DPD-SLD model are inconsistent. Even when T grows with n, it can be shown that the conditional QMLE of ρ has a bias of order $O(T^{-1})$ instead of the desired order $O((nT)^{-1})$. Some modifications are thus necessary whether T is fixed or not, and the adjusted quasi score function is,

$$S_{\mathtt{STL}}^{*}(\psi) = \begin{cases} \frac{1}{\sigma_v^2}\Delta X'\mathbf{C}^{-1}\Delta v(\theta), \\ \frac{1}{2\sigma_v^4}\Delta v(\theta)'\mathbf{C}^{-1}\Delta v(\theta) - \frac{n(T-1)}{2\sigma_v^2}, \\ \frac{1}{\sigma_v^2}\Delta v(\theta)'\mathbf{C}^{-1}\Delta Y_{-1} + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1}), \\ \frac{1}{\sigma_v^2}\Delta v(\theta)'\mathbf{C}^{-1}\mathbf{W}_1\Delta Y + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}\mathbf{W}_1), \\ \frac{1}{\sigma_v^2}\Delta v(\theta)'\mathbf{C}^{-1}\mathbf{W}_2\Delta Y_{-1} + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1}\mathbf{W}_2). \end{cases} \tag{1.14}$$

The M-estimator for the FE-SDPD-SLD model is thus defined as $\hat{\psi}_{\mathtt{SL}} = \arg\{S_{\mathtt{M}}^{*}(\psi) = 0\}$. The root-finding process can be simplified by first solving the equations for β and σ_v^2, given $\delta = (\rho, \lambda_1, \lambda_2)'$, leading to the constrained M-estimators $\hat{\beta}(\delta) = (\Delta X'\mathbf{C}^{-1}\Delta X)^{-1}\Delta X'\mathbf{C}^{-1}\Delta Y(\delta)$

and $\hat{\sigma}_v^2(\delta) = \frac{1}{n(T-1)}\Delta\hat{v}(\delta)'\mathbf{C}^{-1}\Delta\hat{v}(\delta)$, where $\Delta Y(\delta) = \mathbf{B}_1\Delta Y - (\rho I_n + \lambda_2\mathbf{W}_2)\Delta Y_{-1}$ and $\Delta\hat{v}(\delta) = \Delta v(\hat{\beta}(\delta), \delta)$. Substituting $\hat{\beta}(\delta)$ and $\hat{\sigma}_v^2(\delta)$ into the last two components of (1.14) gives the concentrated AQS function of δ:

$$S^{*c}_{\mathtt{STL}}(\delta) = \begin{cases} \frac{1}{\tilde{\sigma}^2_{v,\mathtt{M}}(\delta)}\tilde{v}(\delta)'\mathbf{C}^{-1}\Delta\Delta Y_{-1} + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1}), \\ \frac{1}{\tilde{\sigma}^2_{v,\mathtt{M}}(\delta)}\Delta\tilde{v}(\delta)'\mathbf{C}^{-1}\mathbf{W}_1\Delta Y + \mathrm{tr}(\mathbf{W}_1\mathbf{C}^{-1}\mathbf{D}), \\ \frac{1}{\tilde{\sigma}^2_{v,\mathtt{M}}(\delta)}\Delta\tilde{v}(\delta)'\mathbf{C}^{-1}\mathbf{W}_2\Delta Y_{-1} + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1}\mathbf{W}_2). \end{cases} \tag{1.15}$$

Solving the concentrated equations, $S^{*c}_{\mathtt{SL}}(\delta) = 0$, gives the M-estimator $\hat{\delta}_{\mathtt{M}}$ of δ. The M-estimators of β and σ_v^2 are, thus, $\hat{\beta}_{\mathtt{M}} \equiv \hat{\beta}(\hat{\delta}_{\mathtt{M}})$ and $\hat{\sigma}^2_{v,\mathtt{M}} \equiv \hat{\sigma}_v^2(\hat{\delta}_{\mathtt{M}})$.

The FE-SDPD model with SL and SE effects. Setting $\lambda_2 = 0$ in Model (3.1) yields an FE-SDPD model with both SL and SE dependence. The conditional quasi Gaussian loglikelihood of $\psi = (\beta', \sigma_v^2, \rho, \lambda_1, \lambda_3)'$ reduces to,

$$\ell_{\mathtt{SLE}}(\psi) = -\tfrac{n(T-1)}{2}\log(\sigma_v^2) + \log|\mathbf{B}_1| - \tfrac{1}{2}\log|\Omega| - \tfrac{1}{2\sigma_v^2}\Delta u(\theta)'\Omega^{-1}\Delta u(\theta), \tag{1.16}$$

where $\theta = (\beta', \rho, \lambda_1)'$ and $\Delta u(\theta) = \mathbf{B}_1\Delta Y - \rho\Delta Y_{-1} - \Delta X\beta$. Given $\lambda = (\lambda_1, \lambda_3)'$, $\ell_{\mathtt{SLE}}(\psi)$ is maximized at $\tilde{\theta}(\lambda) = (\Delta\mathbb{X}'\Omega^{-1}\Delta\mathbb{X})^{-1}\Delta\mathbb{X}'\Omega^{-1}\mathbf{B}_1\Delta Y$ and $\tilde{\sigma}_v^2(\lambda) = \frac{1}{n(T-1)}\Delta\tilde{u}'(\lambda)\Omega^{-1}\Delta\tilde{u}(\lambda)$, where $\Delta\tilde{u}(\lambda) = \mathbf{B}_1\Delta Y - \Delta\mathbb{X}\,\tilde{\theta}(\lambda)$, and $\Delta\mathbb{X} = (\Delta X, \Delta Y_{-1})$. Substituting $\tilde{\theta}(\lambda)$ and $\tilde{\sigma}_v^2(\lambda)$ back into $\ell_{\mathtt{SLE}}(\psi)$ gives the concentrated loglikelihood function of λ,

$$\ell^c_{\mathtt{SLE}}(\lambda) = \log|\mathbf{B}_1| - \tfrac{n(T-1)}{2}\log(\tilde{\sigma}_v^2(\lambda)) - \tfrac{1}{2}\log|\Omega|. \tag{1.17}$$

Maximizing $\ell^c_{\mathtt{SLE}}(\lambda)$ gives the CQMLE $\tilde{\lambda}$, and thus the CQMLEs $\tilde{\theta} \equiv \tilde{\theta}(\hat{\lambda})$ and $\tilde{\sigma}_v^2 \equiv \tilde{\sigma}_v^2(\tilde{\lambda})$.

The CQS function $S_{\mathtt{SLE}}(\psi)$ has the components: $\frac{1}{\sigma_v^2}\Delta X'\Omega^{-1}\Delta u(\theta)$, $\frac{1}{2\sigma_v^4}\Delta u(\theta)'\Omega^{-1}\Delta u(\theta) - \frac{n(T-1)}{2\sigma_v^2}$, $\frac{1}{\sigma_v^2}\Delta u(\theta)'\Omega^{-1}\Delta Y_{-1}$, $\frac{1}{\sigma_v^2}\Delta u(\theta)'\Omega^{-1}\mathbf{W}_1\Delta Y - \mathrm{tr}(\mathbf{B}_1^{-1}\mathbf{W}_1)$, $\frac{1}{2\sigma_v^2}\Delta u(\theta)'(C^{-1}\otimes A_3)\Delta u(\theta) - (T-1)\mathrm{tr}(G_3)$. The β, σ_v^2 and λ_3 components of $\mathrm{E}[S_{\mathtt{SLE}}(\psi_0)]$ are zero, but the ρ and λ_1 components are not as seen from Lemma 3.1: $\mathrm{E}(\Delta u'\Omega^{-1}\Delta Y_{-1}) = -\sigma_{v0}^2\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-10})$ and $\mathrm{E}(\Delta u'\Omega^{-1}\mathbf{W}_1\Delta Y) = -\sigma_{v0}^2\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_0\mathbf{W}_1)$, which are of identical forms as those for the SLD model. The results show that the CQMLEs are not consistent unless T also approaches infinity. To achieve consistency, the conditional quasi score function should be modified as:

$$S^*_{\mathtt{SLE}}(\psi) = \begin{cases} \frac{1}{\sigma_v^2}\Delta X'\Omega^{-1}\Delta u(\theta), \\ \frac{1}{2\sigma_v^4}\Delta u(\theta)'\Omega^{-1}\Delta u(\theta) - \frac{n(T-1)}{2\sigma_v^2}, \\ \frac{1}{\sigma_v^2}\Delta u(\theta)'\Omega^{-1}\Delta Y_{-1} + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1}), \\ \frac{1}{\sigma_v^2}\Delta u(\theta)'\Omega^{-1}\mathbf{W}_1\Delta Y + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}\mathbf{W}_1), \\ \frac{1}{2\sigma_v^2}\Delta u(\theta)'(C^{-1}\otimes A_3)\Delta u(\theta) - (T-1)\mathrm{tr}(G_3). \end{cases} \tag{1.18}$$

The M-estimator of the FE-DPD-SLE model is defined as $\hat{\psi}_{\mathtt{M}} = \arg\{S^*_{\mathtt{SLE}}(\psi) = 0\}$. The root-finding process can be simplified by first solving the equations for β and σ_v^2, resulting in the constrained M-estimators $\hat{\beta}(\delta) = (\Delta X'\Omega^{-1}\Delta X)^{-1}\Delta X'\Omega^{-1}\Delta Y(\rho, \lambda_1)$ and $\hat{\sigma}_v^2(\delta) = \frac{1}{n(T-1)}\Delta\hat{u}(\delta)'\Omega^{-1}\Delta\hat{u}(\delta)$, given $\delta = (\rho, \lambda_1, \lambda_3)'$, where $\Delta Y(\rho, \lambda_1) = \mathbf{B}_1\Delta Y - \rho\Delta Y_{-1}$ and

$\Delta\hat{u}(\delta) = \Delta u(\hat{\beta}(\delta), \rho, \lambda_1)$. Substituting $\hat{\beta}(\delta)$ and $\hat{\sigma}^2_{v,\mathtt{M}}(\delta)$ into the last three components of $S^*_{\mathtt{SLE}}(\psi)$ gives the concentrated AQS function:

$$S^{*c}_{\mathtt{SLE}}(\delta) = \begin{cases} \frac{1}{\hat{\sigma}^2_{v,\mathtt{M}}(\delta)}\Delta\hat{u}(\delta)'\Omega^{-1}\Delta Y_{-1} + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1}), \\ \frac{1}{\hat{\sigma}^2_{v,\mathtt{M}}(\delta)}\Delta\hat{u}(\delta)'\Omega^{-1}\mathbf{W}_1\Delta Y + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}\mathbf{W}_1), \\ \frac{1}{2\hat{\sigma}^2_{v,\mathtt{M}}(\delta)}\Delta\hat{u}(\delta)'(C^{-1}\otimes A_3)\Delta\hat{u}(\delta) - (T-1)\mathrm{tr}(G_3). \end{cases} \quad (1.19)$$

Solving the concentrated equations, $S^{*c}_{\mathtt{SLE}}(\delta) = 0$, gives the M-estimator $\hat{\delta}_{\mathtt{M}}$ of δ. The M-estimators of β and σ^2_v are thus $\hat{\beta}_{\mathtt{M}} \equiv \hat{\beta}(\hat{\delta}_{\mathtt{M}})$ and $\hat{\sigma}^2_{v,\mathtt{M}} \equiv \hat{\sigma}^2_v(\hat{\delta}_{\mathtt{M}})$.

2 Monte Carlo Results

Tables 1-5 presented here are the much extended versions of Tables 1-5 reported in the main paper, which contain more cases corresponding to the values of the dynamic parameter ρ, sample sized n and T, etc. Most of the cases with $T = 7$ are reported here. To summarize, the results show that when T is small our approach is comparable with the standard full QML approach if the initial model is correctly specified, but if the the initial model is misspecified, out approach performs better as it is free from the specification of initial conditions. When T is large, our approach has a clear advantage over the full QML approach as the latter faces the 'choice of m value' and 'choice of predictors' problems, besides the issue on its applicability to a FE-SDPD model with SL term (see Footnotes 7 and 8 in the main paper). When T is either small or large, our approach clearly outperforms the CQML approach.

3 An Empirical Illustration

To facilitate the practical applications of the proposed methods, we provide an empirical illustration using the well known data set on public capital productivity of Munnell (1990). The dataset gives indicators related to public capital productivity for 48 US states observed over 17 years (1970-1986).[1,2] In Munnel (1990), the empirical model specified is a Cobb-Douglas production function of the form:

$$\ln Y = \beta_0 + \beta_1 \ln K_1 + \beta_2 \ln K_2 + \beta_3 \ln L + \beta_4 \mathrm{Unemp} + \epsilon,$$

with state specific fixed effects, where Y is the gross social product of a given state, K_1 is public capital, K_2 is private capital, L is labour input and Unemp is the state unemployment rate. This model is now extended by adding the dynamic effect and one or more spatial effects. The spatial weights matrix W takes a contiguity form with its (i, j)th element being

[1] The dataset can be downloaded from http://pages.stern.nyu.edu/~wgreene/Text/Edition6/tablelist6.htm

[2] This dataset has been extensively used for illustrating the applications of the regular panel data models (see, e.g., Baltagi, 2013). In the spatial framework, it was used by Millo and Piras (2012) for illustrating the QML and GMM estimation of fixed effects and random effects spatial panel data models, and by Yang et al. (2016) for illustrating the bias-correction and refined inferences for fixed effects spatial panel data models.

1 if states i and j share a common border, otherwise 0. The final W is row normalised. For models with more than one spatial term, the corresponding $W's$ are taken to be the same.

Each of the five models discussed in the paper is estimated using (a) full data, (b) data from the last six years ($T + 1 = 6$), and (c) data from first six years. Table 6a summarize the CQMLE, FQMLE, M-Estimate (M-Est) and the standard error of the M-Est for the `SE` model, as for this model the full QMLE is available (Su and Yang, 2015). From the results we see that (i) the dynamic and `SE` effects are highly significant in all models, (ii) the three methods give quite different estimates of the dynamic effect, and (iii) the FQMLE of ρ improves over CQMLE in that it is much closer to the M-est in particular when T is small.[3]

Table 6b summarize the results for the other four models. The results show that, for any model estimated and data used, (i) the dynamic effect is alway significant, (ii) there is alway at least one spatial effect that is significant, and (iii) the CQMLE is always significantly smaller than the corresponding M-estimate. The empirical results are consistent with the theories and Monte Carlo results. Thus, it is recommended that in the practical applications of spatial dynamic panel data models, the proposed methods should be used, in particular when T is small. To facilitate the practitioners, the full set of **matlab codes** producing the results in Tables 6a and 6b are made available at the same website as this supplement: `http://www.mysmu.edu/faculty/zlyang/`, under the '`Publications`' category.

Appendix A: Some Basic Lemmas

The following lemmas are essential for the proofs of the main results in this paper.

Lemma A.1 *(Kelejian and Prucha, 1999; Lee, 2002): Let $\{A_n\}$ and $\{B_n\}$ be two sequences of $n \times n$ matrices that are uniformly bounded in both row and column sums. Let C_n be a sequence of conformable matrices whose elements are uniformly $O(h_n^{-1})$. Then*

(i) the sequence $\{A_n B_n\}$ are uniformly bounded in both row and column sums,
(ii) the elements of A_n are uniformly bounded and $tr(A_n) = O(n)$, and
(iii) the elements of $A_n C_n$ and $C_n A_n$ are uniformly $O(h_n^{-1})$.

Lemma A.2 *(Lee, 2004, p.1918): For W_1 and B_1 defined in Model (3.1), if $\|W_1\|$ and $\|B_{10}^{-1}\|$ are uniformly bounded, where $\|\cdot\|$ is a matrix norm, then $\|B_1^{-1}\|$ is uniformly bounded in a neighborhood of λ_{10}.*

Lemma A.3 *(Lee, 2004, p.1918): Let X_n be an $n \times p$ matrix. If the elements X_n are uniformly bounded and $\lim_{n\to\infty} \frac{1}{n} X_n' X_n$ exists and is nonsingular, then $P_n = X_n(X_n' X_n)^{-1} X_n'$ and $M_n = I_n - P_n$ are uniformly bounded in both row and column sums.*

Lemma A.4 *(Lemma B.4, Yang, 2015, extended): Let $\{A_n\}$ be a sequence of $n \times n$ matrices that are uniformly bounded in either row or column sums. Suppose that the elements $a_{n,ij}$ of A_n are $O(h_n^{-1})$ uniformly in all i and j. Let v_n be a random n-vector of iid elements*

[3]FQMLE uses $m = 6$, and the time mean of the regressors as the predictor for the initial differences. The results are quite robust to the value of m, but not quite to the choice of the predictors.

with mean zero, variance σ^2 *and finite 4th moment, and* b_n *a constant n-vector of elements of uniform order* $O(h_n^{-1/2})$. *Then*

$$\begin{array}{ll} (i) \quad \mathrm{E}(v_n' A_n v_n) = O(\frac{n}{h_n}), & (ii)\ \mathrm{Var}(v_n' A_n v_n) = O(\frac{n}{h_n}), \\ (iii)\ \mathrm{Var}(v_n' A_n v_n + b_n' v_n) = O(\frac{n}{h_n}), & (iv)\ v_n' A_n v_n = O_p(\frac{n}{h_n}), \\ (v) \quad v_n' A_n v_n - \mathrm{E}(v_n' A_n v_n) = O_p((\frac{n}{h_n})^{\frac{1}{2}}), & (vi)\ v_n' A_n b_n = O_p((\frac{n}{h_n})^{\frac{1}{2}}), \end{array}$$

and (vii), the results (iii) *and* (vi) *remain valid if* b_n *is a random n-vector independent of* v_n *such that* $\{\mathrm{E}(b_{ni}^2)\}$ *are of uniform order* $O(h_n^{-1})$.

Proof of Lemma A.4: The results in (vii) extend the results (iii) and (vi) of Lemma B.5 of Yang (2015) by allowing b_n to be a random vector. Proofs are done similarly. ■

Lemma A.5 *(***Central Limit Theorem for bilinear quadratic forms***). Let* $\{\Phi_n\}$ *be a sequence of* $n \times n$ *matrices with row and column sums uniformly bounded, and elements of uniform order* $O(h_n^{-1})$. *Let* $v_n = (v_1, \cdots, v_n)'$ *be a random vector of iid elements with mean zero, variance* σ_v^2, *and finite* $(4 + 2\epsilon_0)$*th moment for some* $\epsilon_0 > 0$. *Let* $b_n = \{b_{ni}\}$ *be an* $n \times 1$ *random vector, independent of* v_n, *such that* (i) $\{\mathrm{E}(b_{ni}^2)\}$ *are of uniform order* $O(h_n^{-1})$, (ii) $sup_i E|b_{ni}|^{2+\epsilon_0} < \infty$, (iii) $\frac{h_n}{n}\sum_{i=1}^n[\phi_{n,ii}(b_{ni} - \mathrm{E}b_{ni})] = o_p(1)$ *where* $\{\phi_{n,ii}\}$ *are the diagonal elements of* Φ_n, *and* (iv) $\frac{h_n}{n}\sum_{i=1}^n[b_{ni} - \mathrm{E}(b_{ni}^2)] = o_p(1)$. *Define the bilinear-quadratic form:*

$$Q_n = b_n' v_n + v_n' \Phi_n v_n - \sigma_v^2 tr(\Phi_n),$$

and let $\sigma_{Q_n}^2$ *be the variance of* Q_n. *If* $lim_{n\to\infty} h_n^{1+2/\epsilon_0}/n = 0$ *and* $\{\frac{h_n}{n}\sigma_{Q_n}^2\}$ *are bounded away from zero, then* $Q_n/\sigma_{Q_n} \xrightarrow{d} N(0, 1)$.

Proof of Lemma A.5: The proof proceeds by assuming (W.L.O.G.) Φ_n being symmetric, with elements denoted by $\phi_{n,ij}$. Write $Q_n = \sum_{i=1}^n[b_{ni}v_i + v_i\xi_{ni} + \phi_{n,ii}(v_i^2 - \sigma_v^2)] \equiv \sum_{i=1}^n Y_{ni}$, where $\xi_{ni} = 2\sum_{j=1}^{i-1}\phi_{n,ij}v_j$. Consider the σ-fields $\mathcal{G}_i = \sigma(v_1, \ldots, v_i)$ generated by $(v_1, \ldots, v_i), i = 1, \ldots, n$. By construction, $\mathcal{G}_{i-1} \subseteq \mathcal{G}_i$. Define the σ-field $\mathcal{F}_{n0}$ generated by b_n. By independence between b_n and v_n, $\mathcal{F}_{ni} = \mathcal{F}_{n0} \times \mathcal{G}_i$ is the σ-field generated by $(b_n, v_1, \ldots, v_i)$. Clearly, Y_{ni} is $\mathcal{F}_{ni}$-measurable and ξ_{ni} is $\mathcal{F}_{n,i-1}$-measurable. It follows that $\mathrm{E}(Y_{ni}|\mathcal{F}_{n,i-1}) = b_{ni}\mathrm{E}(v_i) + \mathrm{E}(v_i)\xi_{ni} + \phi_{n,ii}\mathrm{E}(v_i^2 - \sigma_v^2) = 0$, and hence $\{Y_{ni}, \mathcal{F}_{ni},\ 1 \le i \le n\}$ forms a martingale difference (M.D.) array, and $\sigma_{Q_n}^2 = \sum_{i=1}^n E(Y_{ni}^2)$. Define $Z_{ni} = Y_{ni}/\sigma_{Q_n}$. Then, $\{Z_{ni}, \mathcal{F}_{ni},\ 1 \le i \le n\}$ also forms a M.D. array. The proof of the lemma thus amounts to verify the conditions for the central limit theorem (CLT) for M.D. arrays, e.g., the condition (A.1) or (A.3) and condition (A.2) of Theorem A.1 in Kelejian and Prucha (2001):

$$\begin{array}{ll} (a) & \sum_{i=1}^n \mathrm{E}[\mathrm{E}(|Z_{ni}|^{2+\delta}|\mathcal{F}_{n,i-1})] \longrightarrow 0, \quad \text{for some } \delta > 0; \\ (b) & \sum_{i=1}^n \mathrm{E}(Z_{ni}^2|\mathcal{F}_{n,i-1}) \xrightarrow{p} 1. \end{array}$$

The details for the proof of (a) follow closely that of Theorem 1 of Kelejian and Prucha (2001), where the quantities $|b_{ni}|$, b_{ni}^2, $|b_{ni}|^q$ are replaced by their expectations, and references are made to the proof of Lemma A.13 of Lee (2004s) to take care unbounded h_n.

To prove (b), we have $\sum_{i=1}^n \mathrm{E}[Z_{ni}^2|\mathcal{F}_{n,i-1}] - 1 = \sigma_{Q_n}^{-2}\sum_{i=1}^n[\mathrm{E}(Y_{ni}^2|\mathcal{F}_{n,i-1}) - \mathrm{E}(Y_{ni}^2)]$, and

$$
\begin{aligned}
& \tfrac{h_n}{n}\textstyle\sum_{i=1}^n[\mathrm{E}(Y_{ni}^2|\mathcal{F}_{n,i-1})-\mathrm{E}(Y_{ni}^2)] \\
= \; & \sigma_v^2\tfrac{h_n}{n}\textstyle\sum_{i=1}^n(\xi_{ni}^2-\tau_{ni}^2)+2\sigma_v^2\tfrac{h_n}{n}\sum_{i=1}^n(b_{ni}\xi_{ni})+2\mu_3\tfrac{h_n}{n}\sum_{i=1}^n(\phi_{n,ii}\xi_{ni}) \\
& +2\mu_3\tfrac{h_n}{n}\textstyle\sum_{i=1}^n[\phi_{n,ii}(b_{ni}-\mathrm{E}b_{ni})]+\sigma_v^2\tfrac{h_n}{n}\sum_{i=1}^n[b_{ni}^2-\mathrm{E}(b_{ni}^2)] \\
\equiv \; & \sigma_v^2 Q_1+2\sigma_v^2 Q_2+2\mu_3 Q_3+2\mu_3 Q_4+\sigma_v^2 Q_5,
\end{aligned}
$$

where $\tau_{ni}^2=\mathrm{Var}(\xi_{ni})=4\sigma_v^2\sum_{j=1}^{i-1}\phi_{n,ij}^2$. To show $Q_1\xrightarrow{p}0$, we have $Q_1=\frac{h_n}{n}\sum_{i=1}^n(\xi_{ni}^2-\tau_{ni}^2)=4\frac{h_n}{n}\sum_{j=1}^{n-1}a_{nj}(v_j^2-\sigma_v^2)+8\frac{h_n}{n}\sum_{j=1}^{n-1}v_j\varepsilon_{nj}$, where $a_{nj}=\sum_{i=j+1}^n\phi_{n,ij}^2$, $\varepsilon_{nj}=\sum_{k=1}^{j-1}c_{n,ik}v_k$, and $c_{n,ik}=\sum_{i=j+1}^n\phi_{n,ij}\phi_{n,ik}$. Clearly, both $\{(v_j^2-\sigma_v^2),\mathcal{G}_i\}$ and $\{v_j\varepsilon_{nj},\mathcal{G}_i\}$ are M.D. arrays, and hence both terms in Q_1 convergence in probability to zero by the weak law of large numbers (WLLN) for M.D. arrays of Davidson (1994, p. 299).

By applying Chebyshev inequality, we show that $Q_2\xrightarrow{p}0$. Now, it is easy to see that $Q_3=\frac{h_n}{n}\sum_{j=1}^{n-1}d_{n,j}v_j$ where $d_{n,j}=\sum_{i=j+1}^n\phi_{n,ii}\phi_{ij}$. Thus, the convergence of Q_3 is proved by applying Chebyshev inequality. Finally, by Assumptions (iii) and (iv) stated in Lemma A.5, both Q_4 and Q_5 converge to zero in probability. This completes the proof Lemma A.5 (details, in particular the proof of (a), are available from the author upon request). ■

Appendix B: Proofs of Lemmas 3.1-3.3

Proof of Lemma 3.1: Using $\Delta y_t=\mathcal{B}_0\Delta y_{t-1}+B_{10}^{-1}\Delta X_t+B_{10}^{-1}B_{30}^{-1}\Delta v_t,\;\; t=2,\ldots,T$ given in (3.8), we have under Assumption A: if $m\geq 1$, then

$$\mathrm{E}(\Delta y_1\Delta v_2')=B_{10}^{-1}B_{30}^{-1}\mathrm{E}(\Delta v_1\Delta v_2')=-\sigma_{v0}^2B_{10}^{-1}B_{30}^{-1};$$

if $m=0$, then $\mathrm{E}(\Delta y_1\Delta v_2')=B_{10}^{-1}B_{30}^{-1}\mathrm{E}(y_1\Delta v_2')=B_{10}^{-1}B_{30}^{-1}\mathrm{E}(v_1\Delta v_2')=-\sigma_{v0}^2B_{10}^{-1}B_{30}^{-1}$.

Now, for $t\geq 2$, we have, $\mathrm{E}(\Delta y_t\Delta v_{t+1}')=B_{10}^{-1}B_{30}^{-1}\mathrm{E}(v_t\Delta v_{t+1}')=-\sigma_{v0}^2B_{10}^{-1}B_{30}^{-1}$,
$\mathrm{E}(\Delta y_t\Delta v_t')=\mathcal{B}_0\mathrm{E}(\Delta y_{t-1}\Delta v_t')+B_{10}^{-1}B_{30}^{-1}\mathrm{E}(\Delta v_t\Delta v_t')=\sigma_{v0}^2(2I_n-\mathcal{B}_0)B_{10}^{-1}B_{30}^{-1}$, and
$\mathrm{E}(\Delta y_{t+1}\Delta v_t')=\mathcal{B}_0\mathrm{E}(\Delta y_t\Delta v_t')+B_{10}^{-1}B_{30}^{-1}\mathrm{E}(\Delta v_{t+1}\Delta v_t')=-\sigma_{v0}^2(I_n-\mathcal{B}_0)^2B_{10}^{-1}B_{30}^{-1}$.
Furthermore, for $t\geq 3$, we have, $\mathrm{E}(\Delta y_t\Delta v_2')=-\sigma_{v0}^2\mathcal{B}_0^{t-3}(I_n-\mathcal{B}_0)^2B_{10}^{-1}B_{30}^{-1}$; and for $t\geq 4$, we have, $\mathrm{E}(\Delta y_t\Delta v_3')=-\sigma_{v0}^2\mathcal{B}_0^{t-4}(I_n-\mathcal{B}_0)^2B_{10}^{-1}B_{30}^{-1}$, etc., leading to

$$\mathrm{E}(\Delta y_t\Delta v_s')=-\sigma_{v0}^2\mathcal{B}_0^{t-(s+1)}(I_n-\mathcal{B}_0)^2B_{10}^{-1}B_{30}^{-1},\;\text{for } t\geq s+1 \text{ and } s\geq 2.$$

Finally, all the terms $\mathrm{E}(\Delta y_t\Delta v_{t+2}'),t\geq 1$, are zero. Putting these together gives the results of Lemma 3.1. ■

Proof of Lemma 3.2: By (3.8), continuous substitution gives, for $t=2,\ldots,T$,

$$
\begin{aligned}
\Delta y_t \; = \; & \mathcal{B}_0^{t-1}\Delta y_1+\mathcal{B}_0^{t-2}B_{10}^{-1}\Delta X_2\beta_0+\ldots+B_{10}^{-1}\Delta X_t\beta_0 \\
& +\mathcal{B}_0^{t-2}B_{10}^{-1}B_{30}^{-1}\Delta v_2+\ldots+B_{10}^{-1}B_{30}^{-1}\Delta v_t \\
= \; & \mathcal{B}_0^{t-1}\Delta y_1+\{\mathcal{B}_0^{t-2},\mathcal{B}_0^{t-3},\ldots,I_n,0,\ldots,0\}\mathbf{B}_{10}^{-1}\Delta X\beta_0 \\
& +\{\mathcal{B}_0^{t-2},\mathcal{B}_0^{t-3},\ldots,I_n,0,\ldots,0\}\mathbf{B}_{10}^{-1}\mathbf{B}_{30}^{-1}\Delta v.
\end{aligned}
$$

The results of Lemma 3.2 thus follow. ■

Proof of Lemma 3.3: First, for the terms linear in Δv, we have,

$$\begin{aligned}\Pi'\Delta v &= \textstyle\sum_{t=2}^{T}\Pi_t'\Delta v_t\\ &= \textstyle\sum_{t=2}^{T}\sum_{i=1}^{n}\Pi_{it}'\Delta v_{it}\\ &= \textstyle\sum_{i=1}^{n}\sum_{t=2}^{T}\Pi_{it}'\Delta v_{it}\equiv\sum_{i=1}^{n}g_{1i}.\end{aligned}$$

Clearly, $\{g_{1i}\}$ are independent with mean zero, and thus form a vector M.D. sequence. Now, for the terms quadratic in Δv, we have,

$$\begin{aligned}\mathrm{E}(\Delta v'\Phi\Delta v) &= \sigma_{v0}^2\mathrm{tr}[(C\otimes I_n)\Phi]\\ &= \textstyle\sigma_{v0}^2\sum_{t=2}^{T}\sum_{s=2}^{T}\mathrm{tr}(c_{ts}\Phi_{st})\\ &= \textstyle\sigma_{v0}^2\sum_{i=1}^{n}\sum_{t=2}^{T}\sum_{s=2}^{T}(c_{ts}\Phi_{ii,st})\equiv\sigma_{v0}^2\sum_{i=1}^{n}\sum_{t=2}^{T}d_{it},\end{aligned}$$

where $\{c_{ts}, t, s = 2,\ldots,T\}$ are the elements of the matrix C given in Section 3.1, $\{\Phi_{ii,ts}, i = 1,\ldots,n\}$ are the diagonal elements of Φ_{ts}, and $d_{it}=\sum_{s=2}^{T}(c_{ts}\Phi_{ii,st})$; and

$$\begin{aligned}\Delta v'\Phi\Delta v-\mathrm{E}(\Delta v'\Phi\Delta v) &= \textstyle\sum_{t=2}^{T}\sum_{s=2}^{T}\Delta v_t'\Phi_{ts}\Delta v_s-\sigma_{v0}^2\sum_{i=1}^{n}\sum_{t=2}^{T}d_{it}\\ &= \textstyle\sum_{t=2}^{T}\sum_{s=2}^{T}\Delta v_t'(\Phi_{ts}^u+\Phi_{ts}^l+\Phi_{ts}^d)\Delta v_s-\sigma_{v0}^2\sum_{i=1}^{n}\sum_{t=2}^{T}d_{it}\\ &= \textstyle\sum_{t=2}^{T}\sum_{s=2}^{T}\left[\Delta v_s'\Phi_{ts}^{u\prime}\Delta v_t+\Delta v_t'(\Phi_{ts}^l+\Phi_{ts}^d)\Delta v_s\right]-\sigma_{v0}^2\sum_{i=1}^{n}\sum_{t=2}^{T}d_{it}\\ &= \textstyle\sum_{t=2}^{T}\Delta v_t'\Delta\xi_t+\sum_{t=2}^{T}\Delta v_t'\Delta v_t^*-\sigma_{v0}^2\sum_{i=1}^{n}\sum_{t=2}^{T}d_{it},\\ &= \textstyle\sum_{i=1}^{n}\sum_{t=2}^{T}\left(\Delta v_{it}\Delta\xi_{it}+\Delta v_{it}\Delta v_{it}^*-\sigma_{v0}^2d_{it}\right)\\ &\equiv \textstyle\sum_{i=1}^{n}g_{2i},\end{aligned}$$

where $\Delta\xi_t=\sum_{s=2}^{T}(\Phi_{st}^{u\prime}+\Phi_{ts}^l)\Delta v_s$, and $\Delta v_t^*=\sum_{s=2}^{T}\Phi_{ts}^d\Delta v_s$. Noting that $\Delta\xi_{it}$ is $\mathcal{G}_{n,i-1}$-measurable, it is easy to see that $\mathrm{E}(g_{2i}|\mathcal{G}_{n,i-1})=0$. Thus, $\{g_{2i},\mathcal{G}_{n,i}\}$ form a M.D. sequence.

Finally, for the terms bilinear in Δv and $\Delta\mathbf{y}_1 = 1_{T-1}\otimes\Delta y_1$, we have,

$$\begin{aligned}\Delta v'\Psi\mathbf{y}_1 &= \textstyle\sum_{t=2}^{T}\sum_{s=2}^{T}\Delta v_t'\Psi_{ts}\Delta y_1\\ &= \textstyle\sum_{t=2}^{T}\Delta v_t'\big(\sum_{s=2}^{T}\Psi_{ts}\big)\Delta y_1\\ &= \textstyle\sum_{t=2}^{T}\Delta v_t'\Psi_{t+}\Delta y_1\\ &= \textstyle\Delta v_2'\Psi_{2+}\Delta y_1+\sum_{t=3}^{T}\Delta v_t'\Psi_{t+}\Delta y_1\\ &= \textstyle\Delta v_2'\Theta\Delta y_1^\circ+\sum_{t=3}^{T}\Delta v_t'\Delta y_{1t}^*,\end{aligned}$$

where $\Delta y_1^\circ = B_{30}B_{10}\Delta y_1$ and $\Delta y_{1t}^*=\Psi_{t+}\Delta y_1$. The second term equals $\sum_{i=1}^{n}(\sum_{t=3}^{T}\Delta v_{it}\Delta y_{1it}^*)$, which is the sum of n uncorrelated terms of mean zero, due to the fact that Δy_1 is independent of $\Delta v_t, t\geq 3$. The term $\Delta v_2'\Theta\Delta y_1^\circ$ needs some special attention. Noting that

$$\Delta y_1^\circ = B_{30}B_{10}\Delta y_1 = B_{30}B_{20}\Delta y_0+B_{30}\Delta x_1\beta_0+\Delta v_1, \tag{B.1}$$

and as Δy_0 is independent of $v_t, t\geq 1$ by Assumption A, $\mathrm{E}(\Delta v_2'\Theta\Delta y_1^\circ)=-\sigma_{v0}^2\mathrm{tr}(\Theta)$, and

$$\begin{aligned}\Delta v_2'\Theta\Delta y_1^\circ-\mathrm{E}(\Delta v_2'\Theta\Delta y_1^\circ) &= \Delta v_2'(\Theta^u+\Theta^l+\Theta^d)\Delta y_1^\circ+\sigma_{v0}^2\mathrm{tr}(\Theta)\\ &= \Delta v_2'(\Theta^u+\Theta^l)\Delta y_1^\circ+\Delta v_2'\Theta^d\Delta y_1^\circ+\sigma_{v0}^2\mathrm{tr}(\Theta)\\ &= \textstyle\sum_{i=1}^{n}\Delta v_{2i}\Delta\zeta_i+\sum_{i=1}^{n}\Theta_{ii}(\Delta v_{2i}\Delta y_{1i}^\circ+\sigma_{v0}^2),\end{aligned}$$

where $\{\Delta\zeta_i\} = \Delta\zeta = (\Theta^u + \Theta^l)\Delta y_1^\circ$. As $\Delta\zeta_i$ is measurable w.r.t. $\mathcal{F}_{n,i-1}$ and $\{\Delta v_{1,i+1}, \ldots \Delta v_{1,n}\}$, the first term is the sum of a M.D. sequence. The second term is easily seen to be the sum of n uncorrelated terms by (B.1). It follows that $\Delta v'\Psi\Delta\mathbf{y}_1 - \mathrm{E}(\Delta v'\Psi\Delta\mathbf{y}_1) = \sum_{i=1}^n g_{3i}$, where

$$g_{3i} = \Delta v_{2i}\Delta\zeta_i + \Theta_{ii}(\Delta v_{2i}\Delta y_{1i}^\circ + \sigma_{v0}^2) + \textstyle\sum_{t=3}^T \Delta v_{it}\Delta y_{1it}^*.$$

It is easy to see that $\mathrm{E}(g_{3i}|\mathcal{F}_{n,i-1}) = 0$. Hence, $\{g_{3i}, \mathcal{F}_{n,i}\}$ form a M.D. sequence. Finally, it is easy to verify that $\mathrm{E}[(g_{1i}', g_{2i}, g_{3i})|\mathcal{F}_{n,i-1}] = 0$. Hence, $\{(g_{1i}', g_{2i}, g_{3i})', \mathcal{F}_{n,i}\}$ form a vector M.D. sequence. ■

Appendix C: Proofs of Theorems 3.1-3.3

In proving the theorems, the following matrix results are used: (i) the eigenvalues of a projection matrix are either 0 or 1; (ii) the eigenvalues of a positive definite (p.d.) matrix are strictly positive; (iii) $\gamma_{\min}(A)\mathrm{tr}(B) \leq \mathrm{tr}(AB) \leq \gamma_{\max}(A)\mathrm{tr}(B)$ for symmetric matrix A and positive semidefinite (p.s.d.) matrix B; (iv) $\gamma_{\max}(A+B) \leq \gamma_{\max}(A) + \gamma_{\max}(B)$ for symmetric matrices A and B; and (v) $\gamma_{\max}(AB) \leq \gamma_{\max}(A)\gamma_{\max}(B)$ for p.s.d. matrices A and B. See, e.g, Bernstein (2009).

Proof of Theorem 3.1: From (3.17) and (3.20), we have

$$S^{*c}_{\mathtt{STLE}}(\delta) - \bar{S}^{*c}_{\mathtt{STLE}}(\delta) = \begin{cases} \frac{1}{\hat{\sigma}^2_{v,\mathtt{M}}(\delta)}\Delta\hat{u}(\delta)'\Omega^{-1}\Delta Y_{-1} - \frac{1}{\bar{\sigma}^2_{v,\mathtt{M}}(\delta)}\mathrm{E}[\Delta\bar{u}(\delta)'\Omega^{-1}\Delta Y_{-1}], \\ \frac{1}{\hat{\sigma}^2_{v,\mathtt{M}}(\delta)}\Delta\hat{u}(\delta)'\Omega^{-1}\mathbf{W}_1\Delta Y - \frac{1}{\bar{\sigma}^2_{v,\mathtt{M}}(\delta)}\mathrm{E}[\Delta\bar{u}(\delta)'\Omega^{-1}\mathbf{W}_1\Delta Y], \\ \frac{1}{\hat{\sigma}^2_{v,\mathtt{M}}(\delta)}\Delta\hat{u}(\delta)'\Omega^{-1}\mathbf{W}_2\Delta Y_{-1} - \frac{1}{\bar{\sigma}^2_{v,\mathtt{M}}(\delta)}\mathrm{E}[\Delta\bar{u}(\delta)'\Omega^{-1}\mathbf{W}_2\Delta Y_{-1}], \\ \frac{1}{\hat{\sigma}^2_{v,\mathtt{M}}(\delta)}\Delta\hat{u}(\delta)'\Upsilon\Delta\hat{u}(\delta) - \frac{1}{\bar{\sigma}^2_{v,\mathtt{M}}(\delta)}\mathrm{E}[\Delta\bar{u}(\delta)'\Upsilon\Delta\bar{u}(\delta)], \end{cases}$$

where $\Upsilon = \frac{1}{2}(C^{-1} \otimes A_3)$. With Assumption G, consistency of $\hat{\delta}_{\mathtt{M}}$ follows from:

- (**a**) $\inf_{\delta\in\boldsymbol{\Delta}}\bar{\sigma}^2_{v,\mathtt{M}}(\delta)$ is bounded away from zero,
- (**b**) $\sup_{\delta\in\boldsymbol{\Delta}}\big|\hat{\sigma}^2_{v,\mathtt{M}}(\delta) - \bar{\sigma}^2_{v,\mathtt{M}}(\delta)\big| = o_p(1)$,
- (**c**) $\sup_{\delta\in\boldsymbol{\Delta}}\frac{1}{n(T-1)}\big|\Delta\hat{u}(\delta)'\Omega^{-1}\Delta Y_{-1} - \mathrm{E}[\Delta\bar{u}(\delta)'\Omega^{-1}\Delta Y_{-1}]\big| = o_p(1)$,
- (**d**) $\sup_{\delta\in\boldsymbol{\Delta}}\frac{1}{n(T-1)}\big|\Delta\hat{u}(\delta)'\Omega^{-1}\mathbf{W}_1\Delta Y - \mathrm{E}[\Delta\bar{u}(\delta)'\Omega^{-1}\mathbf{W}_1\Delta Y]\big| = o_p(1)$,
- (**e**) $\sup_{\delta\in\boldsymbol{\Delta}}\frac{1}{n(T-1)}\big|\Delta\hat{u}(\delta)'\Omega^{-1}\mathbf{W}_2\Delta Y_{-1} - \mathrm{E}[\Delta\bar{u}(\delta)'\Omega^{-1}\mathbf{W}_2\Delta Y_{-1}]\big| = o_p(1)$,
- (**f**) $\sup_{\delta\in\boldsymbol{\Delta}}\frac{1}{n(T-1)}\big|\Delta\hat{u}(\delta)'\Upsilon\Delta\hat{u}(\delta) - \mathrm{E}[\Delta\bar{u}(\delta)'\Upsilon\Delta\bar{u}(\delta)]\big| = o_p(1)$.

Proof of (a). By $\Delta\bar{u}^*(\delta) = \mathbf{M}(\mathbf{B}_1^*\Delta Y - \mathbf{B}_2^*\Delta Y_{-1}) + \mathbf{P}(\mathbf{B}_1^*\Delta Y^\circ - \mathbf{B}_2^*\Delta Y_{-1}^\circ)$ given in (3.21), and the orthogonality between the two projection matrices $\mathbf{M}$ and $\mathbf{P}$, we have,

$$\begin{aligned} \bar{\sigma}^2_{v,\mathtt{M}}(\delta) &= \tfrac{1}{n(T-1)}\mathrm{E}[\Delta\bar{u}^{*\prime}(\delta)\Delta\bar{u}^*(\delta)] \\ &= \tfrac{1}{n(T-1)}\mathrm{E}[(\mathbf{B}_1^*\Delta Y - \mathbf{B}_2^*\Delta Y_{-1})'\mathbf{M}(\mathbf{B}_1^*\Delta Y - \mathbf{B}_2^*\Delta Y_{-1})] \\ &\quad + \tfrac{1}{n(T-1)}\mathrm{E}[(\mathbf{B}_1^*\Delta Y^\circ - \mathbf{B}_2^*\Delta Y_{-1}^\circ)'\mathbf{P}(\mathbf{B}_1^*\Delta Y^\circ - \mathbf{B}_2^*\Delta Y_{-1}^\circ)] \\ &= \tfrac{1}{n(T-1)}\mathrm{tr}[\mathrm{Var}(\mathbf{B}_1^*\Delta Y - \mathbf{B}_2^*\Delta Y_{-1})] \\ &\quad + \tfrac{1}{n(T-1)}(\mathbf{B}_1^*\mathrm{E}\Delta Y - \mathbf{B}_2^*\mathrm{E}\Delta Y_{-1})'\mathbf{M}(\mathbf{B}_1^*\mathrm{E}\Delta Y - \mathbf{B}_2^*\mathrm{E}\Delta Y_{-1}). \end{aligned}$$

As $\mathbf{M}$ is p.s.d., the second term is nonnegative uniformly in $\delta \in \mathbf{\Delta}$. The first term is $\frac{1}{n(T-1)}\text{tr}[\Omega^{-1}\text{Var}(\mathbf{B}_1\Delta Y - \mathbf{B}_2\Delta Y_{-1})] \geq \frac{1}{n(T-1)}\gamma_{\min}(C^{-1})\gamma_{\min}(B_3'B_3)\text{tr}[\text{Var}(\mathbf{B}_1\Delta Y - \mathbf{B}_2\Delta Y_{-1})]$ $> c > 0$, uniformly in $\delta \in \mathbf{\Delta}$, by the definition of the matrix C, Assumption E(iv) and the assumption given in the theorem. It follows that $\inf_{\delta\in\mathbf{\Delta}}\bar{\sigma}^2_{v,\mathtt{M}}(\delta) > c > 0$.

Proof of (b). Noting that $\Delta\hat{u}^*(\delta) = \Omega^{-\frac{1}{2}}\Delta\hat{u}(\delta) = \mathbf{M}(\mathbf{B}_1^*\Delta Y - \mathbf{B}_2^*\Delta Y_{-1})$, we have,

$$\hat{\sigma}^2_{v,\mathtt{M}}(\delta) = \tfrac{1}{n(T-1)}\Delta\hat{u}^{*\prime}(\delta)\Delta\hat{u}^*(\delta) = \tfrac{1}{n(T-1)}(\mathbf{B}_1^*\Delta Y - \mathbf{B}_2^*\Delta Y_{-1})'\mathbf{M}(\mathbf{B}_1^*\Delta Y - \mathbf{B}_2^*\Delta Y_{-1}).$$

It follows that

$$\begin{aligned}
\hat{\sigma}^2_{v,\mathtt{M}}(\delta) - \bar{\sigma}^2_{v,\mathtt{M}}(\delta) &= \tfrac{1}{n(T-1)}(\mathbf{B}_1^*\Delta Y - \mathbf{B}_2^*\Delta Y_{-1})'\mathbf{M}(\mathbf{B}_1^*\Delta Y - \mathbf{B}_2^*\Delta Y_{-1}) \\
&\quad -\tfrac{1}{n(T-1)}\text{E}[(\mathbf{B}_1^*\Delta Y - \mathbf{B}_2^*\Delta Y_{-1})'\mathbf{M}(\mathbf{B}_1^*\Delta Y - \mathbf{B}_2^*\Delta Y_{-1})] \\
&\quad -\tfrac{1}{n(T-1)}\text{E}[(\mathbf{B}_1^*\Delta Y^\circ - \mathbf{B}_2^*\Delta Y^\circ_{-1})'\mathbf{P}(\mathbf{B}_1^*\Delta Y^\circ - \mathbf{B}_2^*\Delta Y^\circ_{-1})] \\
&= \tfrac{1}{n(T-1)}[\Delta Y'\mathbf{B}_1^{*\prime}\mathbf{M}\mathbf{B}_1^*\Delta Y - \text{E}(\Delta Y'\mathbf{B}_1^{*\prime}\mathbf{M}\mathbf{B}_1^*\Delta Y)] \\
&\quad +\tfrac{1}{n(T-1)}[\Delta Y_{-1}'\mathbf{B}_2^{*\prime}\mathbf{M}\mathbf{B}_2^*\Delta Y_{-1} - \text{E}(\Delta Y_{-1}'\mathbf{B}_2^{*\prime}\mathbf{M}\mathbf{B}_2^*\Delta Y_{-1})] \\
&\quad -\tfrac{2}{n(T-1)}[\Delta Y'\mathbf{B}_1^{*\prime}\mathbf{M}\mathbf{B}_2^*\Delta Y_{-1} - \text{E}(\Delta Y'\mathbf{B}_1^{*\prime}\mathbf{M}\mathbf{B}_2^*\Delta Y_{-1})] \\
&\quad -\tfrac{1}{n(T-1)}\text{E}[(\mathbf{B}_1^*\Delta Y^\circ - \mathbf{B}_2^*\Delta Y^\circ_{-1})'\mathbf{P}(\mathbf{B}_1^*\Delta Y^\circ - \mathbf{B}_2^*\Delta Y^\circ_{-1})] \\
&\equiv (Q_1 - \text{E}Q_1) + (Q_2 - \text{E}Q_2) - 2(Q_3 - \text{E}Q_3) - \text{E}Q_4.
\end{aligned}$$

The results follows if $Q_j - \text{E}Q_j \xrightarrow{p} 0, j = 1, 2, 3$, and $\text{E}Q_4 \longrightarrow 0$, uniformly in $\delta \in \mathbf{\Delta}$.

The uniform convergence of $Q_j - \text{E}Q_j, j = 1, 2, 3$, to zero in probability follows from the pointwise convergence for each $\delta \in \mathbf{\Delta}$ and stochastic equicontinuity of Q_j, according to Theorem 1 of Andrews (1992). Let $\mathbf{M}^* = \Omega^{-\frac{1}{2}}\mathbf{M}\Omega^{-\frac{1}{2}}$. We have, by the two identities: $\Delta Y = \mathbb{R}\,\Delta\mathbf{y}_1 + \boldsymbol{\eta} + \mathbb{S}\Delta v$ and $\Delta Y_{-1} = \mathbb{R}_{-1}\Delta\mathbf{y}_1 + \boldsymbol{\eta}_{-1} + \mathbb{S}_{-1}\Delta v$, given in Lemma 3.2,

$$\begin{aligned}
Q_1 &= \tfrac{1}{n(T-1)}\Delta Y'\mathbf{B}_1^{*\prime}\mathbf{M}\mathbf{B}_1^*\Delta Y \\
&= \tfrac{1}{n(T-1)}\big(\Delta\mathbf{y}_1'\mathbb{R}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_1\mathbb{R}\Delta\mathbf{y}_1 + \boldsymbol{\eta}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_1\boldsymbol{\eta} + \Delta v'\mathbb{S}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_1\mathbb{S}\Delta v \\
&\quad +2\Delta\mathbf{y}_1'\mathbb{R}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_1\boldsymbol{\eta} + 2\Delta\mathbf{y}_1'\mathbb{R}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_1\mathbb{S}\Delta v + 2\boldsymbol{\eta}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_1\mathbb{S}\Delta v\big),
\end{aligned}$$

which gives $Q_1 - \text{E}Q_1 = \sum_{\ell=1}^5(Q_{1,\ell} - \text{E}Q_{1,\ell})$, where $Q_{1,\ell}, \ell = 1, \ldots, 5$, denote the five stochastic terms of Q_1, and $\text{E}Q_{1,5} = 0$;

$$\begin{aligned}
Q_2 &= \tfrac{1}{n(T-1)}\Delta Y_{-1}'\mathbf{B}_2^{*\prime}\mathbf{M}\mathbf{B}_2^*\Delta Y_{-1} \\
&= \tfrac{1}{n(T-1)}\big(\Delta\mathbf{y}_1'\mathbb{R}_{-1}'\mathbf{B}_2'\mathbf{M}^*\mathbf{B}_2\mathbb{R}_{-1}\Delta\mathbf{y}_1 + \boldsymbol{\eta}_{-1}'\mathbf{B}_2'\mathbf{M}^*\mathbf{B}_2\boldsymbol{\eta}_{-1} + \Delta v'\mathbb{S}_{-1}'\mathbf{B}_2'\mathbf{M}^*\mathbf{B}_2\mathbb{S}_{-1}\Delta v \\
&\quad +2\Delta\mathbf{y}_1'\mathbb{R}_{-1}'\mathbf{B}_2'\mathbf{M}^*\mathbf{B}_2\boldsymbol{\eta}_{-1} + 2\Delta\mathbf{y}_1'\mathbb{R}'\mathbf{B}_2'\mathbf{M}^*\mathbf{B}_2\mathbb{S}_{-1}\Delta v + 2\boldsymbol{\eta}_{-1}'\mathbf{B}_2'\mathbf{M}^*\mathbf{B}_2\mathbb{S}_{-1}\Delta v\big),
\end{aligned}$$

leading to $Q_2 - \text{E}Q_2 = \sum_{\ell=1}^5(Q_{2,\ell} - \text{E}Q_{2,\ell})$, where $Q_{2,\ell}, \ell = 1, \ldots, 5$, denote the five stochastic terms of Q_2, and $\text{E}Q_{2,5} = 0$; and

$$\begin{aligned}
Q_3 &= \tfrac{1}{n(T-1)}\Delta Y'\mathbf{B}_1^{*\prime}\mathbf{M}\mathbf{B}_2^*\Delta Y_{-1} \\
&= \tfrac{1}{n(T-1)}\big(\Delta\mathbf{y}_1'\mathbb{R}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_2\mathbb{R}_{-1}\Delta\mathbf{y}_1 + \boldsymbol{\eta}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_2\boldsymbol{\eta}_{-1} + \Delta v'\mathbb{S}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_2\mathbb{S}_{-1}\Delta v \\
&\quad +\Delta\mathbf{y}_1'\mathbb{R}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_2\boldsymbol{\eta}_{-1} + \boldsymbol{\eta}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_2\mathbb{R}_{-1}\Delta\mathbf{y}_1 + \Delta\mathbf{y}_1'\mathbb{R}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_2\mathbb{S}_{-1}\Delta v \\
&\quad +\Delta v'\mathbb{S}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_2\mathbb{R}_{-1}\Delta\mathbf{y}_1' + \boldsymbol{\eta}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_2\mathbb{S}_{-1}\Delta v + \Delta v'\mathbb{S}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_2\boldsymbol{\eta}_{-1}\big),
\end{aligned}$$

leading to $Q_3 - \mathrm{E}Q_3 = \sum_{\ell=1}^{8}(Q_{3,\ell} - \mathrm{E}Q_{3,\ell})$, where $Q_{3,\ell}$, $\ell = 1, \ldots, 8$, denote the eight random terms in Q_3 and the last two terms have expectations zero.

Thus, $Q_k, k = 1, 2, 3$, are decomposed into sums of terms of the forms: $\frac{1}{n(T-1)}\Delta\mathbf{y}_1'\Phi\Delta\mathbf{y}_1$, $\frac{1}{n(T-1)}\Delta v'\Pi\Delta v$, $\frac{1}{n(T-1)}\Delta\mathbf{y}_1'\Psi\Delta v$, $\frac{1}{n(T-1)}\Delta\mathbf{y}_1'\phi$, and $\frac{1}{n(T-1)}\Delta v'\xi$, where the matrices Φ, Π and Ψ, and the vectors ϕ and ξ are defined in terms of $\mathbb{R}$, $\mathbb{R}_{-1}$, $\mathbb{S}$, $\mathbb{S}_{-1}$, $\boldsymbol{\eta}$, $\boldsymbol{\eta}_{-1}$, $\mathbf{B}_1$, $\mathbf{B}_2$ and $\mathbf{M}^*$. Note that $\mathbb{R}$, $\mathbb{R}_{-1}$, $\mathbb{S}$, $\mathbb{S}_{-1}$, $\boldsymbol{\eta}$ and $\boldsymbol{\eta}_{-1}$ depend on true parameter values, whereas $\mathbf{B}_1$ depends on λ_1, $\mathbf{B}_2$ depends on ρ and λ_2, and $\mathbf{M}^*$ depends on λ_3.

For the terms quadratic in $\Delta\mathbf{y}_1$, they can be written as $\frac{1}{n}\Delta y_1'\Phi_{++}(\delta)\Delta y_1$ where $\Phi_{++}(\delta) = \frac{1}{T-1}\sum_t\sum_s \Phi_{t,s}(\delta)$. It can easily be seen by Lemma A.1 and Lemma A.3 that for each $\delta \in \boldsymbol{\Delta}$, $\Phi_{t,s}(\delta)$ are uniformly bounded in either row or column sums. The pointwise convergence of $\frac{1}{n}[\Delta y_1'\Phi_{++}(\delta)\Delta y_1 - \mathrm{E}(\Delta y_1'\Phi_{++}(\delta)\Delta y_1)]$ thus follows from Assumption F(iii). For the terms quadratic in Δv, they can be written as $\frac{1}{n(T-1)}\sum_{t=1}^{T}\sum_{s=1}^{T} v_t'\Pi_{ts}v_s$. The pointwise convergence of $\frac{1}{n}[v_t'\Pi_{ts}v_s - \mathrm{E}(v_t'\Pi_{ts}v_s)]$ follows from Lemma A.4 (v), for each $t, s = 1, \ldots, T$. The pointwise convergence of $\frac{1}{n(T-1)}[\Delta\mathbf{y}_1'\Psi\Delta v - \mathrm{E}(\Delta\mathbf{y}_1'\Psi\Delta v)]$ follows by writing $\Delta\mathbf{y}_1'\Psi\Delta v = \sum_s \Delta y_1\Psi_{+s}\Delta v_s$ and then applying Lemma A.4 (vii) and Assumption F(iv). The pointwise convergence of $\frac{1}{n(T-1)}[\Delta\mathbf{y}_1'\phi - \mathrm{E}(\Delta\mathbf{y}_1'\phi)]$ follows from Assumption F(ii), and of $\frac{1}{n(T-1)}\Delta v'\xi$ from Chebyshev inequality. Thus, $Q_{k,\ell}(\delta) - \mathrm{E}Q_{k,\ell}(\delta) \xrightarrow{p} 0$, for each $\delta \in \boldsymbol{\Delta}$, and all k and ℓ.

Now, for all the $Q_{k,\ell}(\delta)$ terms, let δ_1 and δ_2 be in $\boldsymbol{\Delta}$. We have by the mean value theorem:

$$Q_{k,\ell}(\delta_2) - Q_{k,\ell}(\delta_1) = \tfrac{\partial}{\partial\delta'}Q_{k,\ell}(\bar{\delta})(\delta_2 - \delta_1),$$

where $\bar{\delta}$ lies between δ_1 and δ_2 elementwise. Note that $Q_{k,\ell}(\delta)$ is linear or quadratic in ρ, λ_1 and λ_2, and thus the corresponding partial derivatives takes simple form. It is easy to show that $\sup_{\delta\in\boldsymbol{\Delta}}|\frac{\partial}{\partial\omega}Q_{k,\ell}(\delta)| = O_p(1)$, for $\omega = \rho, \lambda_1, \lambda_2$. For $\frac{\partial}{\partial\lambda_3}Q_{k,\ell}(\delta)$, note that only the matrix M^* involves λ_3. Some algebra leads to the following simple expression for its derivative:

$$\tfrac{d}{d\lambda_3}M^* = \mathbf{M}^*\Omega\dot{\Omega}^{-1}\Omega\mathbf{M}^*,$$

where $\dot{\Omega}^{-1} = \frac{d}{d\lambda_3}\Omega^{-1} = C^{-1}\otimes A_3$. Thus, the results $\sup_{\delta\in\boldsymbol{\Delta}}|\frac{\partial}{\partial\lambda_3}Q_{k,\ell}(\delta)| = O_p(1)$ can be easily proved for all the $Q_{k,\ell}(\delta)$ quantities. For example, for $Q_{1,1}(\delta)$, noting that $\gamma_{\max}(\mathbf{M}) = 1$,

$$\begin{aligned}
\sup_{\delta\in\boldsymbol{\Delta}}|\tfrac{\partial}{\partial\lambda_3}Q_{1,1}(\delta)| &= \sup_{\delta\in\boldsymbol{\Delta}}|\tfrac{1}{n(T-1)}\tfrac{\partial}{\partial\lambda_3}\Delta\mathbf{y}_1'\mathbb{R}'\mathbf{B}_1'\mathbf{M}^*\mathbf{B}_1\mathbb{R}\Delta\mathbf{y}_1| \\
&= \sup_{\delta\in\boldsymbol{\Delta}}\tfrac{1}{n(T-1)}|\Delta\mathbf{y}_1'\mathbb{R}'\mathbf{B}_1'\mathbf{M}^*\Omega\dot{\Omega}^{-1}\Omega\mathbf{M}^*\mathbf{B}_1\mathbb{R}\Delta\mathbf{y}_1| \\
&\leq \sup_{\delta\in\boldsymbol{\Delta}}\tfrac{1}{n(T-1)}|\Delta\mathbf{y}_1'\mathbb{R}'\mathbf{B}_1'\dot{\Omega}^{-1}\mathbf{B}_1\mathbb{R}\Delta\mathbf{y}_1| \\
&\leq \gamma_{\max}(\dot{\Omega}^{-1})\gamma_{\max}(\mathbf{B}_1'\mathbf{B}_1)\tfrac{1}{n(T-1)}|\Delta\mathbf{y}_1'\mathbb{R}'\mathbb{R}\Delta\mathbf{y}_1| \\
&= O(1)\times O(1)\times O_p(1) = O_p(1), \text{ by Assumption F(i).}
\end{aligned}$$

It follows that $Q_{k,\ell}(\delta)$ are stochastically equicontinuous. Hence, by Theorem 1 of Andrews (1992), $Q_{k,\ell}(\delta) - \mathrm{E}Q_{k,\ell}(\delta) \xrightarrow{p} 0$, uniformly in $\delta \in \boldsymbol{\Delta}$ for all k and ℓ. It follows that $Q_k(\delta) - \mathrm{E}Q_k(\delta) \xrightarrow{p} 0$, uniformly in $\delta \in \boldsymbol{\Delta}$, $k = 1, 2, 3$.

It left to show that $\mathrm{E}Q_4(\delta) \to 0$, uniformly in $\delta \in \boldsymbol{\Delta}$. We have,

$$
\begin{aligned}
\mathrm{E}Q_4 &= \tfrac{1}{n(T-1)}\mathrm{E}[(\mathbf{B}_1^*\Delta Y^\circ - \mathbf{B}_2^*\Delta Y_{-1}^\circ)'\mathbf{P}(\mathbf{B}_1^*\Delta Y^\circ - \mathbf{B}_2^*\Delta Y_{-1}^\circ)]\\
&= \tfrac{1}{n(T-1)}\mathrm{tr}[\Omega^{-1}\Delta X(\Delta X'\Omega^{-1}\Delta X)^{-1}\Delta X'\Omega^{-1}\mathrm{Var}(\mathbf{B}_1\Delta Y - \mathbf{B}_2\Delta Y_{-1})]\\
&\le \tfrac{1}{n(T-1)}\gamma_{\max}(\Omega^{-2})\gamma_{\min}^{-1}(\Delta X'\Omega^{-1}\Delta X)\ \mathrm{tr}[\Delta X'\mathrm{Var}(\mathbf{B}_1\Delta Y - \mathbf{B}_2\Delta Y_{-1})\Delta X]\\
&= \tfrac{1}{n(T-1)}\gamma_{\max}(\Omega^{-2})\gamma_{\min}^{-1}\big(\tfrac{\Delta X'\Omega^{-1}\Delta X}{n(T-1)}\big)\ \tfrac{1}{n(T-1)}\mathrm{tr}[\Delta X'\mathrm{Var}(\mathbf{B}_1\Delta Y - \mathbf{B}_2\Delta Y_{-1})\Delta X].
\end{aligned}
$$

As $\Omega^{-1} = C^{-1}\otimes B_3'B_3$, we have by the matrix C defined at the beginning of Section 3.1 and Assumption E(iv), $0 < \underline{c}_w \le \inf_{\lambda_3\in\Lambda_3}\gamma_{\min}(\Omega^{-1}) \le \sup_{\lambda_3\in\Lambda_3}\gamma_{\min}(\Omega^{-1}) \le \bar{c}_w < \infty$. By Assumption D, we have, $0 < \underline{c}_x \le \inf_{\lambda_3\in\Lambda_3}\gamma_{\min}(\Omega^{-1})\gamma_{\min}\big(\frac{\Delta X'\Delta X}{n(T-1)}\big) \le \gamma_{\min}\big(\frac{\Delta X'\Omega^{-1}\Delta X}{n(T-1)}\big) \le \gamma_{\max}\big(\frac{\Delta X'\Omega^{-1}\Delta X}{n(T-1)}\big) \le \sup_{\lambda_3\in\Lambda_3}\gamma_{\max}(\Omega^{-1})\gamma_{\max}\big(\frac{\Delta X'\Delta X}{n(T-1)}\big) \le \bar{c}_x < \infty$. It follows that

$$
\begin{aligned}
\mathrm{E}Q_4 &\le \tfrac{1}{n(T-1)}\bar{c}_w^2\underline{c}_x\tfrac{1}{n(T-1)}\mathrm{tr}[\Delta X'\mathrm{Var}(\mathbf{B}_1\Delta Y - \mathbf{B}_2\Delta Y_{-1})\Delta X]\\
&\le \tfrac{1}{n(T-1)}\bar{c}_w^2\underline{c}_x\bar{c}_y\ \tfrac{1}{n(T-1)}\mathrm{tr}[\Delta X'\Delta X],\ \text{ by the assumption in Theorem 3.1}\\
&= O(n^{-1}),\ \text{ by Assumption D.}
\end{aligned}
$$

Hence, $\hat{\sigma}^2_{v,\mathtt{M}}(\delta) - \bar{\sigma}^2_{v,\mathtt{M}}(\delta) \xrightarrow{p} 0$, uniformly in $\delta\in\mathbf{\Delta}$, completing the proof of **(b)**.

Proofs of (c)-(f). By the expressions of $\Delta\hat{u}(\delta)$ and $\Delta\bar{u}(\delta)$ given earlier, we have,

$$
\begin{aligned}
&\Delta\hat{u}(\delta)'\Omega^{-1}\Delta Y_{-1} - \mathrm{E}[\Delta\bar{u}(\delta)'\Omega^{-1}\Delta Y_{-1}]\\
=\ &[\Delta Y'\mathbf{B}_1'\mathbf{M}^*\Delta Y_{-1} - \mathrm{E}(\Delta Y'\mathbf{B}_1'\mathbf{M}^*\Delta Y_{-1})] - [\Delta Y_{-1}'\mathbf{B}_2'\mathbf{M}^*\Delta Y_{-1} - \mathrm{E}(\Delta Y_{-1}'\mathbf{B}_2'\mathbf{M}^*\Delta Y_{-1})]\\
&-\mathrm{E}(\Delta Y^{\circ\prime}\mathbf{B}_1'\mathbf{P}^*\Delta Y_{-1}^\circ) + \mathrm{E}(\Delta Y_{-1}^{\circ\prime}\mathbf{B}_2'\mathbf{P}^*\Delta Y_{-1}^\circ);\\
&\Delta\hat{u}(\delta)'\Omega^{-1}\mathbf{W}_1\Delta Y - \mathrm{E}[\Delta\bar{u}(\delta)'\Omega^{-1}\mathbf{W}_1\Delta Y]\\
=\ &\Delta Y'\mathbf{B}_1'\mathbf{M}^*\mathbf{W}_1\Delta Y - \mathrm{E}(\Delta Y'\mathbf{B}_1'\mathbf{M}^*\mathbf{W}_1\Delta Y) - \Delta Y_{-1}'\mathbf{B}_2'\mathbf{M}^*\mathbf{W}_1\Delta Y\\
&-\mathrm{E}(\Delta Y_{-1}'\mathbf{B}_2'\mathbf{M}^*\mathbf{W}_1\Delta Y] - \mathrm{E}(\Delta Y^{\circ\prime}\mathbf{B}_1'\mathbf{P}^*\mathbf{W}_1\Delta Y^\circ) + \mathrm{E}(\Delta Y_{-1}^{\circ\prime}\mathbf{B}_2'\mathbf{P}^*\mathbf{W}_1\Delta Y^\circ);\\
&\Delta\hat{u}(\delta)'\Omega^{-1}\mathbf{W}_2\Delta Y_{-1} - \mathrm{E}[\Delta\bar{u}(\delta)'\Omega^{-1}\mathbf{W}_2\Delta Y_{-1}]\\
=\ &\Delta Y'\mathbf{B}_1'\mathbf{M}^*\mathbf{W}_2\Delta Y_{-1} - \mathrm{E}(\Delta Y'\mathbf{B}_1'\mathbf{M}^*\mathbf{W}_2\Delta Y_{-1}) - \Delta Y_{-1}'\mathbf{B}_2'\mathbf{M}^*\mathbf{W}_2\Delta Y_{-1}\\
&-\mathrm{E}(\Delta Y_{-1}'\mathbf{B}_2'\mathbf{M}^*\mathbf{W}_2\Delta Y_{-1}] - \mathrm{E}(\Delta Y^{\circ\prime}\mathbf{B}_1'\mathbf{P}^*\mathbf{W}_2\Delta Y_{-1}^\circ) + \mathrm{E}(\Delta Y_{-1}^{\circ\prime}\mathbf{B}_2'\mathbf{P}^*\mathbf{W}_2\Delta Y_{-1}^\circ);\ \text{and}\\
&\Delta\hat{u}(\delta)'\Upsilon\Delta\hat{u}(\delta) - \mathrm{E}[\Delta\bar{u}(\delta)'\\
=\ &\Delta Y'\mathbf{B}_1'\mathbf{M}^{\circ\prime}\Upsilon\mathbf{M}^\circ\mathbf{B}_1\Delta Y - \mathrm{E}(\Delta Y'\mathbf{B}_1'\mathbf{M}^{\circ\prime}\Upsilon\mathbf{M}^\circ\mathbf{B}_1\Delta Y)\\
&+\Delta Y_{-1}'\mathbf{B}_2'\mathbf{M}^{\circ\prime}\Upsilon\mathbf{M}^\circ\mathbf{B}_2\Delta Y_{-1} - \mathrm{E}(\Delta Y_{-1}'\mathbf{B}_2'\mathbf{M}^{\circ\prime}\Upsilon\mathbf{M}^\circ\mathbf{B}_2\Delta Y_{-1})\\
&-2\Delta Y'\mathbf{B}_1'\mathbf{M}^{\circ\prime}\Upsilon\mathbf{M}^\circ\mathbf{B}_2\Delta Y_{-1} + 2\mathrm{E}(\Delta Y'\mathbf{B}_1'\mathbf{M}^{\circ\prime}\Upsilon\mathbf{M}^\circ\mathbf{B}_2\Delta Y_{-1})\\
&+2\mathrm{E}[(\mathbf{B}_1\Delta Y^\circ - \mathbf{B}_2\Delta Y_{-1}^\circ)'\mathbf{M}^{\circ\prime}\Upsilon\mathbf{P}^\circ(\mathbf{B}_1\Delta Y^\circ - \mathbf{B}_2\Delta Y_{-1}^\circ)]\\
&+2\mathrm{E}[(\mathbf{B}_1\Delta Y^\circ - \mathbf{B}_2\Delta Y_{-1}^\circ)'\mathbf{P}^{\circ\prime}\Upsilon\mathbf{P}^\circ(\mathbf{B}_1\Delta Y^\circ - \mathbf{B}_2\Delta Y_{-1}^\circ)],
\end{aligned}
$$

where $\mathbf{M}^\circ = \mathbf{M}\Omega^{-\frac{1}{2}}$. By Lemma 3.2, all the quantities involving ΔY and ΔY_{-1} can be decomposed into sums of quadratic, bilinear and linear forms in $\Delta\mathbf{y}_1$ and/or Δv, and all the quantities involving ΔY° and ΔY_{-1}° can be handled in a similar manner as for Q_4 in **(b)**. The rest of the proof proceeds in a similar manner as for the proof of **(b)**. ■

Proof of Theorem 3.2: We have by the mean value theorem,

$$
0 = \tfrac{1}{\sqrt{n(T-1)}}S^*_{\mathtt{STLE}}(\hat{\psi}_{\mathtt{STLE}}) = \tfrac{1}{\sqrt{n(T-1)}}S^*_{\mathtt{STLE}}(\psi_0) + \big[\tfrac{1}{n(T-1)}\tfrac{\partial}{\partial\psi'}S^*_{\mathtt{STLE}}(\bar{\psi})\big]\sqrt{n(T-1)}(\hat{\psi}_{\mathtt{M}} - \psi_0),
$$

where $\bar{\psi}$ lies elementwise between $\hat{\psi}_{\mathtt{M}}$ and ψ_0. The result of the theorem follows if

(a) $\frac{1}{\sqrt{n(T-1)}} S^*_{\mathtt{STLE}}(\psi_0) \xrightarrow{D} N\big[0, \lim_{n\to\infty} \Gamma^*_{\mathtt{STLE}}(\psi_0)\big]$,

(b) $\frac{1}{n(T-1)}\big[\frac{\partial}{\partial\psi'} S^*_{\mathtt{STLE}}(\bar{\psi}) - \frac{\partial}{\partial\psi'} S^*_{\mathtt{STLE}}(\psi_0)\big] \xrightarrow{p} 0$, and

(c) $\frac{1}{n(T-1)}\big[\frac{\partial}{\partial\psi'} S^*_{\mathtt{STLE}}(\psi_0) - \mathrm{E}\big(\frac{\partial}{\partial\psi'} S^*_{\mathtt{STLE}}(\psi_0)\big)\big] \xrightarrow{p} 0$.

Proof of (a). From (3.24), we see that $S^*_{\mathtt{STLE}}(\psi_0)$ consists of three types of elements: $\Pi'\Delta v$, $\Delta v'\Phi\Delta v$ and $\Delta v'\Psi\Delta\mathbf{y}_1$, which can be written as

$$\Pi'\Delta v = \textstyle\sum_{t=1}^T \Pi_t^{*\prime} v_t,\;\; \Delta v\Phi\Delta v = \sum_{t=1}^T\sum_{s=1}^T v_t'\Phi^*_{ts} v_s,\;\; \text{and } \Delta v'\Psi\Delta\mathbf{y}_1 = \sum_{t=1}^T v_t'\Psi_t^*\Delta y_1,$$

where Π_t^*, Φ^*_{ts} and Ψ^*_t are formed by the elements of the partitioned Π, Φ and Ψ, respectively. By (2.1), $y_1 = B_{10}^{-1}B_{20}y_0 + \eta_1 + B_{10}^{-1}B_{30}^{-1}v_1$, leading to $\sum_{t=1}^T v_t'\Psi_t^*\Delta y_1 = \sum_{t=1}^T v_t'\Psi_t^{**}y_0 + \sum_{t=1}^T v_t'\Psi_t^{*+}v_1 + \sum_{t=1}^T v_t'\Psi_t^*\eta_1$, for suitably defined non-stochastic quantities η_1, Ψ_t^{**} and Ψ_t^{*+}. These show that, for every non-zero $(p+5)\times 1$ vector of constants c, $c'S^*_{\mathtt{STLE}}(\psi_0)$ can be expressed as

$$c'S^*_{\mathtt{STLE}}(\psi_0) = \sum_{t=1}^T\sum_{s=1}^T v_t'A_{ts}v_s + \sum_{t=1}^T v_t'B_t v_1 + \sum_{t=1}^T v_t' g(y_0) - c'\mu^*,$$

for suitably defined non-stochastic matrices A_{ts} and B_t, vector μ^*, and the function $g(y_0)$ linear in y_0. As, $\{y_0, v_1, \ldots, v_T\}$ are independent, the asymptotic normality of $\frac{1}{\sqrt{n(T-1)}}c'S^*_{\mathtt{STLE}}(\psi_0)$ follows from Lemma A.5. Finally, the Cramér-Wold devise leads to the joint asymptotic normality.

Proof of (b). The Hessian matrix, $H^*_{\mathtt{STLE}}(\psi) = \frac{\partial}{\partial\psi'}S^*_{\mathtt{STLE}}(\psi)$, has the elements:

$H^*_{\beta\beta} = -\frac{1}{\sigma_v^2}\Delta X'\Omega^{-1}\Delta X$,

$H^*_{\beta\sigma_v^2} = -\frac{1}{\sigma_v^4}\Delta X'\Omega^{-1}\Delta u(\theta)$,

$H^*_{\beta\rho} = -\frac{1}{\sigma_v^2}\Delta X'\Omega^{-1}\Delta Y_{-1}$,

$H^*_{\beta\lambda_1} = -\frac{1}{\sigma_v^2}\Delta X'\Omega^{-1}\mathbf{W}_1\Delta Y$,

$H^*_{\beta\lambda_2} = -\frac{1}{\sigma_v^2}\Delta X'\Omega^{-1}\mathbf{W}_2\Delta Y_{-1}$,

$H^*_{\beta\lambda_3} = \frac{1}{\sigma_v^2}\Delta X'\dot{\Omega}^{-1}\Delta u(\theta)$,

$H^*_{\sigma_v^2\rho} = -\frac{1}{\sigma_v^4}\Delta Y_{-1}'\Omega^{-1}\Delta u(\theta)$,

$H^*_{\sigma_v^2\lambda_1} = -\frac{1}{\sigma_v^4}\Delta Y'\mathbf{W}_1'\Omega^{-1}\Delta u(\theta)$,

$H^*_{\rho\lambda_3} = \frac{1}{\sigma_v^2}\Delta Y_{-1}'\dot{\Omega}^{-1}\Delta u(\theta)$,

$H^*_{\lambda_1\lambda_3} = \frac{1}{\sigma_v^2}\Delta Y'\mathbf{W}_1'\dot{\Omega}^{-1}\Delta u(\theta)$,

$H^*_{\lambda_2\lambda_3} = \frac{1}{\sigma_v^2}\Delta Y_{-1}'\mathbf{W}_2'\dot{\Omega}^{-1}\Delta u(\theta)$,

$H^*_{\sigma_v^2\sigma_v^2} = -\frac{1}{\sigma_v^6}\Delta u(\theta)'\Omega^{-1}\Delta u(\theta) + \frac{n(T-1)}{2\sigma_v^4}$,

$H^*_{\sigma_v^2\lambda_2} = -\frac{1}{\sigma_v^4}\Delta Y_{-1}'\mathbf{W}_2'\Omega^{-1}\Delta u(\theta)$,

$H^*_{\sigma_v^2\lambda_3} = \frac{1}{2\sigma_v^4}\Delta u(\theta)'\dot{\Omega}^{-1}\Delta u(\theta)$,

$H^*_{\rho\rho} = -\frac{1}{\sigma_v^2}\Delta Y_{-1}'\Omega^{-1}\Delta Y_{-1} + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1,\rho})$,

$H^*_{\rho\lambda_1} = -\frac{1}{\sigma_v^2}\Delta Y_{-1}'\Omega^{-1}\mathbf{W}_1\Delta Y + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1,\lambda_1})$,

$H^*_{\rho\lambda_2} = -\frac{1}{\sigma_v^2}\Delta Y_{-1}'\Omega^{-1}\mathbf{W}_2\Delta Y_{-1} + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1,\lambda_2})$,

$H^*_{\lambda_1\lambda_1} = -\frac{1}{\sigma_v^2}\Delta Y'\mathbf{W}_1'\Omega^{-1}\mathbf{W}_1\Delta Y + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_1}\mathbf{W}_1)$,

$H^*_{\lambda_1\lambda_2} = -\frac{1}{\sigma_v^2}\Delta Y'\mathbf{W}_1'\Omega^{-1}\mathbf{W}_2\Delta Y_{-1} + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_2}\mathbf{W}_1)$,

$H^*_{\lambda_2\lambda_2} = -\frac{1}{\sigma_v^2}\Delta Y_{-1}'\mathbf{W}_2'\Omega^{-1}\mathbf{W}_2\Delta Y_{-1} + \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1,\lambda_2}\mathbf{W}_2)$,

$H^*_{\lambda_3\lambda_3} = -\frac{1}{\sigma_v^2}\Delta u(\theta)'[C^{-1}\otimes(W_3'W_3)]\Delta u(\theta) - \mathrm{tr}(\mathbf{G}_3^2)$.

where $\dot{\Omega}^{-1} = \frac{\partial}{\partial\lambda_3}\Omega^{-1}$, $\mathbf{D}_{-1,\omega} = \frac{\partial}{\partial\omega}\mathbf{D}_{-1}$ and $\mathbf{D}_\omega = \frac{\partial}{\partial\omega}\mathbf{D}$, $\omega = \rho, \lambda_1, \lambda_2$, and $\mathbf{G}_3 = \mathbf{W}_3\mathbf{B}_3^{-1}$.

It is easy to show that $\frac{1}{n(T-1)}H^*_{\mathtt{STLE}}(\psi_0) = O_p(1)$ by Lemma A.1 and the model assumptions. Thus, $\frac{1}{n(T-1)}H^*_{\mathtt{STLE}}(\bar{\psi}) = O_p(1)$ because $\bar{\psi} - \psi_0 = o_p(1)$ which is implied by $\hat{\psi}_{\mathtt{M}} \xrightarrow{p} \psi_0$.

As $\bar{\sigma}^2 \xrightarrow{p} \sigma_{v0}^2$, $\bar{\sigma}^{-r} = \sigma_{v0}^{-r} + o_p(1), r = 2, 4, 6$. Noting that σ^r appears in $H^*_{\mathtt{STLE}}(\psi)$ multiplicatively, $\frac{1}{n(T-1)} H^*_{\mathtt{STLE}}(\bar{\psi}) = \frac{1}{n(T-1)} H^*_{\mathtt{STLE}}(\bar{\beta}, \sigma_{v0}^2, \bar{\rho}, \bar{\lambda}) + o_p(1)$, i.e., replacing $\bar{\sigma}^2$ by σ_{v0}^2 results in an asymptotically negligible error. The proof of (**b**) is thus equivalent to the proof of

$$\tfrac{1}{n(T-1)} \big[H^*_{\mathtt{STLE}}(\bar{\beta}, \sigma_{v0}^2, \bar{\rho}, \bar{\lambda}) - H^*_{\mathtt{STLE}}(\psi_0)\big] \xrightarrow{p} 0.$$

From $\Delta u(\theta) = \Delta u - (\lambda_1 - \lambda_{10})\mathbf{W}_1 \Delta Y - (\rho - \rho_0)\Delta Y_{-1} - (\lambda_2 - \lambda_{20})\mathbf{W}_2 \Delta Y_{-1} - \Delta X(\beta - \beta_0)$, $\Omega^{-1}(\lambda_3) - \Omega^{-1}(\lambda_{30}) = (\lambda_3^2 - \lambda_{30}^2) C^{-1} \otimes (W_3' W_3) - (\lambda_3 - \lambda_{30}) C^{-1} \otimes (W_3' + W_3)$, and $\dot{\Omega}^{-1} = -C^{-1} \otimes (W_3' B_3 + B_3' W_3)$, we see that all the random elements of $H^*_{\mathtt{STLE}}(\psi)$ are linear, bilinear, or quadratic in ΔY, ΔY_{-1} or Δu, and linear or quadratic in β, ρ, and λ. This means that all the corresponding elements in $\frac{1}{n(T-1)} \big[H^*_{\mathtt{STLE}}(\bar{\beta}, \sigma_{v0}^2, \bar{\rho}, \bar{\lambda}) - H^*_{\mathtt{STLE}}(\psi_0)\big]$ are linear, bilinear, or quadratic in ΔY, ΔY_{-1} or Δu, and linear, bilinear or quadratic in $\bar{\beta} - \beta_0, \bar{\rho} - \rho_0$, and $\bar{\lambda} - \lambda_0$, and thus are all $o_p(1)$ by the consistency of $\hat{\psi}_{\mathtt{M}}$, Lemma 3.2, Lemma A.1 and Assumption F.

This can be easily seen as follows. First, for all the terms linear in ΔY or ΔY_{-1}, quadratic in ΔY or ΔY_{-1}, or bilinear in ΔY and ΔY_{-1}. For example, for the term corresponding to $H^*_{\lambda_1 \lambda_1}$, we have, by the consistency of $\hat{\lambda}_{\mathtt{M}}$, Lemma 3.2, Lemma A.1, and Assumption F,

$$\begin{aligned}
& \tfrac{1}{n(T-1)}[-\tfrac{1}{\sigma_v^2} \Delta Y' \mathbf{W}_1' \Omega^{-1}(\bar{\lambda}_3) \mathbf{W}_1 \Delta Y + \tfrac{1}{\sigma_v^2} \Delta Y' \mathbf{W}_1' \Omega_0^{-1}(\lambda_{30}) \mathbf{W}_1 \Delta Y] \\
= \; & \tfrac{1}{n(T-1)} \tfrac{1}{\sigma_{v0}^2} \Delta Y' \mathbf{W}_1' [-\Omega^{-1}(\bar{\lambda}_3) + \Omega_0^{-1}(\lambda_{30})] \mathbf{W}_1 \Delta Y] \\
= \; & (\bar{\lambda}_3 - \lambda_{30}) \tfrac{1}{n(T-1)\sigma_{v0}^2} \Delta Y' \mathbf{W}_1' [C^{-1} \otimes (W_3' + W_3)] \mathbf{W}_1 \Delta Y \\
& -(\bar{\lambda}_3^2 - \lambda_{30}^2) \tfrac{1}{n(T-1)\sigma_{v0}^2} \Delta Y' \mathbf{W}_1' [C^{-1} \otimes (W_3' W_3)] \mathbf{W}_1 \Delta Y \\
= \; & o_p(1) \times O_p(1) - o_p(1) \times O_p(1) = o_p(1).
\end{aligned}$$

Now, all the remaining terms involve $\Delta u(\theta)$. We have, for example,

$$\begin{aligned}
& H^*_{\sigma_v^2 \lambda_1}(\bar{\beta}, \sigma_{v0}^2, \bar{\rho}, \bar{\lambda}) - H^*_{\sigma_v^2 \lambda_1}(\psi_0) \\
= \; & -\tfrac{1}{\sigma_{v0}^4} \Delta Y' \mathbf{W}_1' [\Omega^{-1}(\bar{\lambda}_3) \Delta u(\bar{\theta}) - \Omega_0^{-1} \Delta u] \\
= \; & -\tfrac{1}{\sigma_{v0}^4} \Delta Y' \mathbf{W}_1' \big\{[\Omega_0^{-1} + (\bar{\lambda}_3^2 - \lambda_{30}^2) C^{-1} \otimes (W_3' W_3) - (\bar{\lambda}_3 - \lambda_{30}) C^{-1} \otimes (W_3' + W_3)] \\
& \times [\Delta u - (\bar{\lambda}_1 - \lambda_{10}) \mathbf{W}_1 \Delta Y - (\bar{\rho} - \rho_0) \Delta Y_{-1} - (\bar{\lambda}_2 - \lambda_{20}) \mathbf{W}_2 \Delta Y_{-1} - \Delta X(\bar{\beta} - \beta_0] - \Omega_0^{-1} \Delta u\big\},
\end{aligned}$$

from which one sees clearly that it is linear, bilinear or quadratic in ΔY, ΔY_{-1}, or Δu, and linear, bilinear or quadratic in $\bar{\beta} - \beta_0, \bar{\rho} - \rho_0$, and $\bar{\lambda} - \lambda_0$. The proof of

$$\tfrac{1}{n(T-1)}[H^*_{\sigma_v^2 \lambda_1}(\bar{\beta}, \sigma_{v0}^2, \bar{\rho}, \bar{\lambda}) - H^*_{\sigma_v^2 \lambda_1}(\psi_0)] = o_p(1)$$

boils down to show that the quantities $\frac{1}{n(T-1)} \Delta Y' \mathbf{W}_1' \Omega_0^{-1} \mathbf{W}_1 \Delta Y$, $\frac{1}{n(T-1)} \Delta Y' \mathbf{W}_1' \Omega_0^{-1} \Delta Y_{-1}$, $\frac{1}{n(T-1)} \Delta Y' \mathbf{W}_1' \Omega_0^{-1} \Delta X$, etc., are all $O_p(1)$, which can be done easily by Lemma 3.2, Lemma A.1 and Assumption F. The proofs for the other terms involving $\Delta u(\theta)$ proceed in the same manner. It left to show that

(a) $\frac{1}{n(T-1)}[\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1,\rho}(\bar\rho,\bar\lambda_1,\bar\lambda_2)) - \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1,\rho}(\rho_0,\lambda_{10},\lambda_{20}))] = o_p(1)$
(b) $\frac{1}{n(T-1)}[\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1,\lambda_1}(\bar\rho,\bar\lambda_1,\bar\lambda_2)) - \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1,\lambda_1}(\rho_0,\lambda_{10},\lambda_{20}))] = o_p(1)$
(c) $\frac{1}{n(T-1)}[\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1,\lambda_2}(\bar\rho,\bar\lambda_1,\bar\lambda_2)) - \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1,\lambda_2}(\rho_0,\lambda_{10},\lambda_{20}))] = o_p(1)$
(d) $\frac{1}{n(T-1)}[\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_1}(\bar\rho,\bar\lambda_1,\bar\lambda_2)\mathbf{W}_1) - \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_1}(\rho_0,\lambda_{10},\lambda_{20})\mathbf{W}_1)] = o_p(1)$
(e) $\frac{1}{n(T-1)}[\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_2}(\bar\rho,\bar\lambda_1,\bar\lambda_2)\mathbf{W}_1) - \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_2}(\rho_0,\lambda_{10},\lambda_{20})\mathbf{W}_1)] = o_p(1)$
(f) $\frac{1}{n(T-1)}[\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1,\lambda_2}(\bar\rho,\bar\lambda_1,\bar\lambda_2)\mathbf{W}_2)) - \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{-1,\lambda_2}(\rho_0,\lambda_{10},\lambda_{20})\mathbf{W}_2))] = o_p(1)$
(g) $\frac{1}{n(T-1)}[\mathrm{tr}(\mathbf{G}(\bar\lambda_3)^2) - \mathrm{tr}(\mathbf{G}(\lambda_{30})^2)] = o_p(1)$.

It is easy to show the last result. By the mean value theorem, $\mathrm{tr}(\mathbf{G}(\bar\lambda_3)^2) - \mathrm{tr}(\mathbf{G}(\lambda_{30})^2) = 2(\bar\lambda_3 - \lambda_{30})\mathrm{tr}(\mathbf{G}(\lambda_3^*)^3)$, where λ_3^* lies between $\bar\lambda_3$ and λ_{30}. By Lemmas A.1 and A.2, the elements of $\mathbf{G}(\lambda_3^*)^3$ is uniformly bounded. Thus, $\frac{1}{n(T-1)}\mathrm{tr}(\mathbf{G}(\lambda_3^*)^3) = O_p(1)$, leading to (g). The proofs of (a)-(f) are similar, and some details are given for the most complicate case (d). Let $(\rho^*,\lambda_1^*,\lambda_2^*)$ be between $(\bar\rho,\bar\lambda_1,\bar\lambda_2)$ and $(\rho_0,\lambda_{10},\lambda_{20})$. By the mean value theorem,

$$\begin{aligned}&\frac{1}{n(T-1)}[\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_1}(\bar\rho,\bar\lambda_1,\bar\lambda_2)\mathbf{W}_1) - \mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_1}(\rho_0,\lambda_{10},\lambda_{20})\mathbf{W}_1)]\\ =\ & \frac{\bar\rho-\rho_0}{n(T-1)}\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_1}^{\rho^*}\mathbf{W}_1) + \frac{\bar\lambda_1-\lambda_{10}}{n(T-1)}\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_1}^{\lambda_1^*}\mathbf{W}_1) + \frac{\bar\lambda_2-\lambda_{20}}{n(T-1)}\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_1}^{\lambda_2^*}\mathbf{W}_1),\end{aligned}$$

where $\mathbf{D}_{\lambda_1}^{\rho^*}$, $\mathbf{D}_{\lambda_1}^{\lambda_1^*}$ and $\mathbf{D}_{\lambda_1}^{\lambda_2^*}$ are the partial derivatives of $\mathbf{D}_{\lambda_1}$ evaluated at $(\rho^*,\lambda_1^*,\lambda_2^*)$. Consider, W.L.O.G., $T=3$. Recall $B_1 = I_n - \lambda W_1$, $B_2 = \rho I_n + \lambda_2 W_2$ and $\mathcal{B} = B_2^{-1}B_2$. We have

$$\mathbf{D}(\rho,\lambda_1,\lambda_2) = \begin{pmatrix} B_1^{-1}B_2B_1^{-1}, & B_1^{-1} \\ (I_n - B_1^{-1}B_2)^2B_1^{-1}, & B_1^{-1}B_2B_1^{-1}\end{pmatrix}.$$

This shows that the elements of $\mathbf{D}_{\lambda_1}$ are the multiplications of the matrices W_1, B_1^{-1} and B_2. Subsequently, $\mathbf{D}_{\lambda_1}^{\rho}$, $\mathbf{D}_{\lambda_1}^{\lambda_1}$ and $\mathbf{D}_{\lambda_1}^{\lambda_2}$ have elements being the multiplications of the matrices W_1, W_2, $B_1^{-1}(\lambda_1)$, and $B_2(\rho,\lambda_2)$, and hence are uniformly bounded in a matrix norm, in the neighborhood of $(\rho_0,\lambda_{10},\lambda_{20})$ by Lemmas A.1 and A.2. Therefore, $\frac{1}{n(T-1)}\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_1}^{\rho^*}\mathbf{W}_1) = O_p(1)$, $\frac{1}{n(T-1)}\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_1}^{\lambda_1^*}\mathbf{W}_1) = O_p(1)$, and $\frac{1}{n(T-1)}\mathrm{tr}(\mathbf{C}^{-1}\mathbf{D}_{\lambda_1}^{\lambda_2^*}\mathbf{W}_1) = O_p(1)$, leading to (d).

Proof of (c). First, for the terms involving only Δu (linear or quadratic), the results follows Lemma A.4(v)-(vi), noticing $\Delta u = \mathbf{B}_{30}^{-1}\mathbf{F}v$ where $\mathbf{F}v = \Delta v$. For example,

$$H^*_{\sigma_v^2\lambda_3}(\psi_0) - \mathrm{E}[H^*_{\sigma_v^2\lambda_3}(\psi_0)] = \frac{1}{2\sigma_{v0}^4}[\Delta u'\dot\Omega_0^{-1}\Delta u - \mathrm{E}(\Delta u'\dot\Omega_0^{-1}\Delta u)] = \frac{1}{2\sigma_{v0}^4}[v'\mathbf{A}v - \mathrm{E}(v'\mathbf{A}v)],$$

where $\mathbf{A} = \mathbf{F}'\mathbf{B}_{30}'^{-1}\dot\Omega_0^{-1}\mathbf{B}_{30}^{-1}\mathbf{F}$, which is easily seen to be uniformly bounded in both row and column sums. Thus, Lemma A.4(v) leads to $\frac{1}{n(T-1)}\{H^*_{\sigma_v^2\lambda_3}(\psi_0) - \mathrm{E}[H^*_{\sigma_v^2\lambda_3}(\psi_0)]\} = o_p(1)$.

Second, by Lemma 3.2 all the terms involving ΔY and ΔY_{-1} can be written as sums of the terms linear in $\Delta\mathbf{y}$, quadratic in $\Delta\mathbf{y}$, bilinear in $\Delta\mathbf{y}$ and Δv, or quadratic in Δv. Thus, the results follow by repeatedly applying Lemma A.1, Lemma A.4, and Assumption F. ■

Proof of Theorem 3.3: First, the result $\Sigma^*_{\mathtt{STLE}}(\hat\psi_{\mathtt{M}}) - \Sigma^*_{\mathtt{STLE}}(\psi_0) \xrightarrow{p} 0$ is implied by the result **(b)** in the proof of Theorem 3.2. The result $\frac{1}{n(T-1)}\sum_{i=1}^n[\hat g_i\hat g_i' - \mathrm{E}(g_ig_i')] \xrightarrow{p} 0$ follows from $\frac{1}{n(T-1)}\sum_{i=1}^n(\hat{g}_i\hat{g}_i' - g_ig_i') \xrightarrow{p} 0$ and $\frac{1}{n(T-1)}\sum_{i=1}^n[g_ig_i' - \mathrm{E}(g_ig_i')] \xrightarrow{p} 0$. The proof of the former is straightforward by applying the mean value theorem. We focus on the proof of the latter result. As the elements of $S^*_{\mathtt{STLE}}(\psi_0)$ are mixtures of terms of the forms $\Pi'\Delta v = \sum_{i=1}^n g_{1i}$, $\Delta v'\Phi\Delta v - \mathrm{E}(\Delta v'\Phi\Delta v) = \sum_{i=1}^n g_{2i}$ and $\Delta v'\Psi\Delta\mathbf{y}_1 - \mathrm{E}(\Delta v'\Psi\Delta\mathbf{y}_1) = \sum_{i=1}^n g_{3i}$, it suffices to show that

$$\frac{1}{n(T-1)}\sum_{i=1}^n[g_{ki}g_{ri}' - \mathrm{E}(g_{ki}g_{ri}')] = o_p(1),\ k,r = 1,2,3.$$

To facilitate the proof, the following *dot* notation is convenient: (a) for an $n(T-1)\times 1$ vector Δv with elements $\{\Delta v_{it}\}$ double indexed by $i = 1,\ldots,n$ for each $t = 2,\ldots,T$, $\Delta v_{\cdot t}$ is the subvector that contains all the elements with the same t, and $\Delta v_{i\cdot}$ is the subvector that picks up the elements with the same i; (b) for an $n(T-1)\times n(T-1)$ matrix Φ with elements $\{\Phi_{it,js},\ i,j = 1,\ldots,n;\ t,s = 2,\ldots,T\}$, where it is the double index for the rows and js the double index for the columns, $\Phi_{\cdot t,\cdot s}$ is the $n\times n$ submatrix corresponding to the (t,s) periods, $\Phi_{i\cdot,j\cdot}$ the $(T-1)\times(T-1)$ submatrix corresponding to the (i,j) units, $\Phi_{it,j\cdot}$ the $(T-1)\times 1$ subvector that picks up the element from the itth row corresponding to $s = 2,\ldots,T$.

With the vector dot notation, the $g_{ri}, r = 1,2,3$, defined in Lemma 3.3 can be written as $g_{1i} = \Pi_{i\cdot}'\Delta v_{i\cdot}$, $g_{2i} = \Delta v_{i\cdot}'\Delta\xi_{i\cdot} + \Delta v_{i\cdot}'\Delta v_{i\cdot}^* - 1_{T-1}'d_{i\cdot}$, and $g_{3i} = \Delta v_{2i}\Delta\zeta_i + \Theta_{ii}(\Delta v_{2i}\Delta y_{1i}^{\circ} + \sigma_{v0}^2) + \Delta v_{i-}'\Delta y_{1i-}^*$ where '$-$' plays the same role as '$\cdot$' but corresponds to $t = 3,\ldots,T$. Note that under Assumptions D and E, one can easily see by Lemma A.1 that the elements of all the Π's, Φ's,, and Ψ's, defined in (3.24), are uniformly bounded. The proofs proceed by applying the weak law of large numbers (WLLN) for M.D. arrays, see, e.g., Davidson (1994, p. 299).

First, with $g_{1i} = \Pi_{i\cdot}'\Delta v_{i\cdot}$, $\frac{1}{n(T-1)}\sum_{i=1}^n[g_{1i}g_{1i}' - \mathrm{E}(g_{1i}g_{1i}')] = \frac{1}{n(T-1)}\sum_{i=1}^n\Pi_{i\cdot}'(\Delta v_{i\cdot}\Delta v_{i\cdot}' - \sigma_{v0}^2C)\Pi_{i\cdot} \equiv \frac{1}{n(T-1)}\sum_{i=1}^n U_{n,i}$, where C is defined below (3.2). Without loss of generality, assume U_{ni} is a scalar, as if not we can work on each element of it. Clearly, $\{U_{n,i}\}$ are independent, thus form a M.D. array. By Assumption B and using the fact that the elements of $\Pi_{i\cdot}$ are uniformly bounded, it is easy to show that $\mathrm{E}|U_{n,i}|^{1+\epsilon} \le K_u < \infty$, for $\epsilon > 0$. Thus, $\{U_{n,i}\}$ are uniformly integrable. With the constant coefficients $\frac{1}{n(T-1)}$ the other two conditions of WLLN for M.D. arrays of Davidson are satisfied. Thus, $\frac{1}{n(T-1)}\sum_{i=1}^n U_{n,i} \xrightarrow{p} 0$.

Second, with $g_{2i} = \Delta v_{i\cdot}'\Delta\xi_{i\cdot} + \Delta v_{i\cdot}'\Delta v_{i\cdot}^* - 1_{T-1}'d_{i\cdot}$, we have,

$$\begin{aligned}
&\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^n[g_{2i}^2 - \mathrm{E}(g_{2i}^2)]\\
=\ &\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^n[(\Delta v_{i\cdot}'\Delta\xi_{i\cdot})^2 - \mathrm{E}((\Delta v_{i\cdot}'\Delta\xi_{i\cdot})^2)]\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^n[(\Delta v_{i\cdot}'\Delta v_{i\cdot}^*)^2 - \mathrm{E}((\Delta v_{i\cdot}'\Delta v_{i\cdot}^*)^2)]\\
&+\tfrac{2}{n(T-1)}\textstyle\sum_{i=1}^n(\Delta v_{i\cdot}'\Delta\xi_{i\cdot})(\Delta v_{i\cdot}'\Delta v_{i\cdot}^*) - \tfrac{2}{n(T-1)}\sum_{i=1}^n(1_{T-1}'d_{i\cdot})(\Delta v_{i\cdot}'\Delta\xi_{i\cdot})\\
&-\tfrac{2}{n(T-1)}\textstyle\sum_{i=1}^n[(1_{T-1}'d_{i\cdot})(\Delta v_{i\cdot}'\Delta v_{i\cdot}^* - \mathrm{E}(\Delta v_{i\cdot}'\Delta v_{i\cdot}^*))] \equiv \sum_{r=1}^5 H_r.
\end{aligned}$$

Now, $H_1 = \frac{1}{n(T-1)}\sum_{i=1}^n[\Delta\xi_{i\cdot}'(\Delta v_{i\cdot}\Delta v_{i\cdot}' - \sigma_{vo}^2C)\Delta\xi_{i\cdot}] + \frac{\sigma_{v0}^2}{n(T-1)}\sum_{i=1}^n[\Delta\xi_{i\cdot}'C\Delta\xi_{i\cdot} - \mathrm{E}(\Delta\xi_{i\cdot}'C\Delta\xi_{i\cdot})]$. For the first term, let $V_{n,i} = \Delta\xi_{i\cdot}'(\Delta v_{i\cdot}\Delta v_{i\cdot}' - \sigma_{v0}^2C)\Delta\xi_{i\cdot}$. As $\Delta\xi_{i\cdot}$ is $\mathcal{G}_{n,i-1}$-measurable, $\mathrm{E}(V_{n,i}|\mathcal{G}_{n,i-1}) = 0$. Thus, $\{V_{n,i},\mathcal{G}_{n,i}\}$ form a M.D. array. It is easy to see that $\mathrm{E}|V_{n,i}^{1+\epsilon}| \le K_v < \infty$, for some $\epsilon > 0$. Thus, $\{V_{n,i}\}$ is uniformly integrable. The other two conditions of the WLLN for M.D. arrays of Davidson are satisfied. Thus, $\frac{1}{n(T-1)}\sum_{i=1}^n V_{n,i} \xrightarrow{p} 0$.

For the second term of H_1, recall $\xi_t = \sum_{s=2}^T(\Phi_{st}^{u\prime} + \Phi_{ts}^{\ell})\Delta v_s$. We have,

$\Delta\xi_{it} = \sum_{s=2}^{T}\sum_{j=1}^{i-1}(\Phi_{js,it}+\Phi_{it,js})\Delta v_{js} = \sum_{j=1}^{i-1}\sum_{s=2}^{T}(\Phi_{js,it}+\Phi_{it,js})\Delta v_{js} = \sum_{j=1}^{i-1}\phi'_{ijt}\Delta v_{j\cdot}$,

where $\phi_{ijt} = (\Phi_{j\cdot,it}+\Phi_{it,j\cdot})$. Thus, $(\Delta\xi_{it})^2 - \mathrm{E}[(\Delta\xi_{it})^2] = \sum_{j=1}^{i-1}[\phi'_{ijt}(\Delta v_{j\cdot}\Delta v'_{j\cdot} - \sigma_{v0}^2 C)\phi_{ijt}] + 2\sum_{j=1}^{i-1}\sum_{k=1}^{j-1}\Delta v'_{j\cdot}\phi_{ijt}\phi'_{ikt}\Delta v_{k\cdot}$. It follows that

$$\begin{aligned}
&\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}\{(\Delta\xi_{it})^2 - \mathrm{E}[(\Delta\xi_{it})^2]\}\\
=\ &\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}\sum_{j=1}^{i-1}[\phi'_{ijt}(\Delta v_{j\cdot}\Delta v'_{j\cdot} - \sigma_{v0}^2 C)\phi_{ijt}]\\
&+2\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}\sum_{j=1}^{i-1}\sum_{k=1}^{j-1}\Delta v'_{j\cdot}\phi_{ijt}\phi'_{ikt}\Delta v_{k\cdot}\\
=\ &\tfrac{1}{n(T-1)}\textstyle\sum_{j=1}^{n-1}\{\sum_{i=j+1}^{n}[\phi'_{ijt}(\Delta v_{j\cdot}\Delta v'_{j\cdot} - \sigma_{v0}^2 C)\phi_{ijt}]\}\\
&+2\tfrac{1}{n(T-1)}\textstyle\sum_{j=1}^{n-1}\Delta v'_{j\cdot}\{\sum_{i=j+1}^{n}\sum_{k=1}^{j-1}\phi_{ijt}\phi'_{ikt}\Delta v_{k\cdot}\}.
\end{aligned}$$

Clearly, the first term is the 'average' of $n-1$ independent terms, and the second is the 'average' of a M.D. array as the term in the curling brackets is $G_{n,j-1}$-measurable. Conditions of Theorem 19.7 of Davidson (1994) are easily verified, and hence $\frac{1}{n(T-1)}\sum_{i=1}^{n}\{(\Delta\xi_{it})^2 - \mathrm{E}[(\Delta\xi_{it})^2]\} = o_p(1)$. Similarly, one shows that $\frac{1}{n(T-1)}\sum_{i=1}^{n}\{\Delta\xi_{it}\Delta\xi_{is} - \mathrm{E}[(\Delta\xi_{it}\Delta\xi_{is})]\} = o_p(1)$ for $s \neq t$. Thus, $\frac{\sigma_{v0}^2}{n(T-1)}\sum_{i=1}^{n}[\Delta\xi'_{i\cdot}C\Delta\xi_{i\cdot} - \mathrm{E}(\Delta\xi'_{i\cdot}C\Delta\xi_{i\cdot})] = o_p(1)$, and $H_1 = o_p(1)$.

The proofs for H_3 and H_4 can be done in a similar manner as the proof for the second term of H_1. The proofs for H_2 and H_5 are similar to the proof of the first part of H_1, as they each involves a sum of n independent terms.

Third, with $g_{3i} = \Delta v_{2i}\Delta\zeta_i + \Theta_{ii}(\Delta v_{2i}\Delta y_{1i}^{\circ} + \sigma_{v0}^2) + \Delta v'_{i-}\Delta y_{1i-}^{*}$, we obtain,

$$\begin{aligned}
&\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[g_{3i}^2 - \mathrm{E}(g_{3i}^2)]\\
=\ &\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[(\Delta v_{2i}^2 - 2\sigma_{v0}^2)\Delta\zeta_i^2] + \tfrac{2\sigma_{v0}^2}{n(T-1)}\sum_{i=1}^{n}[\Delta\zeta_i^2 - \mathrm{E}(\Delta\zeta_i^2)]\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}\Theta_{ii}^2[(\Delta v_{2i}\Delta y_{1i}^{\circ})^2 - \mathrm{E}((\Delta v_{2i}\Delta y_{1i}^{\circ})^2)]\\
&+\tfrac{2\sigma_{v0}^2}{n(T-1)}\textstyle\sum_{i=1}^{n}\Theta_{ii}^2[\Delta v_{2i}\Delta y_{1i}^{\circ} - \mathrm{E}(\Delta v_{2i}\Delta y_{1i}^{\circ})]\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[(\Delta v'_{i-}\Delta y_{1i-}^{*})^2 - \mathrm{E}((\Delta v'_{i-}\Delta y_{1i-}^{*})^2)]\\
&+\tfrac{2}{n(T-1)}\textstyle\sum_{i=1}^{n}\Theta_{ii}[\Delta v_{2i}^2\Delta\zeta_i\Delta y_{1i}^{\circ} - \mathrm{E}(\Delta v_{2i}^2\Delta\zeta_i\Delta y_{1i}^{\circ})] + \tfrac{2\sigma_{v0}^2}{n(T-1)}\sum_{i=1}^{n}\Theta_{ii}\Delta v_{2i}\Delta\zeta_i\\
&+\tfrac{2}{n(T-1)}\textstyle\sum_{i=1}^{n}[\Delta v_{2i}\Delta\zeta_i(\Delta v'_{i-}\Delta y_{1i-}^{*}) - \mathrm{E}(\Delta v_{2i}\Delta\zeta_i(\Delta v'_{i-}\Delta y_{1i-}^{*}))]\\
&+\tfrac{2}{n(T-1)}\textstyle\sum_{i=1}^{n}\Theta_{ii}[(\Delta v_{2i}\Delta y_{1i}^{\circ})(\Delta v'_{i-}\Delta y_{1i-}^{*}) - \mathrm{E}((\Delta v_{2i}\Delta y_{1i}^{\circ})(\Delta v'_{i-}\Delta y_{1i-}^{*}))]\\
&+\tfrac{2\sigma_{v0}^2}{n(T-1)}\textstyle\sum_{i=1}^{n}\Theta_{ii}[\Delta v'_{i-}\Delta y_{1i-}^{*} - \mathrm{E}(\Delta v'_{i-}\Delta y_{1i-}^{*})] \equiv \sum_{r=1}^{10}Q_r.
\end{aligned}$$

As $\Delta\zeta_i^2$ is $\mathcal{F}_{n,i-1}$-measurable, Q_1 is the average of a M.D. array and its convergence follows from WLLN for M.D. array, and the convergence of Q_7 immediately follows. For Q_2, note that $\Delta\zeta = (\Theta^{w\prime}+\Theta^{\ell})\Delta y_1^{\circ} = (\Theta^{w\prime}+\Theta^{\ell})B_{30}B_{10}\Delta y_1$. It follows that $Q_2 = \frac{2\sigma_{v0}^2}{n(T-1)}\sum_{i=1}^{n}(\Delta y_1' A\Delta y_1 - \mathrm{E}(\Delta y_1' A\Delta y_1)] = o_p(1)$ by Assumption F, where $A = ((\Theta^{w\prime}+\Theta^{\ell})B_{30}B_{10})'(\Theta^{w\prime}+\Theta^{\ell})B_{30}B_{10}$ is easily seen to be uniformly bounded in both row and column sums. Writing $\Delta y_1^{\circ} = B_{30}B_{10}\Delta y_0 + B_{30}\Delta x_1\beta_0 + \Delta v_1 \equiv g(y_0, v_0) + v_1$, the convergence of Q_3, Q_4 and Q_6 can be easily proved though tedious. The results for Q_5 and Q_{10} are proved by the independence between $\Delta v'_{i-}$ and Δy_{1i-}^{*}, $\Delta y_{1t}^{*} = \Phi_{t+}\Delta y_1$, and Assumption F. Finally, the results for Q_8 and Q_9 can be proved by further writing $\Delta y_{1t}^{*} = \Phi_{t+}\Delta y_1 = \Phi_{t+}(B_{30}B_{10})^{-1}\Delta y_1^{\circ} \equiv q(\Delta y_0, v_0) + \Phi_{t+}(B_{30}B_{10})^{-1}v_1$.

Subsequently, for the cross-product terms, we have,

$$\begin{aligned}
&\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[g_{1i}g_{2i}-\mathrm{E}(g_{1i}g_{2i})]\\
=\;&\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[\Pi_{i\cdot}'(\Delta v_{i\cdot}\Delta v_{i\cdot}'-\sigma_{v0}^{2}C)\Delta\xi_{i\cdot}]+\tfrac{\sigma_{v0}^{2}}{n(T-1)}\textstyle\sum_{i=1}^{n}(\Pi_{i\cdot}'C\Delta\xi_{i\cdot})\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}\Pi_{i\cdot}'[\Delta v_{i\cdot}\Delta v_{i\cdot}'\Delta v_{i\cdot}^{*}-\mathrm{E}(\Delta v_{i\cdot}\Delta v_{i\cdot}'\Delta v_{i\cdot}^{*})]+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[(1_{T-1}'d_{i\cdot})\Pi_{i\cdot}'\Delta v_{i\cdot}].
\end{aligned}$$

$$\begin{aligned}
&\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[g_{1i}g_{3i}-\mathrm{E}(g_{1i}g_{3i})]\\
=\;&\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}\Pi_{i\cdot}'[\Delta v_{i\cdot}\Delta v_{2i}\Delta\zeta_{i}-\mathrm{E}(\Delta v_{i\cdot}\Delta v_{2i}\Delta\zeta_{i})]\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}\Theta_{ii}\Pi_{i\cdot}'[\Delta v_{i\cdot}(\Delta v_{2i}\Delta y_{1i}^{\circ}+\sigma_{v0}^{2})-\mathrm{E}(\Delta v_{i\cdot}(\Delta v_{2i}\Delta y_{1i}^{\circ}+\sigma_{v0}^{2}))]\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}\Pi_{i\cdot}'[\Delta v_{i\cdot}\Delta v_{i-}'\Delta y_{1i-}^{*}-\mathrm{E}(\Delta v_{i\cdot}\Delta v_{i-}'\Delta y_{1i-}^{*})].
\end{aligned}$$

$$\begin{aligned}
&\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[g_{2i}g_{3i}-\mathrm{E}(g_{2i}g_{3i})]\\
=\;&\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[(\Delta v_{i\cdot}'\Delta\xi_{i})(\Delta v_{2i}\Delta\zeta_{i})-\mathrm{E}((\Delta v_{i\cdot}'\Delta\xi_{i})(\Delta v_{2i}\Delta\zeta_{i}))]\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}\Theta_{ii}[(\Delta v_{i\cdot}'\Delta\xi_{i})(\Delta v_{2i}\Delta y_{1i}^{\circ}+\sigma_{v0}^{2})-\mathrm{E}((\Delta v_{i\cdot}'\Delta\xi_{i})(\Delta v_{2i}\Delta y_{1i}^{\circ}+\sigma_{v0}^{2}))]\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[(\Delta v_{i\cdot}'\Delta\xi_{i})(\Delta v_{i-}'\Delta y_{1i-}^{*})-\mathrm{E}((\Delta v_{i\cdot}'\Delta\xi_{i})(\Delta v_{i-}'\Delta y_{1i-}^{*}))]\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[(\Delta v_{i\cdot}'\Delta v_{i\cdot}^{*})(\Delta v_{2i}\Delta\zeta_{i})-\mathrm{E}((\Delta v_{i\cdot}'\Delta v_{i\cdot}^{*})(\Delta v_{2i}\Delta\zeta_{i}))]\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[(\Delta v_{i\cdot}'\Delta v_{i\cdot}^{*})(\Delta v_{2i}\Delta y_{1i}^{\circ}+\sigma_{v0}^{2})-\mathrm{E}((\Delta v_{i\cdot}'\Delta v_{i\cdot}^{*})(\Delta v_{2i}\Delta y_{1i}^{\circ}+\sigma_{v0}^{2}))]\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[(\Delta v_{i\cdot}'\Delta v_{i\cdot}^{*})(\Delta v_{i-}'\Delta y_{1i-}^{*})-\mathrm{E}((\Delta v_{i\cdot}'\Delta v_{i\cdot}^{*})(\Delta v_{i-}'\Delta y_{1i-}^{*}))]\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[(1_{T-1}'d_{i\cdot})\Delta v_{2i}\Delta\zeta_{i}]+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[(1_{T-1}'d_{i\cdot})\Theta_{ii}(\Delta v_{2i}\Delta y_{1i}^{\circ}+\sigma_{v0}^{2})]\\
&+\tfrac{1}{n(T-1)}\textstyle\sum_{i=1}^{n}[(1_{T-1}'d_{i\cdot})(\Delta v_{i-}'\Delta y_{1i-}^{*}-\mathrm{E}(\Delta v_{i-}'\Delta y_{1i-}^{*}))]
\end{aligned}$$

The convergence of each of the terms above can be proved in a similarly manner as these terms appear in similar forms as the terms appeared in the H_r and Q_r. ∎

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Table 1a. Empirical Mean(sd) of CQMLE, FQMLE and M-Estimator, `SE` Model, $T = 3, m = 5$

		$n = 50$			$n = 200$		
`err`	ψ	`CQMLE`	`FQMLE`	`M-Est`	`CQMLE`	`FQMLE`	`M-Est`
1	1	1.0152(.096)	1.0017(.100)	1.0015(.100)	1.0109(.050)	1.0021(.052)	1.0020(.053)
	1	0.9154(.135)	0.9678(.148)	0.9719(.154)	0.9080(.065)	0.9960(.079)	0.9962(.080)
	.5	0.3605(.055)	0.4995(.065)	0.5015(.066)	0.2869(.033)	0.5009(.043)	0.5013(.044)
	.5	0.4702(.107)	0.4761(.093)	0.4793(.105)	0.4775(.073)	0.4877(.060)	0.4907(.070)
2	1	1.0142(.098)	1.0007(.102)	1.0002(.102)	1.0099(.050)	1.0015(.053)	1.0014(.053)
	1	0.9176(.266)	0.9662(.284)	0.9785(.307)	0.9045(.128)	0.9920(.152)	0.9935(.155)
	.5	0.3610(.066)	0.4975(.069)	0.5023(.078)	0.2876(.041)	0.5002(.047)	0.5018(.052)
	.5	0.4701(.106)	0.4770(.092)	0.4803(.104)	0.4741(.075)	0.4844(.063)	0.4883(.072)
3	1	1.0133(.099)	1.0001(.103)	0.9997(.103)	1.0090(.047)	1.0003(.049)	1.0003(.049)
	1	0.9192(.198)	0.9678(.212)	0.9771(.227)	0.9060(.099)	0.9938(.119)	0.9947(.121)
	.5	0.3585(.059)	0.4953(.066)	0.4992(.071)	0.2881(.036)	0.5018(.046)	0.5029(.048)
	.5	0.4681(.110)	0.4736(.093)	0.4786(.106)	0.4741(.075)	0.4852(.062)	0.4884(.073)
1	1	1.0525(.100)	1.0035(.103)	1.0012(.104)	1.0517(.052)	1.0009(.053)	0.9999(.053)
	1	0.9204(.138)	0.9255(.126)	0.9702(.154)	0.9313(.069)	0.9712(.066)	0.9915(.078)
	0	-0.1524(.065)	-0.0036(.074)	0.0032(.078)	-0.1825(.035)	-0.0032(.042)	0.0005(.043)
	.5	0.4731(.106)	0.4848(.085)	0.4807(.105)	0.4820(.072)	0.4897(.059)	0.4881(.070)
2	1	1.0528(.099)	1.0042(.102)	1.0006(.104)	1.0479(.053)	0.9979(.055)	0.9962(.055)
	1	0.9230(.265)	0.9032(.241)	0.9764(.299)	0.9327(.133)	0.9596(.129)	0.9940(.150)
	0	-0.1529(.071)	-0.0091(.076)	0.0022(.086)	-0.1821(.039)	-0.0042(.043)	0.0018(.047)
	.5	0.4741(.103)	0.4880(.086)	0.4805(.102)	0.4806(.073)	0.4917(.059)	0.4873(.072)
3	1	1.0515(.100)	1.0021(.103)	0.9990(.104)	1.0497(.053)	0.9998(.054)	0.9985(.054)
	1	0.9250(.200)	0.9194(.185)	0.9767(.224)	0.9319(.102)	0.9661(.100)	0.9924(.115)
	0	-0.1543(.068)	-0.0077(.076)	0.0014(.083)	-0.1831(.037)	-0.0045(.043)	0.0001(.045)
	.5	0.4740(.107)	0.4855(.088)	0.4811(.105)	0.4834(.072)	0.4929(.057)	0.4906(.070)
1	1	1.0484(.103)	0.9987(.104)	1.0015(.104)	1.0418(.053)	0.9989(.054)	0.9997(.054)
	1	0.9504(.139)	0.9552(.135)	0.9764(.147)	0.9641(.072)	0.9849(.071)	0.9909(.075)
	-.5	-0.6034(.059)	-0.4915(.067)	-0.4978(.070)	-0.6070(.030)	-0.4970(.035)	-0.4988(.036)
	.5	0.4798(.108)	0.4830(.090)	0.4815(.108)	0.4889(.072)	0.4880(.060)	0.4905(.072)
2	1	1.0494(.102)	0.9980(.102)	1.0024(.103)	1.0419(.054)	0.9981(.054)	0.9997(.054)
	1	0.9380(.271)	0.9261(.250)	0.9642(.284)	0.9661(.138)	0.9775(.131)	0.9933(.145)
	-.5	-0.6028(.065)	-0.4885(.070)	-0.4981(.075)	-0.6072(.032)	-0.4949(.035)	-0.4989(.037)
	.5	0.4792(.104)	0.4831(.090)	0.4822(.102)	0.4849(.073)	0.4889(.060)	0.4862(.073)
3	1	1.0481(.105)	0.9971(.106)	1.0009(.106)	1.0409(.054)	0.9975(.054)	0.9989(.054)
	1	0.9388(.195)	0.9340(.182)	0.9647(.205)	0.9658(.103)	0.9808(.099)	0.9928(.108)
	-.5	-0.6059(.061)	-0.4924(.068)	-0.5005(.072)	-0.6092(.030)	-0.4974(.034)	-0.5008(.035)
	.5	0.4744(.108)	0.4790(.091)	0.4764(.108)	0.4841(.073)	0.4871(.059)	0.4856(.072)

Note: Par = $\psi = (\beta, \sigma_v^2, \rho, \lambda_3)'$; `err` = 1 (normal), 2 (normal mixture), and 3 (chi-square).
X_t values are generated with $\theta_x = (g, \phi_1, \phi_2, \sigma_1, \sigma_2) = (.01, .5, .5, 1, .5)$, as in Footnote 13.
W_3 is generated according to Group Interaction scheme as in Footnote 14.

Table 1a. Cont'd, $T = \mathbf{7}$

		$n=50$			$n=100$		
err	ψ	CQMLE	FQMLE	M-Est	CQMLE	FQMLE	M-Est
1	1	1.0248(.044)	1.0015(.044)	1.0013(.044)	1.0231(.033)	1.0018(.033)	1.0017(.033)
	1	0.9771(.081)	0.9888(.083)	0.9893(.083)	0.9821(.059)	0.9949(.060)	0.9956(.061)
	.5	0.4456(.028)	0.4987(.029)	0.4990(.029)	0.4407(.021)	0.4990(.022)	0.4994(.022)
	.5	0.4928(.057)	0.4920(.055)	0.4947(.056)	0.4931(.047)	0.4904(.044)	0.4953(.046)
2	1	1.0247(.045)	1.0012(.045)	1.0010(.045)	1.0232(.033)	1.0020(.033)	1.0019(.033)
	1	0.9776(.183)	0.9887(.186)	0.9899(.187)	0.9806(.129)	0.9931(.132)	0.9942(.133)
	.5	0.4461(.028)	0.4988(.029)	0.4992(.029)	0.4412(.022)	0.4991(.022)	0.4996(.022)
	.5	0.4919(.058)	0.4914(.055)	0.4941(.057)	0.4906(.048)	0.4882(.046)	0.4928(.047)
3	1	1.0250(.044)	1.0015(.044)	1.0013(.044)	1.0214(.033)	1.0003(.033)	1.0002(.033)
	1	0.9751(.130)	0.9863(.133)	0.9872(.134)	0.9779(.095)	0.9908(.097)	0.9915(.097)
	.5	0.4458(.028)	0.4986(.029)	0.4990(.029)	0.4413(.020)	0.4996(.021)	0.5000(.021)
	.5	0.4903(.057)	0.4896(.056)	0.4923(.057)	0.4919(.048)	0.4898(.045)	0.4940(.047)
1	1	1.0342(.048)	1.0017(.048)	1.0009(.048)	1.0360(.035)	1.0021(.035)	1.0016(.036)
	1	0.9840(.083)	0.9660(.077)	0.9922(.084)	0.9873(.059)	0.9807(.056)	0.9961(.060)
	0	-0.0603(.036)	-0.0020(.037)	-0.0005(.038)	-0.0632(.026)	-0.0012(.027)	-0.0003(.028)
	.5	0.4932(.055)	0.4953(.052)	0.4945(.055)	0.4932(.047)	0.4946(.044)	0.4941(.047)
2	1	1.0345(.047)	1.0024(.047)	1.0013(.047)	1.0341(.034)	1.0006(.034)	0.9999(.034)
	1	0.9860(.182)	0.9575(.170)	0.9943(.185)	0.9805(.133)	0.9666(.126)	0.9893(.135)
	0	-0.0603(.037)	-0.0028(.038)	-0.0008(.039)	-0.0631(.027)	-0.0018(.027)	-0.0005(.028)
	.5	0.4920(.057)	0.4959(.054)	0.4931(.057)	0.4917(.048)	0.4943(.045)	0.4928(.047)
3	1	1.0327(.046)	1.0004(.046)	0.9994(.047)	1.0343(.035)	1.0005(.035)	0.9999(.035)
	1	0.9882(.134)	0.9642(.125)	0.9966(.135)	0.9863(.093)	0.9759(.088)	0.9951(.094)
	0	-0.0606(.038)	-0.0025(.039)	-0.0007(.039)	-0.0637(.027)	-0.0018(.028)	-0.0007(.029)
	.5	0.4933(.056)	0.4958(.052)	0.4942(.056)	0.4930(.047)	0.4953(.044)	0.4941(.046)
1	1	1.0244(.047)	0.9977(.047)	1.0002(.047)	1.0251(.034)	0.9988(.034)	1.0003(.034)
	1	0.9822(.082)	0.9705(.078)	0.9867(.082)	0.9906(.058)	0.9859(.056)	0.9952(.059)
	-.5	-0.5454(.037)	-0.4945(.038)	-0.4992(.039)	-0.5472(.027)	-0.4973(.028)	-0.5002(.029)
	.5	0.4919(.057)	0.4920(.054)	0.4923(.057)	0.4959(.047)	0.4952(.044)	0.4963(.047)
2	1	1.0262(.047)	0.9983(.047)	1.0019(.047)	1.0241(.036)	0.9969(.036)	0.9992(.036)
	1	0.9859(.177)	0.9655(.165)	0.9906(.179)	0.9945(.128)	0.9834(.121)	0.9992(.129)
	-.5	-0.5460(.037)	-0.4931(.039)	-0.5000(.040)	-0.5466(.027)	-0.4950(.029)	-0.4995(.029)
	.5	0.4903(.059)	0.4924(.055)	0.4904(.059)	0.4944(.046)	0.4955(.043)	0.4947(.046)
3	1	1.0251(.047)	0.9977(.047)	1.0008(.047)	1.0246(.035)	0.9978(.035)	0.9998(.035)
	1	0.9791(.132)	0.9630(.123)	0.9837(.133)	0.9892(.095)	0.9816(.090)	0.9939(.096)
	-.5	-0.5462(.036)	-0.4943(.039)	-0.5002(.039)	-0.5468(.027)	-0.4962(.029)	-0.4998(.029)
	.5	0.4924(.055)	0.4932(.051)	0.4927(.055)	0.4956(.046)	0.4952(.043)	0.4958(.046)

Table 1b. Empirical sd and average of standard errors of M-Estimator
`SE` Model, $T = 3, m = 5$, Parameter configurations as in Table 1a.

		$n = 50$				$n = 100$				$n = 200$			
`err`	ψ	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$
1	1	.100	.112	.099	.096	.071	.073	.070	.069	.053	.053	.051	.051
	1	.154	.165	.150	.146	.113	.114	.110	.109	.080	.081	.079	.080
	.5	.066	.068	.064	.065	.059	.054	.054	.056	.044	.040	.042	.044
	.5	.105	.111	.099	.096	.083	.086	.081	.080	.070	.070	.068	.068
2	1	.102	.124	.099	.093	.071	.078	.069	.068	.053	.055	.051	.050
	1	.307	.117	.152	.263	.209	.076	.110	.198	.155	.050	.079	.147
	.5	.078	.071	.064	.070	.065	.053	.054	.063	.052	.037	.042	.051
	.5	.104	.126	.099	.090	.089	.095	.082	.078	.072	.074	.068	.067
3	1	.103	.117	.099	.095	.070	.075	.069	.069	.049	.053	.051	.051
	1	.227	.133	.151	.203	.162	.089	.110	.153	.121	.061	.079	.113
	.5	.071	.070	.064	.066	.062	.053	.054	.060	.048	.039	.042	.047
	.5	.106	.118	.099	.093	.088	.091	.082	.079	.073	.072	.068	.067
1	1	.104	.113	.102	.100	.072	.074	.071	.071	.053	.054	.052	.052
	1	.154	.165	.149	.144	.111	.112	.107	.106	.078	.078	.076	.076
	.0	.078	.081	.075	.075	.056	.057	.055	.055	.043	.042	.042	.042
	.5	.105	.111	.099	.094	.087	.086	.082	.081	.070	.071	.069	.068
2	1	.104	.126	.103	.131	.074	.078	.071	.070	.055	.056	.052	.052
	1	.299	.117	.157	.568	.211	.073	.107	.196	.150	.048	.076	.143
	.0	.086	.086	.077	.173	.065	.058	.055	.060	.047	.041	.042	.046
	.5	.102	.126	.099	.094	.087	.094	.082	.078	.072	.075	.069	.067
3	1	.104	.120	.102	.099	.073	.076	.071	.071	.054	.054	.052	.052
	1	.224	.132	.150	.203	.156	.086	.107	.149	.115	.058	.076	.109
	.0	.083	.084	.075	.077	.059	.057	.055	.057	.045	.042	.042	.044
	.5	.105	.117	.099	.093	.084	.091	.082	.079	.070	.072	.068	.067
1	1	.104	.115	.104	.101	.072	.075	.072	.071	.054	.054	.053	.052
	1	.147	.161	.145	.140	.103	.108	.103	.101	.075	.075	.073	.072
	-.5	.070	.077	.069	.067	.048	.051	.048	.047	.036	.036	.035	.035
	.5	.108	.111	.099	.095	.086	.087	.082	.081	.072	.070	.068	.068
2	1	.103	.126	.103	.099	.072	.081	.072	.071	.054	.056	.053	.052
	1	.284	.107	.143	.255	.215	.067	.104	.196	.145	.043	.073	.142
	-.5	.075	.085	.069	.067	.051	.054	.048	.049	.037	.037	.035	.036
	.5	.102	.127	.099	.089	.084	.095	.082	.077	.073	.075	.069	.067
3	1	.106	.120	.103	.100	.074	.077	.072	.071	.054	.055	.053	.053
	1	.205	.124	.143	.194	.151	.081	.103	.147	.108	.054	.073	.106
	-.5	.072	.080	.069	.068	.049	.052	.048	.048	.035	.037	.035	.035
	.5	.108	.118	.100	.094	.088	.092	.083	.079	.072	.073	.069	.067

Table 1b. Cont'd, $T = \mathbf{7}$

		$n = 50$				$n = 100$				$n = 200$			
err	ψ	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$
1	1	.044	.047	.044	.043	.033	.034	.032	.032	.025	.026	.025	.025
	1	.083	.090	.084	.082	.061	.062	.059	.058	.042	.043	.042	.042
	.5	.029	.031	.028	.028	.022	.022	.021	.021	.016	.016	.016	.015
	.5	.056	.060	.055	.054	.046	.048	.046	.045	.039	.040	.039	.039
2	1	.045	.050	.044	.043	.033	.035	.032	.032	.025	.026	.025	.025
	1	.187	.050	.084	.172	.133	.032	.059	.127	.093	.021	.042	.092
	.5	.029	.032	.028	.028	.022	.023	.021	.021	.017	.016	.016	.016
	.5	.057	.066	.055	.052	.047	.051	.046	.045	.040	.041	.039	.038
3	1	.044	.049	.044	.043	.033	.034	.032	.032	.026	.026	.025	.025
	1	.134	.062	.083	.128	.097	.041	.059	.092	.069	.028	.042	.066
	.5	.029	.031	.028	.028	.021	.022	.021	.021	.017	.016	.016	.016
	.5	.057	.063	.056	.053	.047	.049	.046	.046	.039	.040	.039	.038
1	1	.048	.051	.047	.047	.036	.036	.035	.035	.027	.027	.027	.027
	1	.084	.090	.083	.082	.060	.061	.059	.058	.040	.042	.042	.042
	0	.038	.042	.039	.038	.028	.029	.028	.028	.021	.021	.021	.021
	.5	.055	.061	.055	.054	.047	.048	.046	.046	.040	.040	.039	.039
2	1	.047	.054	.047	.046	.034	.038	.035	.034	.027	.028	.027	.027
	1	.185	.050	.084	.174	.135	.031	.058	.126	.095	.021	.042	.092
	0	.039	.045	.039	.037	.028	.030	.028	.028	.021	.022	.021	.021
	.5	.057	.066	.055	.052	.047	.051	.046	.045	.040	.041	.039	.038
3	1	.047	.052	.047	.047	.035	.037	.035	.035	.027	.028	.027	.027
	1	.135	.062	.084	.128	.094	.040	.059	.093	.068	.027	.042	.066
	0	.039	.043	.039	.038	.029	.030	.028	.028	.021	.021	.021	.021
	.5	.056	.063	.055	.052	.046	.049	.046	.045	.039	.040	.039	.039
1	1	.047	.051	.047	.046	.034	.036	.035	.035	.027	.027	.027	.027
	1	.082	.089	.083	.081	.059	.061	.059	.058	.041	.042	.041	.041
	-.5	.039	.043	.040	.040	.029	.030	.029	.028	.021	.021	.021	.021
	.5	.057	.061	.056	.054	.047	.048	.046	.045	.039	.040	.039	.039
2	1	.047	.054	.047	.046	.036	.038	.035	.035	.027	.028	.027	.026
	1	.179	.048	.083	.174	.129	.031	.059	.127	.095	.020	.041	.091
	-.5	.040	.048	.040	.038	.029	.032	.029	.028	.021	.022	.021	.020
	.5	.059	.067	.056	.052	.046	.051	.046	.045	.038	.041	.039	.038
3	1	.047	.052	.047	.046	.035	.037	.035	.034	.026	.027	.027	.027
	1	.133	.061	.082	.125	.096	.040	.059	.093	.067	.027	.041	.066
	-.5	.039	.045	.040	.039	.029	.031	.029	.028	.021	.021	.021	.021
	.5	.055	.063	.056	.053	.046	.049	.046	.045	.039	.040	.039	.039

Table 2a. Empirical Mean(sd) of CQMLE and M-Estimator, `SL` Model, $T = 3, m = 5$

		$n = 50$		$n = 100$		$n = 200$	
`err`	ψ	`CQMLE`	`M-Est`	`CQMLE`	`M-Est`	`CQMLE`	`M-Est`
1	1	1.0190(.053)	.9992(.055)	1.0024(.035)	1.0004(.036)	1.0112(.025)	.9998(.025)
	1	.9365(.133)	.9695(.143)	.9620(.096)	.9855(.100)	.9657(.068)	.9949(.072)
	.5	.4279(.042)	.5015(.047)	.4467(.024)	.5007(.026)	.4310(.020)	.4995(.022)
	.2	.2331(.060)	.1933(.064)	.2114(.049)	.1953(.052)	.2048(.039)	.1980(.040)
2	1	1.0193(.052)	.9992(.054)	1.0005(.034)	.9984(.034)	1.0116(.026)	1.0003(.026)
	1	.9391(.260)	.9743(.280)	.9558(.184)	.9797(.194)	.9635(.137)	.9929(.145)
	.5	.4289(.045)	.5031(.048)	.4474(.027)	.5012(.028)	.4318(.022)	.5000(.023)
	.2	.2335(.061)	.1938(.065)	.2124(.050)	.1967(.053)	.2030(.037)	.1962(.039)
3	1	1.0180(.055)	.9980(.056)	1.0019(.035)	.9998(.036)	1.0111(.024)	.9997(.025)
	1	.9388(.203)	.9730(.218)	.9581(.147)	.9817(.155)	.9623(.102)	.9913(.108)
	.5	.4277(.043)	.5018(.047)	.4461(.027)	.5000(.029)	.4319(.021)	.4999(.023)
	.2	.2354(.060)	.1960(.064)	.2121(.050)	.1962(.052)	.2054(.037)	.1986(.039)
1	1	1.0349(.056)	.9978(.058)	1.0159(.035)	.9990(.036)	1.0260(.026)	1.0005(.027)
	1	.9398(.138)	.9691(.146)	.9626(.098)	.9844(.102)	.9684(.068)	.9921(.071)
	0	-.0801(.047)	-.0010(.052)	-.0670(.031)	-.0004(.033)	-.0718(.023)	.0003(.025)
	.2	.2301(.089)	.1892(.092)	.1879(.061)	.1944(.061)	.1909(.046)	.1973(.047)
2	1	1.0367(.056)	.9999(.058)	1.0157(.036)	.9989(.037)	1.0253(.026)	.9997(.027)
	1	.9389(.268)	.9695(.284)	.9587(.195)	.9809(.204)	.9694(.139)	.9935(.146)
	0	-.0759(.050)	.0030(.055)	-.0667(.034)	-.0005(.035)	-.0726(.025)	-.0004(.026)
	.2	.2273(.090)	.1869(.094)	.1892(.062)	.1959(.062)	.1925(.046)	.1990(.047)
3	1	1.0344(.056)	.9977(.058)	1.0158(.035)	.9989(.036)	1.0245(.026)	.9988(.027)
	1	.9404(.202)	.9703(.214)	.9647(.150)	.9869(.157)	.9693(.103)	.9933(.108)
	0	-.0770(.048)	.0019(.053)	-.0662(.032)	.0005(.034)	-.0714(.023)	.0009(.024)
	.2	.2316(.092)	.1916(.095)	.1876(.061)	.1941(.061)	.1913(.046)	.1975(.046)
1	1	1.0261(.057)	.9978(.059)	1.0188(.037)	.9993(.037)	1.0226(.027)	.9992(.027)
	1	.9600(.136)	.9753(.141)	.9761(.099)	.9891(.102)	.9797(.068)	.9917(.070)
	-.5	-.5577(.046)	-.4976(.050)	-.5543(.031)	-.4989(.034)	-.5513(.022)	-.4991(.024)
	.2	.1898(.096)	.1841(.097)	.1855(.059)	.1978(.059)	.1824(.044)	.1975(.044)
2	1	1.0254(.056)	.9971(.058)	1.0179(.037)	.9985(.037)	1.0228(.028)	.9993(.028)
	1	.9557(.281)	.9719(.290)	.9745(.201)	.9878(.206)	.9857(.140)	.9980(.143)
	-.5	-.5557(.048)	-.4958(.052)	-.5542(.033)	-.4990(.035)	-.5518(.023)	-.4995(.024)
	.2	.1985(.093)	.1920(.094)	.1853(.058)	.1974(.058)	.1815(.043)	.1966(.043)
3	1	1.0261(.057)	.9978(.059)	1.0184(.037)	.9989(.037)	1.0225(.028)	.9990(.028)
	1	.9487(.198)	.9640(.204)	.9752(.155)	.9884(.159)	.9808(.105)	.9929(.108)
	-.5	-.5562(.048)	-.4971(.052)	-.5547(.034)	-.4992(.037)	-.5514(.023)	-.4991(.025)
	.2	.1938(.097)	.1877(.098)	.1824(.058)	.1943(.058)	.1827(.043)	.1974(.044)

Note: Par = $\psi = (\beta, \sigma_v^2, \rho, \lambda_1)'$; `err` = 1 (normal), 2 (normal mixture), and 3 (chi-square).
X_t values are generated with $\theta_x = (g, \phi_1, \phi_2, \sigma_1, \sigma_2) = (.01, .5, .5, 2, 1)$, as in Footnote 13.
W_1 is generated according to Queen Contiguity scheme.

Table 2a. Cont'd, $T = \mathbf{7}$

		$n = 50$		$n = 100$		$n = 200$	
`err`	ψ	`CQMLE`	`M-Est`	`CQMLE`	`M-Est`	`CQMLE`	`M-Est`
1	1	1.0069(.024)	.9994(.024)	1.0064(.017)	.9994(.017)	1.0094(.012)	.9998(.012)
	1	.9833(.080)	.9885(.080)	.9879(.059)	.9935(.060)	.9926(.040)	.9976(.040)
	.5	.4787(.017)	.4999(.018)	.4775(.013)	.5001(.013)	.4807(.008)	.4997(.009)
	.2	.1995(.034)	.1984(.034)	.2025(.025)	.1992(.025)	.2063(.014)	.2002(.014)
2	1	1.0075(.024)	.9999(.024)	1.0071(.018)	1.0001(.018)	1.0098(.012)	1.0003(.012)
	1	.9843(.182)	.9896(.184)	.9910(.131)	.9968(.132)	.9928(.091)	.9978(.092)
	.5	.4780(.017)	.4991(.017)	.4771(.013)	.4997(.013)	.4807(.008)	.4997(.008)
	.2	.1986(.033)	.1975(.033)	.2017(.024)	.1982(.024)	.2056(.014)	.1996(.014)
3	1	1.0080(.024)	1.0003(.024)	1.0078(.017)	1.0007(.017)	1.0091(.012)	.9996(.012)
	1	.9837(.132)	.9890(.133)	.9893(.094)	.9950(.095)	.9905(.066)	.9954(.067)
	.5	.4774(.018)	.4986(.018)	.4771(.013)	.4997(.013)	.4808(.008)	.4998(.008)
	.2	.2008(.033)	.1997(.034)	.2021(.024)	.1987(.024)	.2059(.014)	.1998(.014)
1	1	1.0149(.026)	1.0012(.026)	1.0137(.019)	1.0000(.020)	1.0143(.013)	.9998(.014)
	1	.9863(.082)	.9902(.082)	.9905(.058)	.9946(.059)	.9930(.040)	.9971(.040)
	.0	-.0284(.025)	-.0019(.025)	-.0266(.018)	.0002(.018)	-.0261(.012)	-.0001(.012)
	.2	.2011(.043)	.1992(.043)	.2017(.030)	.1989(.030)	.2037(.021)	.1994(.020)
2	1	1.0146(.026)	1.0009(.026)	1.0139(.020)	1.0003(.020)	1.0144(.014)	.9999(.014)
	1	.9885(.178)	.9926(.179)	.9860(.129)	.9901(.130)	.9934(.093)	.9975(.094)
	.0	-.0272(.025)	-.0006(.025)	-.0279(.018)	-.0013(.018)	-.0259(.013)	.0001(.013)
	.2	.1991(.042)	.1973(.042)	.2014(.031)	.1986(.031)	.2037(.020)	.1994(.020)
3	1	1.0145(.026)	1.0008(.026)	1.0137(.019)	1.0000(.020)	1.0143(.013)	.9998(.013)
	1	.9875(.136)	.9915(.137)	.9873(.092)	.9913(.093)	.9932(.067)	.9972(.067)
	.0	-.0281(.025)	-.0015(.026)	-.0276(.018)	-.0009(.018)	-.0258(.013)	.0002(.013)
	.2	.2001(.043)	.1981(.042)	.2011(.031)	.1983(.031)	.2043(.021)	.2001(.020)
1	1	1.0123(.026)	1.0001(.026)	1.0130(.020)	.9996(.020)	1.0130(.014)	1.0000(.014)
	1	.9841(.082)	.9864(.082)	.9910(.058)	.9936(.058)	.9955(.040)	.9981(.040)
	-.5	-.5248(.028)	-.5006(.029)	-.5258(.020)	-.5001(.021)	-.5250(.014)	-.5000(.015)
	.2	.1957(.045)	.1970(.045)	.1983(.035)	.1986(.035)	.2004(.023)	.2003(.023)
2	1	1.0133(.027)	1.0010(.027)	1.0128(.020)	.9994(.020)	1.0135(.013)	1.0005(.013)
	1	.9849(.184)	.9872(.185)	.9979(.129)	1.0005(.130)	.9930(.093)	.9956(.093)
	-.5	-.5240(.028)	-.4999(.028)	-.5255(.020)	-.4997(.021)	-.5255(.014)	-.5006(.015)
	.2	.1924(.046)	.1936(.046)	.1974(.036)	.1977(.036)	.1996(.023)	.1995(.023)
3	1	1.0109(.027)	.9986(.027)	1.0131(.020)	.9997(.020)	1.0129(.013)	.9999(.013)
	1	.9832(.133)	.9855(.134)	.9916(.094)	.9942(.094)	.9935(.068)	.9960(.069)
	-.5	-.5238(.028)	-.4996(.029)	-.5250(.020)	-.4992(.021)	-.5248(.014)	-.4998(.014)
	.2	.1954(.047)	.1967(.047)	.1979(.035)	.1981(.035)	.2000(.023)	.1998(.023)

Note: Par = $\psi = (\beta, \sigma_v^2, \rho, \lambda_1)'$; `err` = 1 (normal), 2 (normal mixture), and 3 (chi-square). X_t values are generated with $\theta_x = (g, \phi_1, \phi_2, \sigma_1, \sigma_2) = (.01, .5, .5, 2, 1)$, as in Footnote 13. W_1 is generated according to Queen Contiguity scheme.

Table 2b. Empirical sd and average of standard errors of M-Estimator
`SL` Model, $T = 3, m = 5$, Parameter configurations as in Table 2a.

		$n = 50$				$n = 100$				$n = 200$			
`err`	ψ	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$
1	1	.055	.060	.054	.052	.036	.036	.035	.034	.025	.026	.026	.025
	1	.143	.158	.142	.138	.100	.106	.101	.100	.072	.075	.073	.072
	.5	.047	.049	.044	.044	.026	.028	.026	.026	.022	.021	.021	.021
	.2	.064	.070	.063	.061	.052	.048	.052	.059	.040	.037	.039	.042
2	1	.054	.066	.054	.052	.034	.039	.034	.034	.026	.027	.026	.025
	1	.280	.105	.143	.255	.194	.063	.101	.190	.145	.042	.073	.141
	.5	.048	.052	.044	.046	.028	.029	.026	.027	.023	.021	.021	.022
	.2	.065	.077	.063	.061	.053	.051	.052	.058	.039	.039	.039	.042
3	1	.056	.062	.054	.052	.036	.037	.034	.034	.025	.027	.026	.025
	1	.218	.122	.143	.196	.155	.078	.101	.144	.108	.053	.072	.105
	.5	.047	.050	.044	.045	.029	.028	.026	.027	.023	.021	.021	.022
	.2	.064	.074	.063	.060	.052	.049	.052	.058	.039	.038	.039	.042
1	1	.058	.063	.057	.056	.036	.037	.036	.035	.027	.027	.027	.027
	1	.146	.157	.142	.138	.102	.106	.101	.099	.071	.074	.072	.072
	0	.052	.055	.051	.050	.033	.034	.033	.033	.025	.025	.024	.024
	.2	.092	.101	.092	.090	.061	.065	.062	.061	.047	.047	.046	.045
2	1	.058	.069	.057	.055	.037	.040	.035	.035	.027	.028	.027	.027
	1	.284	.104	.142	.252	.204	.063	.101	.191	.146	.042	.072	.141
	0	.055	.059	.050	.052	.035	.036	.033	.033	.026	.025	.024	.025
	.2	.094	.110	.091	.088	.062	.069	.061	.060	.047	.048	.045	.045
3	1	.058	.066	.057	.055	.036	.039	.036	.035	.027	.028	.027	.026
	1	.214	.120	.142	.195	.157	.078	.101	.145	.108	.053	.072	.106
	0	.053	.056	.051	.051	.034	.035	.033	.033	.024	.025	.024	.024
	.2	.095	.105	.091	.088	.061	.067	.062	.061	.046	.047	.046	.045
1	1	.059	.063	.057	.056	.037	.039	.037	.036	.027	.028	.027	.027
	1	.141	.155	.140	.137	.102	.106	.100	.099	.070	.073	.071	.071
	-.5	.050	.056	.051	.049	.034	.036	.034	.034	.024	.024	.024	.023
	.2	.097	.099	.096	.100	.059	.059	.059	.060	.044	.041	.043	.045
2	1	.058	.069	.057	.055	.037	.041	.037	.036	.028	.029	.027	.027
	1	.290	.099	.140	.251	.206	.062	.100	.192	.143	.040	.072	.142
	-.5	.052	.061	.050	.049	.035	.038	.034	.034	.024	.025	.024	.024
	.2	.094	.109	.095	.096	.058	.063	.058	.059	.043	.043	.043	.045
3	1	.059	.066	.057	.055	.037	.039	.037	.036	.028	.029	.027	.027
	1	.204	.118	.139	.191	.159	.077	.100	.145	.108	.051	.071	.105
	-.5	.052	.058	.050	.049	.037	.037	.034	.034	.025	.024	.024	.024
	.2	.098	.104	.096	.098	.058	.061	.059	.060	.044	.042	.043	.045

Table 2b. Cont'd, $T = \mathbf{7}$

		$n = 50$				$n = 100$				$n = 200$			
err	ψ	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$
1	1	.024	.025	.024	.023	.017	.018	.017	.017	.012	.012	.012	.012
	1	.080	.087	.081	.080	.060	.060	.058	.057	.040	.042	.041	.041
	.5	.018	.019	.018	.017	.013	.013	.013	.013	.009	.009	.008	.008
	.2	.034	.034	.034	.035	.025	.023	.024	.026	.014	.014	.014	.014
2	1	.024	.027	.023	.023	.018	.019	.017	.017	.012	.013	.012	.012
	1	.184	.045	.081	.173	.132	.029	.058	.127	.092	.020	.041	.091
	.5	.017	.020	.017	.017	.013	.014	.013	.013	.008	.009	.008	.008
	.2	.033	.036	.034	.035	.024	.023	.024	.026	.014	.014	.014	.014
3	1	.024	.026	.024	.023	.017	.018	.017	.017	.012	.012	.012	.012
	1	.133	.058	.081	.127	.095	.039	.058	.092	.067	.026	.041	.066
	.5	.018	.020	.018	.017	.013	.014	.013	.013	.008	.009	.008	.008
	.2	.034	.035	.034	.035	.024	.023	.024	.026	.014	.014	.014	.014
1	1	.026	.028	.026	.026	.020	.020	.019	.019	.014	.014	.013	.013
	1	.082	.087	.081	.080	.059	.060	.058	.057	.040	.042	.041	.041
	.0	.025	.027	.025	.025	.018	.019	.018	.018	.012	.013	.013	.013
	.2	.043	.046	.043	.043	.030	.032	.031	.031	.020	.021	.021	.021
2	1	.026	.030	.026	.026	.020	.021	.019	.019	.014	.014	.013	.013
	1	.179	.045	.081	.174	.130	.029	.057	.125	.094	.019	.041	.091
	0	.025	.029	.025	.025	.018	.019	.018	.018	.013	.013	.013	.013
	.2	.042	.048	.043	.043	.031	.033	.031	.031	.020	.021	.021	.021
3	1	.026	.029	.026	.026	.020	.020	.019	.019	.013	.014	.013	.013
	1	.137	.059	.081	.126	.093	.039	.058	.091	.067	.026	.041	.066
	0	.026	.028	.025	.025	.018	.019	.018	.018	.013	.013	.013	.013
	.2	.042	.047	.043	.043	.031	.032	.031	.031	.020	.021	.021	.021
1	1	.026	.029	.027	.027	.020	.021	.020	.020	.014	.014	.014	.014
	1	.082	.087	.081	.079	.058	.060	.058	.057	.040	.042	.041	.041
	-.5	.029	.031	.029	.028	.021	.022	.021	.021	.015	.015	.015	.015
	.2	.045	.049	.048	.051	.035	.036	.036	.037	.023	.024	.024	.025
2	1	.027	.031	.027	.026	.020	.022	.020	.020	.013	.014	.014	.014
	1	.185	.044	.081	.172	.130	.029	.058	.127	.093	.019	.041	.091
	-.5	.028	.033	.029	.028	.021	.023	.021	.021	.015	.015	.015	.015
	.2	.046	.052	.048	.050	.036	.038	.036	.037	.023	.024	.024	.024
3	1	.027	.030	.027	.026	.020	.021	.020	.020	.013	.014	.014	.014
	1	.134	.058	.081	.125	.094	.038	.058	.092	.069	.026	.041	.066
	-.5	.029	.032	.029	.028	.021	.022	.021	.021	.014	.015	.015	.015
	.2	.047	.051	.048	.051	.035	.037	.036	.037	.023	.024	.024	.024

Table 3a. Empirical Mean(sd) of CQMLE and M-Estimator, SLE Model, $T = 3, m = 5$

		$n = 50$		$n = 100$		$n = 200$	
err	ψ	CQMLE	M-Est	CQMLE	M-Est	CQMLE	M-Est
1	1	1.0139(.057)	1.0000(.058)	1.0050(.037)	1.0002(.038)	.9941(.027)	1.0009(.028)
	1	.9236(.132)	.9538(.141)	.9497(.095)	.9769(.101)	.9577(.070)	.9887(.075)
	.5	.4264(.041)	.4993(.046)	.4323(.028)	.4983(.031)	.4255(.021)	.4999(.023)
	.2	.2063(.066)	.1910(.071)	.1686(.076)	.1907(.080)	.1918(.046)	.1946(.049)
	.2	.1776(.168)	.1871(.166)	.1942(.120)	.1859(.119)	.1952(.082)	.1948(.081)
2	1	1.0137(.056)	.9998(.058)	1.0022(.036)	.9972(.037)	.9948(.027)	1.0015(.028)
	1	.9240(.261)	.9560(.279)	.9471(.193)	.9749(.204)	.9617(.138)	.9933(.147)
	.5	.4281(.046)	.5009(.048)	.4338(.031)	.4997(.032)	.4255(.023)	.5000(.024)
	.2	.2091(.065)	.1942(.070)	.1675(.076)	.1885(.080)	.1910(.047)	.1942(.050)
	.2	.1734(.168)	.1825(.167)	.1935(.119)	.1865(.118)	.1985(.082)	.1967(.082)
3	1	1.0162(.058)	1.0022(.060)	1.0057(.036)	1.0008(.036)	.9937(.027)	1.0005(.027)
	1	.9185(.194)	.9492(.208)	.9501(.143)	.9777(.152)	.9577(.101)	.9888(.108)
	.5	.4273(.044)	.5000(.047)	.4336(.031)	.4998(.033)	.4262(.022)	.5005(.023)
	.2	.2057(.066)	.1908(.070)	.1697(.076)	.1912(.081)	.1926(.046)	.1959(.049)
	.2	.1757(.166)	.1822(.167)	.1917(.117)	.1850(.115)	.1921(.083)	.1914(.083)
1	1	1.0377(.062)	.9988(.064)	1.0235(.038)	1.0001(.039)	1.0175(.028)	1.0003(.029)
	1	.9298(.135)	.9574(.143)	.9590(.098)	.9813(.103)	.9602(.069)	.9882(.073)
	0	-.0779(.047)	.0021(.052)	-.0702(.032)	.0002(.034)	-.0831(.024)	.0003(.027)
	.2	.2075(.087)	.1844(.090)	.1668(.073)	.1883(.073)	.2061(.050)	.1949(.051)
	.2	.1793(.166)	.1872(.165)	.1950(.117)	.1961(.117)	.1928(.086)	.1949(.086)
2	1	1.0383(.063)	.9994(.064)	1.0223(.038)	.9991(.038)	1.0173(.027)	1.0002(.028)
	1	.9337(.270)	.9630(.287)	.9534(.196)	.9759(.205)	.9578(.137)	.9860(.145)
	0	-.0781(.051)	.0016(.054)	-.0713(.034)	-.0017(.035)	-.0834(.026)	-.0003(.028)
	.2	.2115(.083)	.1891(.086)	.1685(.071)	.1895(.070)	.2058(.047)	.1946(.048)
	.2	.1749(.161)	.1818(.159)	.1896(.118)	.1915(.117)	.1955(.081)	.1968(.081)
3	1	1.0412(.061)	1.0023(.063)	1.0227(.038)	.9995(.039)	1.0180(.027)	1.0007(.028)
	1	.9328(.198)	.9609(.210)	.9564(.151)	.9788(.158)	.9598(.104)	.9880(.110)
	0	-.0816(.050)	-.0020(.053)	-.0711(.033)	-.0012(.036)	-.0845(.025)	-.0011(.027)
	.2	.2083(.084)	.1857(.087)	.1720(.072)	.1931(.071)	.2076(.048)	.1961(.049)
	.2	.1817(.167)	.1892(.165)	.1897(.117)	.1912(.116)	.1924(.083)	.1942(.084)
1	1	1.0339(.065)	.9985(.066)	1.0253(.041)	1.0006(.042)	1.0226(.028)	1.0003(.029)
	1	.9363(.135)	.9510(.139)	.9665(.098)	.9796(.100)	.9777(.068)	.9928(.070)
	-.5	-.5576(.047)	-.4984(.051)	-.5546(.033)	-.4997(.036)	-.5608(.024)	-.4996(.026)
	.2	.1820(.100)	.1779(.101)	.1778(.067)	.1888(.068)	.1895(.050)	.1936(.050)
	.2	.1832(.171)	.1817(.171)	.1898(.117)	.1897(.116)	.1955(.083)	.1946(.083)
2	1	1.0337(.066)	.9980(.067)	1.0248(.039)	1.0001(.040)	1.0229(.029)	1.0008(.029)
	1	.9379(.275)	.9536(.283)	.9655(.195)	.9788(.201)	.9759(.139)	.9911(.143)
	-.5	-.5565(.051)	-.4971(.054)	-.5550(.034)	-.5002(.036)	-.5598(.025)	-.4990(.027)
	.2	.1825(.100)	.1782(.101)	.1794(.067)	.1906(.068)	.1896(.052)	.1937(.052)
	.2	.1806(.170)	.1794(.170)	.1953(.116)	.1949(.116)	.1951(.081)	.1948(.081)
3	1	1.0322(.064)	.9967(.065)	1.0237(.038)	.9989(.039)	1.0227(.029)	1.0005(.029)
	1	.9411(.203)	.9564(.209)	.9691(.147)	.9824(.151)	.9736(.104)	.9887(.107)
	-.5	-.5572(.048)	-.4975(.052)	-.5542(.033)	-.4991(.035)	-.5609(.025)	-.5001(.026)
	.2	.1825(.102)	.1785(.103)	.1773(.068)	.1882(.069)	.1895(.051)	.1931(.052)
	.2	.1813(.169)	.1798(.169)	.1926(.114)	.1928(.115)	.1946(.081)	.1944(.080)

Note: Par = $\psi = (\beta, \sigma_v^2, \rho, \lambda_1, \lambda_3)'$; err = 1 (normal), 2 (normal mixture), and 3 (chi-square). X_t values are generated with $\theta_x = (g, \phi_1, \phi_2, \sigma_1, \sigma_2) = (.01, .5, .5, 2, 1)$, as in Footnote 13. W_1 from Group Interaction, and W_3 from Queen; see Footnote 14.

Table 3a. Cont'd, $T = \mathbf{7}$

		$n = 50$		$n = 100$		$n = 200$	
err	ψ	CQMLE	M-Est	CQMLE	M-Est	CQMLE	M-Est
1	1	1.0083(.023)	1.0003(.024)	1.0092(.018)	.9995(.018)	1.0089(.012)	.9998(.012)
	1	.9788(.079)	.9838(.080)	.9877(.058)	.9935(.058)	.9914(.041)	.9961(.041)
	.5	.4794(.017)	.4991(.017)	.4757(.013)	.4998(.014)	.4808(.008)	.4999(.008)
	.2	.2057(.030)	.1994(.030)	.1976(.030)	.1979(.031)	.2050(.018)	.1990(.018)
	.2	.1945(.092)	.1972(.091)	.1960(.066)	.1958(.066)	.1962(.047)	.1970(.047)
2	1	1.0083(.023)	1.0003(.023)	1.0093(.017)	.9996(.017)	1.0094(.012)	1.0002(.012)
	1	.9843(.183)	.9894(.185)	.9857(.128)	.9916(.130)	.9948(.089)	.9996(.090)
	.5	.4795(.017)	.4993(.017)	.4757(.013)	.4997(.013)	.4807(.008)	.4998(.008)
	.2	.2051(.031)	.1989(.031)	.1981(.029)	.1982(.029)	.2047(.018)	.1987(.018)
	.2	.1893(.092)	.1920(.091)	.1972(.066)	.1970(.066)	.1956(.048)	.1966(.047)
3	1	1.0078(.023)	.9998(.023)	1.0103(.018)	1.0006(.018)	1.0092(.012)	1.0000(.012)
	1	.9748(.133)	.9799(.134)	.9863(.093)	.9921(.094)	.9918(.065)	.9966(.066)
	.5	.4801(.017)	.4998(.017)	.4752(.013)	.4992(.013)	.4810(.008)	.5001(.009)
	.2	.2052(.031)	.1989(.032)	.1970(.030)	.1972(.030)	.2051(.018)	.1991(.018)
	.2	.1920(.093)	.1945(.092)	.1961(.064)	.1959(.065)	.1961(.047)	.1968(.047)
1	1	1.0144(.026)	1.0000(.026)	1.0151(.019)	1.0002(.020)	1.0144(.013)	1.0003(.014)
	1	.9824(.082)	.9864(.082)	.9884(.057)	.9922(.058)	.9917(.042)	.9955(.042)
	0	-.0259(.025)	.0004(.025)	-.0283(.018)	-.0003(.019)	-.0258(.012)	-.0004(.012)
	.2	.1994(.047)	.1945(.047)	.1950(.039)	.1974(.038)	.2025(.029)	.1980(.029)
	.2	.1943(.092)	.1961(.092)	.1952(.066)	.1952(.066)	.1972(.048)	.1982(.048)
2	1	1.0140(.026)	.9996(.026)	1.0150(.019)	1.0001(.019)	1.0141(.013)	1.0000(.013)
	1	.9807(.182)	.9848(.184)	.9869(.128)	.9908(.129)	.9915(.094)	.9953(.095)
	0	-.0268(.025)	-.0006(.025)	-.0282(.019)	-.0003(.019)	-.0255(.012)	-.0001(.012)
	.2	.2007(.044)	.1960(.044)	.1929(.039)	.1953(.038)	.2019(.028)	.1975(.028)
	.2	.1960(.093)	.1972(.093)	.1973(.067)	.1970(.067)	.1969(.047)	.1977(.046)
3	1	1.0143(.027)	.9999(.027)	1.0143(.020)	.9993(.020)	1.0142(.013)	1.0001(.013)
	1	.9854(.133)	.9895(.134)	.9935(.092)	.9974(.093)	.9915(.064)	.9952(.065)
	0	-.0268(.025)	-.0005(.025)	-.0283(.018)	-.0002(.018)	-.0260(.012)	-.0006(.013)
	.2	.2003(.045)	.1956(.045)	.1956(.039)	.1981(.039)	.2035(.029)	.1991(.029)
	.2	.1911(.090)	.1926(.090)	.1992(.067)	.1992(.066)	.1975(.048)	.1984(.048)
1	1	1.0126(.027)	.9998(.027)	1.0134(.020)	.9999(.020)	1.0128(.013)	1.0000(.014)
	1	.9842(.082)	.9867(.083)	.9907(.057)	.9929(.058)	.9939(.040)	.9964(.040)
	-.5	-.5252(.029)	-.5004(.030)	-.5261(.021)	-.5001(.022)	-.5243(.014)	-.4994(.015)
	.2	.1919(.050)	.1916(.050)	.1889(.038)	.1957(.038)	.1970(.031)	.1981(.032)
	.2	.1955(.091)	.1956(.091)	.2000(.067)	.1998(.067)	.1979(.046)	.1981(.046)
2	1	1.0112(.027)	.9984(.027)	1.0135(.020)	1.0000(.020)	1.0134(.013)	1.0006(.013)
	1	.9894(.187)	.9919(.188)	.9910(.132)	.9933(.132)	.9885(.094)	.9909(.095)
	-.5	-.5237(.028)	-.4990(.029)	-.5260(.021)	-.5000(.021)	-.5248(.014)	-.5000(.015)
	.2	.1962(.051)	.1960(.051)	.1900(.038)	.1966(.038)	.1954(.031)	.1963(.032)
	.2	.1921(.092)	.1921(.092)	.1992(.066)	.1987(.066)	.1980(.047)	.1984(.047)
3	1	1.0122(.027)	.9994(.028)	1.0139(.020)	1.0003(.020)	1.0128(.013)	1.0000(.014)
	1	.9857(.136)	.9882(.137)	.9891(.094)	.9914(.094)	.9943(.067)	.9968(.067)
	-.5	-.5235(.028)	-.4988(.029)	-.5264(.020)	-.5004(.021)	-.5246(.014)	-.4997(.014)
	.2	.1932(.050)	.1930(.050)	.1890(.038)	.1955(.038)	.1958(.033)	.1968(.033)
	.2	.1951(.092)	.1952(.092)	.1983(.065)	.1979(.065)	.1962(.047)	.1966(.047)

Table 3b. Empirical sd and average of standard errors of M-Estimator
SLE Model, $T = 3, m = 5$, Parameter configurations as in Table 3a.

		$n = 50$				$n = 100$				$n = 200$			
err	ψ	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$
1	1	.058	.063	.058	.057	.038	.038	.037	.036	.028	.028	.027	.027
	1	.141	.158	.140	.136	.101	.107	.101	.100	.075	.075	.073	.072
	.5	.046	.047	.044	.044	.031	.030	.029	.030	.023	.022	.022	.023
	.2	.071	.070	.068	.073	.080	.063	.077	.099	.049	.043	.048	.054
	.2	.166	.179	.160	.159	.119	.123	.116	.117	.081	.085	.082	.081
2	1	.058	.070	.057	.056	.037	.041	.036	.036	.028	.029	.027	.027
	1	.279	.109	.141	.249	.204	.066	.101	.190	.147	.044	.073	.141
	.5	.048	.051	.044	.045	.032	.032	.029	.031	.024	.022	.022	.024
	.2	.070	.079	.068	.070	.080	.068	.077	.097	.050	.046	.048	.054
	.2	.167	.209	.161	.151	.118	.136	.116	.113	.082	.090	.082	.080
3	1	.060	.066	.057	.057	.036	.040	.037	.036	.027	.029	.027	.027
	1	.208	.124	.140	.188	.152	.080	.101	.144	.108	.054	.073	.106
	.5	.047	.049	.044	.045	.033	.030	.029	.032	.023	.022	.022	.023
	.2	.070	.073	.068	.072	.081	.065	.077	.099	.049	.045	.048	.054
	.2	.167	.192	.161	.154	.115	.129	.116	.115	.083	.087	.082	.081
1	1	.064	.068	.062	.061	.039	.040	.038	.038	.029	.029	.028	.028
	1	.143	.158	.140	.137	.103	.107	.101	.099	.073	.075	.072	.072
	0	.052	.056	.051	.051	.034	.035	.034	.034	.027	.027	.026	.026
	.2	.090	.097	.086	.084	.073	.074	.070	.069	.051	.049	.048	.048
	.2	.165	.180	.160	.159	.117	.122	.114	.113	.086	.085	.082	.082
2	1	.064	.075	.062	.061	.038	.043	.038	.038	.028	.030	.028	.028
	1	.287	.108	.142	.249	.205	.066	.101	.190	.145	.043	.072	.140
	0	.054	.061	.051	.052	.035	.037	.034	.034	.028	.027	.026	.027
	.2	.086	.107	.085	.081	.070	.080	.069	.067	.048	.051	.048	.047
	.2	.159	.209	.161	.150	.117	.135	.115	.109	.081	.090	.082	.080
3	1	.063	.071	.062	.062	.039	.042	.038	.038	.028	.029	.028	.028
	1	.210	.124	.141	.192	.158	.080	.101	.143	.110	.054	.072	.106
	0	.053	.058	.051	.051	.036	.036	.034	.034	.027	.027	.026	.027
	.2	.087	.102	.086	.082	.071	.076	.069	.068	.049	.050	.048	.048
	.2	.165	.193	.160	.153	.116	.129	.115	.111	.084	.087	.082	.081
1	1	.066	.070	.064	.063	.042	.042	.040	.039	.029	.029	.029	.028
	1	.139	.154	.138	.134	.100	.106	.100	.099	.070	.074	.072	.071
	-.5	.051	.056	.050	.050	.036	.036	.034	.034	.026	.026	.026	.026
	.2	.101	.102	.096	.101	.068	.069	.065	.065	.050	.050	.051	.054
	.2	.171	.183	.162	.161	.116	.122	.115	.114	.083	.085	.082	.081
2	1	.067	.079	.064	.063	.040	.045	.040	.039	.029	.031	.029	.028
	1	.283	.104	.138	.246	.201	.064	.100	.189	.143	.042	.072	.140
	-.5	.054	.062	.050	.049	.036	.039	.034	.034	.027	.027	.026	.026
	.2	.101	.116	.096	.097	.068	.073	.065	.064	.052	.052	.051	.053
	.2	.170	.211	.163	.152	.116	.135	.115	.109	.081	.090	.082	.079
3	1	.065	.074	.064	.063	.039	.043	.040	.039	.029	.030	.029	.028
	1	.209	.121	.138	.191	.151	.078	.100	.143	.107	.052	.071	.105
	-.5	.052	.059	.050	.050	.035	.037	.034	.034	.026	.027	.026	.026
	.2	.103	.109	.096	.098	.069	.071	.065	.065	.052	.051	.051	.053
	.2	.169	.195	.163	.157	.115	.128	.115	.112	.080	.087	.082	.081

Table 3b. Cont'd, $T = \mathbf{7}$

		$n = 50$				$n = 100$				$n = 200$			
err	ψ	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$
1	1	.024	.025	.023	.023	.018	.018	.018	.017	.012	.012	.012	.012
	1	.080	.088	.081	.080	.058	.060	.058	.057	.041	.042	.041	.041
	.5	.017	.019	.017	.017	.014	.014	.013	.013	.008	.009	.008	.008
	.2	.030	.032	.030	.031	.031	.028	.030	.033	.018	.018	.018	.019
	.2	.091	.099	.091	.089	.066	.069	.066	.065	.047	.048	.047	.047
2	1	.023	.027	.023	.023	.017	.019	.018	.017	.012	.012	.012	.012
	1	.185	.047	.082	.173	.130	.030	.058	.126	.090	.020	.041	.091
	.5	.017	.020	.017	.016	.013	.014	.013	.013	.008	.009	.008	.008
	.2	.031	.034	.030	.031	.029	.029	.030	.033	.018	.018	.018	.019
	.2	.091	.109	.091	.087	.066	.073	.066	.064	.047	.050	.047	.046
3	1	.023	.026	.023	.023	.018	.019	.018	.017	.012	.012	.012	.012
	1	.134	.060	.081	.125	.094	.040	.058	.091	.066	.027	.041	.066
	.5	.017	.019	.017	.017	.013	.014	.013	.013	.009	.009	.008	.008
	.2	.032	.033	.030	.031	.030	.028	.030	.033	.018	.018	.018	.019
	.2	.092	.103	.091	.088	.065	.070	.066	.065	.047	.049	.047	.047
1	1	.026	.028	.026	.026	.020	.020	.019	.019	.014	.013	.013	.013
	1	.082	.089	.081	.080	.058	.060	.058	.057	.042	.042	.041	.041
	0	.025	.027	.025	.025	.019	.019	.018	.018	.012	.013	.012	.012
	.2	.047	.049	.045	.044	.038	.040	.039	.038	.029	.029	.028	.028
	.2	.092	.100	.091	.089	.066	.069	.066	.065	.048	.048	.047	.047
2	1	.026	.030	.026	.026	.019	.021	.019	.019	.013	.014	.013	.013
	1	.184	.046	.081	.172	.129	.030	.058	.126	.095	.020	.041	.091
	0	.025	.029	.025	.024	.019	.020	.018	.018	.012	.013	.012	.012
	.2	.044	.053	.045	.043	.038	.042	.038	.038	.028	.030	.028	.028
	.2	.093	.109	.091	.087	.067	.073	.066	.064	.046	.050	.047	.046
3	1	.027	.029	.026	.026	.020	.021	.019	.019	.013	.014	.013	.013
	1	.134	.059	.082	.128	.093	.039	.058	.093	.065	.027	.041	.066
	0	.025	.028	.025	.025	.018	.019	.018	.018	.013	.013	.012	.012
	.2	.045	.050	.045	.044	.039	.041	.039	.038	.029	.029	.028	.028
	.2	.090	.105	.091	.088	.066	.071	.066	.065	.048	.049	.047	.047
1	1	.027	.029	.027	.026	.020	.021	.020	.020	.014	.014	.013	.013
	1	.083	.089	.081	.080	.058	.060	.058	.057	.040	.042	.041	.041
	-.5	.030	.032	.029	.029	.022	.022	.021	.021	.015	.015	.015	.015
	.2	.050	.053	.050	.052	.038	.038	.038	.038	.032	.032	.032	.033
	.2	.091	.100	.091	.090	.067	.069	.066	.065	.046	.048	.047	.047
2	1	.027	.031	.027	.026	.020	.022	.020	.020	.013	.014	.013	.013
	1	.188	.046	.082	.174	.132	.030	.058	.126	.095	.020	.041	.091
	-.5	.029	.034	.029	.028	.021	.023	.021	.021	.015	.015	.015	.014
	.2	.051	.057	.050	.050	.038	.040	.038	.038	.032	.032	.032	.033
	.2	.092	.111	.092	.087	.066	.073	.066	.063	.047	.050	.047	.046
3	1	.028	.030	.027	.027	.020	.021	.020	.020	.014	.014	.013	.013
	1	.137	.059	.081	.127	.094	.039	.058	.092	.067	.027	.041	.066
	-.5	.029	.032	.029	.029	.021	.023	.021	.021	.014	.015	.015	.015
	.2	.050	.055	.050	.051	.038	.039	.038	.038	.033	.032	.032	.033
	.2	.092	.105	.091	.088	.065	.071	.066	.064	.047	.049	.047	.047

Table 3c. Empirical Mean(sd) of CQMLE and M-Estimator, `SLE` Model, $T = 3, m = 5$

		$n = 50$		$n = 100$		$n = 200$	
`err`	ψ	`CQMLE`	`M-Est`	`CQMLE`	`M-Est`	`CQMLE`	`M-Est`
1	1	1.0028(.051)	1.0012(.052)	.9894(.035)	.9990(.036)	1.0130(.028)	1.0001(.028)
	1	.9268(.133)	.9542(.141)	.9491(.093)	.9800(.100)	.9629(.070)	.9911(.075)
	.5	.4332(.041)	.4993(.044)	.4285(.029)	.4999(.032)	.4337(.019)	.5010(.021)
	.2	.2064(.078)	.1930(.083)	.2190(.069)	.1905(.075)	.1871(.063)	.1938(.066)
	.2	.1383(.185)	.1489(.183)	.1561(.148)	.1582(.146)	.1781(.122)	.1723(.120)
2	1	1.0006(.050)	.9989(.051)	.9908(.036)	1.0005(.037)	1.0123(.027)	.9995(.028)
	1	.9219(.263)	.9505(.280)	.9495(.188)	.9813(.200)	.9626(.138)	.9910(.146)
	.5	.4355(.043)	.5011(.045)	.4291(.032)	.5006(.034)	.4332(.022)	.5001(.023)
	.2	.2016(.078)	.1881(.083)	.2199(.067)	.1921(.072)	.1881(.064)	.1940(.067)
	.2	.1411(.175)	.1525(.171)	.1597(.148)	.1634(.145)	.1778(.122)	.1733(.120)
3	1	1.0001(.052)	.9984(.053)	.9920(.036)	1.0015(.037)	1.0121(.028)	.9993(.028)
	1	.9247(.199)	.9527(.212)	.9461(.143)	.9771(.152)	.9596(.102)	.9875(.108)
	.5	.4345(.042)	.5006(.046)	.4287(.031)	.4996(.034)	.4324(.021)	.4991(.022)
	.2	.2038(.080)	.1901(.086)	.2209(.071)	.1925(.076)	.1898(.063)	.1955(.066)
	.2	.1394(.186)	.1510(.182)	.1598(.148)	.1629(.145)	.1745(.121)	.1695(.118)
1	1	1.0228(.054)	1.0007(.055)	1.0128(.035)	.9991(.036)	1.0279(.028)	.9996(.029)
	1	.9271(.136)	.9511(.143)	.9522(.096)	.9813(.102)	.9635(.068)	.9899(.072)
	0	-.0730(.047)	.0000(.051)	-.0880(.036)	.0000(.039)	-.0795(.024)	.0001(.026)
	.2	.1975(.101)	.1855(.102)	.1901(.080)	.1886(.082)	.1921(.073)	.1896(.074)
	.2	.1396(.191)	.1555(.188)	.1689(.146)	.1747(.144)	.1685(.126)	.1748(.122)
2	1	1.0201(.055)	.9978(.056)	1.0130(.036)	.9994(.037)	1.0280(.029)	.9999(.030)
	1	.9314(.258)	.9568(.272)	.9457(.196)	.9750(.208)	.9634(.136)	.9901(.143)
	0	-.0749(.050)	-.0016(.053)	-.0877(.038)	-.0006(.041)	-.0799(.026)	-.0006(.027)
	.2	.1978(.102)	.1851(.104)	.1904(.080)	.1889(.082)	.1948(.075)	.1922(.076)
	.2	.1407(.188)	.1585(.182)	.1693(.151)	.1750(.151)	.1683(.126)	.1746(.125)
3	1	1.0201(.054)	.9980(.055)	1.0121(.036)	.9985(.037)	1.0272(.028)	.9991(.029)
	1	.9307(.201)	.9554(.212)	.9515(.149)	.9809(.158)	.9618(.100)	.9881(.106)
	0	-.0756(.047)	-.0024(.051)	-.0867(.036)	.0011(.040)	-.0799(.025)	-.0008(.026)
	.2	.1986(.100)	.1865(.101)	.1902(.080)	.1894(.081)	.1932(.075)	.1911(.076)
	.2	.1455(.180)	.1606(.177)	.1650(.145)	.1697(.144)	.1642(.128)	.1697(.128)
1	1	1.0255(.055)	1.0012(.055)	1.0193(.036)	1.0010(.037)	1.0266(.030)	1.0001(.030)
	1	.9386(.136)	.9523(.140)	.9631(.097)	.9784(.100)	.9738(.069)	.9895(.071)
	-.5	-.5556(.046)	-.4978(.050)	-.5624(.034)	-.4995(.038)	-.5644(.024)	-.5005(.027)
	.2	.1858(.093)	.1867(.096)	.1916(.078)	.1914(.081)	.1871(.069)	.1933(.071)
	.2	.1529(.186)	.1533(.188)	.1676(.148)	.1687(.151)	.1774(.123)	.1718(.127)
2	1	1.0231(.057)	.9988(.058)	1.0179(.038)	.9997(.038)	1.0272(.030)	1.0006(.030)
	1	.9575(.282)	.9724(.290)	.9637(.196)	.9794(.202)	.9745(.141)	.9904(.145)
	-.5	-.5575(.049)	-.4991(.053)	-.5609(.036)	-.4982(.039)	-.5645(.025)	-.5006(.027)
	.2	.1865(.092)	.1883(.095)	.1868(.077)	.1862(.079)	.1830(.070)	.1885(.073)
	.2	.1590(.175)	.1598(.176)	.1726(.145)	.1741(.147)	.1816(.124)	.1762(.130)
3	1	1.0232(.056)	.9992(.057)	1.0176(.039)	.9992(.039)	1.0260(.030)	.9994(.031)
	1	.9418(.206)	.9558(.211)	.9669(.145)	.9826(.149)	.9721(.106)	.9879(.109)
	-.5	-.5551(.048)	-.4973(.051)	-.5630(.035)	-.4998(.038)	-.5638(.025)	-.4999(.027)
	.2	.1831(.096)	.1836(.099)	.1879(.078)	.1872(.080)	.1845(.068)	.1906(.071)
	.2	.1614(.180)	.1629(.182)	.1637(.152)	.1655(.153)	.1811(.126)	.1753(.131)

Note: Par = $\psi = (\beta, \sigma_v^2, \rho, \lambda_1, \lambda_3)'$; `err` = 1 (normal), 2 (normal mixture), and 3 (chi-square).
X_t values are generated with $\theta_x = (g, \phi_1, \phi_2, \sigma_1, \sigma_2) = (.01, .5, .5, 2, 1)$, as in Footnote 13.
W_1 **and** W_3 **are from Group Interaction scheme, and not equal**; see Footnote 14.

Table 3c. Cont'd, $T = \mathbf{7}$

		$n = 50$		$n = 100$		$n = 200$	
err	ψ	CQMLE	M-Est	CQMLE	M-Est	CQMLE	M-Est
1	1	1.0076(.024)	1.0006(.024)	1.0070(.017)	1.0004(.017)	1.0103(.012)	1.0004(.012)
	1	.9792(.080)	.9841(.081)	.9869(.057)	.9926(.058)	.9899(.042)	.9947(.042)
	.5	.4791(.017)	.4988(.017)	.4758(.013)	.4998(.013)	.4795(.009)	.4997(.009)
	.2	.2037(.033)	.1983(.033)	.1970(.034)	.1989(.034)	.1968(.023)	.1988(.023)
	.2	.1811(.092)	.1842(.091)	.1888(.078)	.1892(.078)	.1929(.063)	.1932(.062)
2	1	1.0077(.024)	1.0008(.024)	1.0067(.017)	1.0001(.017)	1.0106(.012)	1.0007(.012)
	1	.9811(.187)	.9861(.189)	.9900(.130)	.9958(.132)	.9887(.093)	.9935(.094)
	.5	.4788(.017)	.4985(.017)	.4757(.013)	.4997(.013)	.4796(.009)	.4998(.009)
	.2	.2029(.032)	.1975(.032)	.1965(.034)	.1983(.034)	.1973(.024)	.1993(.024)
	.2	.1816(.088)	.1851(.088)	.1866(.078)	.1872(.078)	.1894(.064)	.1895(.064)
3	1	1.0067(.024)	.9997(.024)	1.0074(.018)	1.0008(.018)	1.0098(.012)	.9999(.012)
	1	.9802(.138)	.9852(.139)	.9888(.095)	.9945(.096)	.9906(.065)	.9955(.065)
	.5	.4797(.017)	.4995(.018)	.4750(.013)	.4989(.013)	.4795(.008)	.4998(.009)
	.2	.2027(.032)	.1974(.033)	.1953(.034)	.1971(.034)	.1972(.024)	.1991(.024)
	.2	.1885(.087)	.1911(.086)	.1884(.079)	.1889(.078)	.1904(.063)	.1906(.062)
1	1	1.0134(.027)	1.0005(.027)	1.0129(.019)	1.0001(.019)	1.0145(.013)	.9998(.013)
	1	.9848(.082)	.9886(.083)	.9872(.057)	.9911(.058)	.9934(.041)	.9972(.042)
	0	-.0270(.025)	-.0017(.025)	-.0276(.018)	-.0003(.018)	-.0263(.013)	.0000(.013)
	.2	.2005(.050)	.1964(.050)	.1999(.042)	.1973(.041)	.2012(.033)	.1988(.033)
	.2	.1844(.090)	.1876(.090)	.1865(.079)	.1893(.079)	.1878(.066)	.1904(.066)
2	1	1.0128(.026)	1.0000(.027)	1.0130(.019)	1.0002(.019)	1.0145(.013)	.9998(.013)
	1	.9828(.185)	.9866(.186)	.9877(.127)	.9917(.128)	.9929(.092)	.9967(.093)
	0	-.0261(.025)	-.0009(.025)	-.0279(.018)	-.0006(.018)	-.0263(.013)	.0000(.013)
	.2	.2006(.048)	.1965(.048)	.1975(.043)	.1950(.042)	.2018(.033)	.1993(.033)
	.2	.1822(.094)	.1855(.094)	.1873(.081)	.1899(.081)	.1868(.067)	.1893(.067)
3	1	1.0135(.026)	1.0007(.026)	1.0124(.019)	.9996(.019)	1.0147(.013)	1.0000(.013)
	1	.9821(.135)	.9858(.136)	.9873(.095)	.9912(.096)	.9931(.067)	.9969(.068)
	0	-.0264(.025)	-.0012(.025)	-.0276(.018)	-.0002(.018)	-.0263(.012)	.0000(.012)
	.2	.1997(.049)	.1957(.049)	.2008(.042)	.1982(.042)	.1999(.033)	.1975(.033)
	.2	.1844(.092)	.1877(.092)	.1891(.077)	.1918(.077)	.1893(.065)	.1917(.065)
1	1	1.0120(.027)	.9994(.027)	1.0113(.020)	.9992(.020)	1.0133(.013)	1.0001(.013)
	1	.9832(.082)	.9857(.082)	.9913(.056)	.9938(.056)	.9929(.042)	.9954(.042)
	-.5	-.5243(.028)	-.4994(.029)	-.5245(.020)	-.4992(.021)	-.5256(.014)	-.5002(.015)
	.2	.1977(.052)	.1954(.053)	.1927(.045)	.1956(.046)	.1963(.037)	.1966(.037)
	.2	.1845(.094)	.1860(.094)	.1914(.082)	.1891(.083)	.1939(.067)	.1937(.067)
2	1	1.0116(.027)	.9991(.027)	1.0121(.019)	1.0000(.019)	1.0133(.014)	1.0001(.014)
	1	.9872(.187)	.9897(.187)	.9890(.131)	.9915(.131)	.9956(.092)	.9982(.092)
	-.5	-.5243(.028)	-.4997(.028)	-.5242(.020)	-.4990(.021)	-.5253(.014)	-.4999(.014)
	.2	.1969(.052)	.1946(.053)	.1920(.045)	.1949(.046)	.1974(.036)	.1978(.036)
	.2	.1858(.092)	.1872(.092)	.1907(.081)	.1882(.082)	.1929(.065)	.1926(.066)
3	1	1.0126(.028)	1.0001(.028)	1.0119(.019)	.9997(.019)	1.0134(.014)	1.0002(.014)
	1	.9805(.130)	.9830(.130)	.9893(.093)	.9917(.094)	.9949(.065)	.9975(.066)
	-.5	-.5236(.029)	-.4989(.030)	-.5247(.020)	-.4994(.020)	-.5256(.015)	-.5002(.015)
	.2	.2009(.052)	.1985(.052)	.1930(.044)	.1958(.045)	.1981(.037)	.1986(.037)
	.2	.1836(.094)	.1854(.094)	.1949(.081)	.1928(.082)	.1906(.067)	.1902(.067)

Table 3d. Empirical sd and average of standard errors of M-Estimator
SLE Model, $T = 3, m = 5$, Parameter configurations **as in Table 3c**.

		$n = 50$				$n = 100$				$n = 200$			
err	ψ	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$
1	1	.052	.056	.051	.050	.036	.038	.036	.035	.028	.029	.028	.028
	1	.141	.157	.140	.137	.100	.108	.102	.100	.075	.075	.072	.072
	.5	.044	.045	.042	.042	.032	.032	.031	.031	.021	.021	.021	.021
	.2	.083	.072	.080	.098	.075	.065	.072	.086	.066	.056	.064	.076
	.2	.183	.179	.160	.163	.146	.143	.135	.138	.120	.116	.114	.117
2	1	.051	.062	.050	.049	.037	.041	.036	.035	.028	.030	.028	.027
	1	.280	.107	.140	.247	.200	.067	.102	.192	.146	.043	.072	.141
	.5	.045	.049	.041	.042	.034	.033	.031	.033	.023	.021	.021	.022
	.2	.083	.080	.079	.095	.072	.070	.072	.085	.067	.059	.064	.074
	.2	.171	.208	.160	.153	.145	.155	.134	.134	.120	.122	.113	.114
3	1	.053	.058	.051	.050	.037	.039	.036	.035	.028	.029	.028	.028
	1	.212	.123	.140	.190	.152	.080	.101	.144	.108	.054	.072	.105
	.5	.046	.046	.041	.043	.034	.031	.031	.033	.022	.021	.021	.022
	.2	.086	.076	.080	.097	.076	.067	.072	.085	.066	.057	.064	.075
	.2	.182	.191	.160	.159	.145	.147	.134	.136	.118	.119	.114	.115
1	1	.055	.058	.053	.052	.036	.038	.036	.036	.029	.030	.029	.029
	1	.143	.156	.139	.135	.102	.108	.102	.100	.072	.075	.072	.072
	0	.051	.053	.049	.048	.039	.040	.038	.039	.026	.026	.026	.026
	.2	.102	.103	.093	.093	.082	.082	.078	.078	.074	.075	.073	.073
	.2	.188	.185	.164	.163	.144	.145	.137	.138	.122	.122	.119	.119
2	1	.056	.064	.053	.052	.037	.041	.036	.035	.030	.031	.029	.029
	1	.272	.108	.140	.247	.208	.067	.101	.188	.143	.043	.072	.140
	0	.053	.058	.048	.049	.041	.042	.038	.039	.027	.026	.026	.027
	.2	.104	.116	.093	.092	.082	.088	.078	.077	.076	.078	.072	.072
	.2	.182	.215	.165	.155	.151	.159	.137	.132	.125	.129	.119	.117
3	1	.055	.061	.053	.052	.037	.039	.036	.035	.029	.030	.029	.029
	1	.212	.123	.140	.191	.158	.080	.102	.145	.106	.054	.072	.105
	0	.051	.055	.049	.049	.040	.040	.038	.040	.026	.026	.026	.026
	.2	.101	.108	.093	.093	.081	.084	.078	.077	.076	.076	.072	.072
	.2	.177	.196	.164	.159	.144	.151	.137	.136	.128	.127	.119	.118
1	1	.055	.060	.055	.055	.037	.039	.037	.037	.030	.030	.030	.029
	1	.140	.155	.138	.134	.100	.106	.100	.098	.071	.074	.071	.071
	-.5	.050	.055	.050	.049	.038	.039	.037	.036	.027	.027	.026	.026
	.2	.096	.096	.090	.095	.081	.075	.075	.080	.071	.067	.069	.073
	.2	.188	.185	.168	.171	.151	.146	.140	.142	.127	.123	.120	.122
2	1	.058	.068	.055	.054	.038	.042	.037	.037	.030	.032	.030	.029
	1	.290	.105	.141	.253	.202	.064	.100	.190	.145	.042	.071	.140
	-.5	.053	.062	.050	.049	.039	.042	.037	.037	.027	.028	.026	.027
	.2	.095	.108	.090	.092	.079	.081	.076	.080	.073	.070	.069	.073
	.2	.176	.213	.168	.161	.147	.160	.138	.136	.130	.130	.120	.120
3	1	.057	.063	.055	.054	.039	.040	.037	.037	.031	.031	.030	.030
	1	.211	.120	.138	.191	.149	.079	.100	.143	.109	.052	.071	.104
	-.5	.051	.058	.050	.049	.038	.040	.037	.037	.027	.027	.026	.027
	.2	.099	.102	.091	.094	.080	.078	.076	.080	.071	.069	.069	.073
	.2	.182	.196	.166	.164	.153	.153	.140	.140	.131	.126	.120	.120

Table 3d. Cont'd, $T = \mathbf{7}$

		$n = 50$				$n = 100$				$n = 200$			
err	ψ	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$
1	1	.024	.026	.024	.024	.017	.018	.017	.017	.012	.012	.012	.012
	1	.081	.088	.081	.080	.058	.060	.058	.057	.042	.042	.041	.041
	.5	.017	.019	.017	.017	.013	.014	.013	.013	.009	.009	.009	.009
	.2	.033	.031	.032	.035	.034	.031	.033	.038	.023	.022	.024	.026
	.2	.091	.095	.087	.085	.078	.078	.075	.075	.062	.064	.062	.062
2	1	.024	.028	.024	.023	.017	.019	.017	.017	.012	.013	.012	.012
	1	.189	.046	.081	.173	.132	.030	.058	.127	.094	.020	.041	.091
	.5	.017	.020	.017	.017	.013	.014	.013	.013	.009	.009	.009	.009
	.2	.032	.034	.032	.035	.034	.032	.033	.038	.024	.023	.024	.025
	.2	.088	.104	.087	.083	.078	.082	.075	.074	.064	.066	.063	.062
3	1	.024	.027	.024	.023	.018	.018	.017	.017	.012	.013	.012	.012
	1	.139	.060	.081	.126	.096	.040	.058	.092	.065	.027	.041	.066
	.5	.018	.019	.017	.017	.013	.014	.013	.013	.009	.009	.009	.009
	.2	.033	.033	.032	.035	.034	.031	.034	.038	.024	.023	.024	.025
	.2	.086	.099	.086	.083	.078	.080	.075	.074	.062	.065	.062	.062
1	1	.027	.029	.026	.026	.019	.020	.019	.019	.013	.014	.013	.013
	1	.083	.089	.081	.080	.058	.060	.058	.057	.042	.042	.041	.041
	0	.025	.027	.025	.024	.018	.019	.018	.018	.013	.013	.013	.013
	.2	.050	.051	.047	.047	.041	.044	.042	.042	.033	.033	.033	.033
	.2	.090	.099	.089	.087	.079	.082	.078	.078	.066	.067	.065	.065
2	1	.027	.031	.026	.026	.019	.020	.019	.019	.013	.014	.013	.013
	1	.186	.047	.081	.173	.128	.030	.058	.126	.093	.020	.041	.091
	0	.025	.029	.025	.024	.018	.020	.018	.018	.013	.013	.013	.013
	.2	.048	.054	.047	.046	.042	.046	.042	.042	.033	.034	.033	.033
	.2	.094	.109	.089	.084	.081	.086	.078	.076	.067	.069	.065	.064
3	1	.026	.030	.026	.026	.019	.020	.019	.019	.013	.014	.013	.013
	1	.136	.059	.081	.127	.096	.039	.058	.092	.068	.026	.041	.066
	0	.025	.027	.025	.024	.018	.019	.018	.018	.012	.013	.013	.013
	.2	.049	.053	.047	.046	.042	.045	.042	.042	.033	.034	.033	.033
	.2	.092	.103	.089	.086	.077	.083	.078	.077	.065	.067	.065	.064
1	1	.027	.030	.028	.027	.020	.020	.020	.019	.013	.014	.014	.014
	1	.082	.089	.081	.079	.056	.060	.058	.057	.042	.042	.041	.041
	-.5	.029	.032	.029	.029	.021	.022	.021	.021	.015	.015	.015	.015
	.2	.053	.056	.053	.055	.046	.045	.045	.047	.037	.037	.037	.039
	.2	.094	.100	.091	.090	.083	.083	.080	.080	.067	.068	.066	.067
2	1	.027	.032	.027	.027	.019	.021	.020	.019	.014	.014	.014	.014
	1	.187	.046	.081	.174	.131	.029	.058	.126	.092	.020	.041	.091
	-.5	.028	.034	.029	.028	.021	.023	.021	.021	.014	.016	.015	.015
	.2	.053	.060	.053	.055	.046	.047	.045	.046	.036	.037	.037	.039
	.2	.092	.109	.091	.087	.082	.088	.080	.079	.066	.070	.066	.066
3	1	.028	.031	.027	.027	.019	.021	.020	.019	.014	.014	.014	.014
	1	.130	.059	.081	.126	.094	.039	.058	.092	.066	.027	.041	.066
	-.5	.030	.033	.029	.029	.020	.022	.021	.021	.015	.015	.015	.015
	.2	.052	.058	.053	.055	.045	.046	.045	.047	.037	.037	.037	.039
	.2	.094	.105	.091	.089	.082	.084	.079	.079	.067	.069	.066	.066

Table 4a. Empirical Mean(sd) of CQMLE and M-Estimator, `STL` Model, $T = 3, m = 5$

		$n = 50$		$n = 100$		$n = 200$	
`err`	ψ	`CQMLE`	`M-Est`	`CQMLE`	`M-Est`	`CQMLE`	`M-Est`
1	1	.9851(.058)	.9979(.060)	1.0078(.037)	.9993(.038)	1.0050(.026)	1.0005(.026)
	1	.9322(.133)	.9596(.141)	.9616(.096)	.9845(.101)	.9619(.069)	.9897(.074)
	.5	.4340(.042)	.5021(.046)	.4438(.027)	.5013(.029)	.4314(.020)	.5006(.023)
	.2	.1912(.102)	.1804(.104)	.1907(.067)	.1924(.067)	.1996(.049)	.1965(.050)
	.2	.2657(.080)	.2135(.082)	.2611(.063)	.2052(.063)	.2643(.045)	.2018(.046)
2	1	.9841(.058)	.9966(.059)	1.0068(.037)	.9982(.038)	1.0046(.026)	1.0000(.027)
	1	.9285(.268)	.9572(.285)	.9610(.197)	.9847(.207)	.9678(.139)	.9963(.147)
	.5	.4332(.045)	.5007(.047)	.4428(.029)	.5005(.030)	.4317(.023)	.5013(.024)
	.2	.1947(.097)	.1840(.099)	.1896(.068)	.1913(.068)	.1984(.049)	.1954(.050)
	.2	.2627(.080)	.2108(.081)	.2632(.065)	.2072(.065)	.2650(.046)	.2022(.046)
3	1	.9858(.059)	.9986(.061)	1.0081(.038)	.9996(.038)	1.0034(.026)	.9988(.026)
	1	.9371(.196)	.9654(.208)	.9606(.145)	.9837(.153)	.9604(.101)	.9883(.107)
	.5	.4345(.043)	.5030(.047)	.4426(.028)	.5001(.030)	.4311(.022)	.5002(.024)
	.2	.1910(.099)	.1803(.101)	.1902(.068)	.1920(.067)	.1977(.050)	.1946(.050)
	.2	.2647(.079)	.2118(.081)	.2631(.064)	.2070(.064)	.2676(.045)	.2055(.046)
1	1	1.0083(.058)	1.0020(.058)	1.0229(.038)	.9993(.039)	1.0258(.027)	1.0005(.028)
	1	.9242(.135)	.9515(.143)	.9606(.096)	.9822(.101)	.9623(.068)	.9889(.072)
	0	-.0849(.050)	-.0014(.055)	-.0663(.031)	-.0001(.034)	-.0793(.024)	.0003(.026)
	.2	.1861(.096)	.1887(.098)	.1896(.070)	.1913(.070)	.1959(.050)	.1961(.050)
	.2	.2166(.112)	.2039(.121)	.2340(.057)	.2016(.059)	.2108(.054)	.2019(.056)
2	1	1.0056(.059)	.9994(.060)	1.0240(.041)	1.0006(.042)	1.0258(.027)	1.0005(.028)
	1	.9251(.260)	.9536(.276)	.9585(.196)	.9806(.205)	.9618(.135)	.9888(.142)
	0	-.0843(.054)	-.0015(.058)	-.0663(.034)	-.0005(.035)	-.0800(.026)	-.0005(.027)
	.2	.1816(.095)	.1839(.098)	.1909(.070)	.1926(.070)	.1941(.050)	.1941(.051)
	.2	.2193(.113)	.2060(.122)	.2350(.060)	.2029(.062)	.2105(.054)	.2015(.057)
3	1	1.0057(.058)	.9996(.059)	1.0243(.040)	1.0007(.041)	1.0260(.027)	1.0006(.028)
	1	.9305(.202)	.9587(.214)	.9587(.147)	.9804(.153)	.9640(.101)	.9909(.107)
	0	-.0839(.051)	.0000(.056)	-.0648(.032)	.0012(.034)	-.0797(.024)	.0001(.026)
	.2	.1862(.094)	.1890(.097)	.1881(.071)	.1898(.071)	.1937(.051)	.1938(.051)
	.2	.2142(.114)	.2022(.123)	.2354(.059)	.2031(.060)	.2112(.054)	.2025(.057)
1	1	1.0135(.059)	1.0004(.059)	1.0255(.040)	1.0001(.041)	1.0241(.028)	.9989(.028)
	1	.9425(.133)	.9585(.138)	.9679(.098)	.9823(.101)	.9744(.068)	.9901(.070)
	-.5	-.5643(.048)	-.5001(.052)	-.5594(.035)	-.5017(.037)	-.5614(.024)	-.5000(.026)
	.2	.1872(.095)	.1869(.095)	.1915(.073)	.1903(.073)	.2023(.052)	.1989(.052)
	.2	.1956(.124)	.1960(.131)	.2109(.067)	.1958(.069)	.1987(.057)	.1990(.060)
2	1	1.0147(.059)	1.0016(.059)	1.0254(.040)	1.0000(.041)	1.0248(.029)	.9999(.029)
	1	.9415(.273)	.9583(.282)	.9671(.199)	.9818(.204)	.9754(.142)	.9912(.146)
	-.5	-.5616(.051)	-.4975(.055)	-.5560(.034)	-.4983(.036)	-.5611(.025)	-.4999(.027)
	.2	.1872(.096)	.1871(.097)	.1941(.072)	.1929(.072)	.1999(.051)	.1963(.051)
	.2	.1992(.119)	.1991(.127)	.2136(.068)	.1991(.070)	.1987(.058)	.1990(.061)
3	1	1.0142(.058)	1.0012(.059)	1.0261(.040)	1.0008(.041)	1.0253(.029)	1.0003(.029)
	1	.9417(.207)	.9580(.214)	.9677(.145)	.9822(.149)	.9716(.104)	.9873(.107)
	-.5	-.5634(.051)	-.4992(.055)	-.5574(.033)	-.4999(.035)	-.5599(.024)	-.4987(.026)
	.2	.1887(.096)	.1887(.096)	.1931(.072)	.1920(.073)	.1983(.050)	.1948(.050)
	.2	.1938(.121)	.1929(.129)	.2140(.065)	.1990(.067)	.1978(.057)	.1987(.060)

Note: Par = $\psi = (\beta, \sigma_v^2, \rho, \lambda_1, \lambda_2)'$; `err` = 1 (normal), 2 (normal mixture), and 3 (chi-square). X_t values are generated with $\theta_x = (g, \phi_1, \phi_2, \sigma_1, \sigma_2) = (.01, .5, .5, 2, 1)$, as in Footnote 13. W_1 and W_2 are from Queen Contiguity, and equal.

Table 4a. Cont'd, $T = \mathbf{7}$

		$n = 50$		$n = 100$		$n = 200$	
err	ψ	CQMLE	M-Est	CQMLE	M-Est	CQMLE	M-Est
1	1	1.0045(.025)	1.0000(.025)	1.0073(.017)	.9998(.017)	1.0078(.012)	1.0002(.012)
	1	.9809(.079)	.9855(.080)	.9897(.058)	.9940(.058)	.9923(.041)	.9966(.041)
	.5	.4780(.018)	.4996(.019)	.4807(.012)	.5001(.012)	.4812(.008)	.5001(.009)
	.2	.1904(.046)	.1950(.046)	.1971(.031)	.1994(.031)	.1974(.023)	.1984(.023)
	.2	.2280(.041)	.2043(.042)	.2208(.030)	.2006(.030)	.2200(.021)	.2013(.021)
2	1	1.0040(.026)	.9994(.026)	1.0078(.017)	1.0003(.018)	1.0074(.012)	.9998(.012)
	1	.9920(.182)	.9968(.184)	.9884(.130)	.9927(.131)	.9934(.090)	.9977(.091)
	.5	.4780(.018)	.4999(.019)	.4805(.013)	.4999(.013)	.4816(.009)	.5005(.009)
	.2	.1908(.046)	.1954(.046)	.1962(.032)	.1985(.032)	.1986(.023)	.1995(.023)
	.2	.2276(.042)	.2038(.042)	.2216(.031)	.2014(.031)	.2186(.022)	.1999(.022)
3	1	1.0050(.025)	1.0006(.025)	1.0076(.018)	1.0001(.018)	1.0075(.012)	.9999(.012)
	1	.9815(.135)	.9861(.137)	.9912(.095)	.9955(.096)	.9903(.067)	.9945(.068)
	.5	.4783(.018)	.4999(.018)	.4805(.012)	.4999(.012)	.4810(.008)	.4999(.009)
	.2	.1905(.048)	.1950(.048)	.1954(.031)	.1977(.031)	.1978(.023)	.1988(.023)
	.2	.2278(.042)	.2041(.042)	.2226(.030)	.2024(.030)	.2198(.022)	.2012(.022)
1	1	1.0121(.026)	1.0006(.026)	1.0124(.019)	1.0001(.019)	1.0136(.014)	1.0001(.014)
	1	.9844(.081)	.9881(.082)	.9884(.055)	.9917(.056)	.9927(.040)	.9963(.040)
	0	-.0272(.025)	-.0009(.026)	-.0247(.017)	-.0005(.018)	-.0260(.013)	-.0002(.013)
	.2	.1945(.049)	.1957(.049)	.1955(.034)	.1972(.034)	.1977(.025)	.1986(.025)
	.2	.2060(.055)	.2018(.056)	.2117(.034)	.2010(.034)	.2084(.026)	.2005(.027)
2	1	1.0120(.027)	1.0005(.027)	1.0131(.019)	1.0009(.019)	1.0137(.014)	1.0002(.014)
	1	.9778(.183)	.9816(.185)	.9915(.129)	.9949(.130)	.9906(.093)	.9942(.094)
	0	-.0262(.025)	-.0001(.025)	-.0251(.017)	-.0010(.017)	-.0260(.012)	-.0002(.012)
	.2	.1958(.049)	.1971(.049)	.1953(.034)	.1969(.034)	.1992(.025)	.2001(.025)
	.2	.2052(.056)	.2011(.057)	.2112(.034)	.2006(.034)	.2080(.027)	.2002(.027)
3	1	1.0127(.027)	1.0011(.027)	1.0119(.019)	.9997(.019)	1.0137(.013)	1.0002(.013)
	1	.9835(.133)	.9872(.134)	.9923(.092)	.9957(.093)	.9920(.067)	.9956(.067)
	0	-.0264(.025)	-.0002(.026)	-.0250(.017)	-.0007(.018)	-.0260(.012)	-.0002(.012)
	.2	.1952(.050)	.1965(.049)	.1962(.034)	.1979(.034)	.1983(.025)	.1992(.025)
	.2	.2056(.054)	.2014(.055)	.2105(.034)	.1998(.034)	.2084(.026)	.2006(.027)
1	1	1.0126(.028)	1.0006(.028)	1.0112(.020)	.9991(.020)	1.0132(.014)	1.0003(.014)
	1	.9831(.080)	.9855(.080)	.9917(.058)	.9941(.058)	.9939(.041)	.9965(.041)
	-.5	-.5232(.027)	-.4996(.028)	-.5233(.019)	-.4994(.020)	-.5261(.014)	-.5009(.015)
	.2	.1959(.049)	.1953(.050)	.1986(.035)	.1976(.035)	.1997(.025)	.1989(.025)
	.2	.2018(.054)	.1995(.054)	.2050(.036)	.1998(.036)	.2054(.027)	.2001(.028)
2	1	1.0116(.027)	.9996(.028)	1.0116(.019)	.9996(.019)	1.0131(.014)	1.0003(.014)
	1	.9885(.182)	.9910(.183)	.9895(.131)	.9919(.131)	.9935(.092)	.9960(.093)
	-.5	-.5232(.028)	-.4995(.028)	-.5233(.019)	-.4996(.020)	-.5253(.014)	-.5002(.015)
	.2	.1985(.050)	.1978(.050)	.1986(.035)	.1975(.035)	.1992(.024)	.1984(.024)
	.2	.2035(.054)	.2012(.055)	.2049(.037)	.1997(.037)	.2062(.028)	.2008(.028)
3	1	1.0124(.027)	1.0005(.027)	1.0126(.019)	1.0006(.019)	1.0122(.014)	.9994(.014)
	1	.9826(.136)	.9851(.136)	.9901(.094)	.9925(.094)	.9946(.066)	.9972(.066)
	-.5	-.5244(.028)	-.5009(.028)	-.5241(.020)	-.5004(.021)	-.5248(.014)	-.4996(.014)
	.2	.2002(.049)	.1996(.049)	.1982(.036)	.1971(.037)	.1991(.026)	.1983(.026)
	.2	.2014(.055)	.1990(.056)	.2062(.037)	.2011(.037)	.2050(.028)	.1997(.028)

Table 4b. Empirical sd and average of standard errors of M-Estimator
`STL` Model, $T = 3, m = 5$, Parameter configurations as in Table 4a.

		$n = 50$				$n = 100$				$n = 200$			
`err`	ψ	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$
1	1	.060	.066	.058	.056	.038	.040	.038	.037	.026	.027	.027	.026
	1	.141	.157	.140	.139	.101	.108	.101	.100	.074	.074	.072	.073
	.5	.046	.040	.044	.056	.029	.026	.028	.033	.023	.019	.022	.028
	.2	.104	.109	.099	.103	.067	.070	.068	.069	.050	.049	.049	.051
	.2	.082	.089	.080	.080	.063	.060	.064	.075	.046	.041	.045	.055
2	1	.059	.072	.058	.055	.038	.043	.038	.037	.027	.028	.027	.027
	1	.285	.105	.140	.251	.207	.065	.101	.193	.147	.043	.073	.141
	.5	.047	.044	.043	.056	.030	.028	.028	.033	.024	.019	.022	.029
	.2	.099	.120	.098	.100	.068	.075	.068	.068	.050	.052	.049	.050
	.2	.081	.099	.079	.078	.065	.064	.064	.074	.046	.043	.045	.055
3	1	.061	.069	.058	.056	.038	.041	.038	.037	.026	.028	.027	.026
	1	.208	.122	.141	.198	.153	.079	.101	.146	.107	.053	.072	.106
	.5	.047	.041	.044	.057	.030	.027	.028	.033	.024	.018	.022	.029
	.2	.101	.115	.099	.102	.067	.073	.068	.068	.050	.051	.049	.050
	.2	.081	.094	.080	.079	.064	.062	.064	.075	.046	.042	.045	.054
1	1	.058	.064	.057	.055	.039	.041	.039	.039	.028	.029	.028	.028
	1	.143	.157	.139	.135	.101	.107	.101	.099	.072	.074	.072	.072
	0	.055	.059	.053	.052	.034	.035	.033	.033	.026	.026	.026	.026
	.2	.098	.105	.095	.096	.070	.074	.070	.071	.050	.053	.051	.051
	.2	.121	.093	.115	.160	.059	.049	.060	.076	.056	.044	.057	.075
2	1	.060	.071	.056	.054	.042	.044	.039	.039	.028	.030	.028	.028
	1	.276	.108	.140	.246	.205	.065	.101	.191	.142	.043	.072	.140
	0	.058	.065	.052	.052	.035	.036	.033	.034	.027	.027	.026	.027
	.2	.098	.118	.095	.093	.070	.080	.070	.069	.051	.055	.051	.050
	.2	.122	.103	.114	.156	.062	.053	.059	.075	.057	.046	.056	.074
3	1	.059	.067	.057	.055	.041	.043	.039	.039	.028	.029	.028	.028
	1	.214	.125	.140	.190	.153	.080	.101	.143	.107	.054	.072	.105
	0	.056	.062	.053	.053	.034	.036	.033	.033	.026	.027	.026	.026
	.2	.097	.110	.095	.095	.071	.077	.070	.070	.051	.054	.051	.050
	.2	.123	.098	.115	.160	.060	.050	.059	.076	.057	.045	.057	.075
1	1	.059	.064	.058	.057	.041	.043	.041	.040	.028	.029	.029	.028
	1	.138	.155	.138	.135	.101	.107	.100	.098	.070	.074	.071	.071
	-.5	.052	.059	.053	.051	.037	.037	.035	.035	.026	.027	.026	.026
	.2	.095	.106	.095	.093	.073	.076	.073	.074	.052	.053	.052	.052
	.2	.131	.109	.126	.162	.069	.063	.067	.077	.060	.053	.060	.070
2	1	.059	.071	.058	.056	.041	.046	.040	.040	.029	.030	.028	.028
	1	.282	.104	.138	.248	.204	.064	.100	.190	.146	.041	.071	.140
	-.5	.055	.066	.052	.051	.036	.040	.035	.035	.027	.027	.026	.026
	.2	.097	.118	.094	.090	.072	.082	.072	.072	.051	.056	.051	.051
	.2	.127	.122	.124	.158	.070	.068	.067	.076	.061	.056	.060	.069
3	1	.059	.067	.058	.056	.041	.044	.041	.040	.029	.030	.028	.028
	1	.214	.122	.138	.187	.149	.079	.100	.143	.107	.052	.071	.104
	-.5	.055	.062	.052	.051	.035	.039	.035	.035	.026	.027	.026	.025
	.2	.096	.111	.094	.091	.073	.078	.072	.073	.050	.054	.052	.051
	.2	.129	.114	.125	.160	.067	.065	.067	.076	.060	.055	.060	.070

Table 4b. Cont'd, $T = \mathbf{7}$

		$n = 50$				$n = 100$				$n = 200$			
`err`	ψ	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$
1	1	.025	.027	.025	.024	.017	.018	.017	.017	.012	.012	.012	.012
	1	.080	.088	.081	.080	.058	.060	.058	.057	.041	.042	.041	.041
	.5	.019	.019	.018	.019	.012	.012	.012	.013	.009	.008	.009	.009
	.2	.046	.049	.047	.048	.031	.032	.032	.032	.023	.023	.023	.024
	.2	.042	.044	.042	.044	.030	.030	.030	.032	.021	.021	.022	.023
2	1	.026	.029	.025	.025	.018	.019	.017	.017	.012	.013	.012	.012
	1	.184	.046	.082	.174	.131	.029	.058	.126	.091	.020	.041	.091
	.5	.019	.021	.019	.019	.013	.013	.012	.013	.009	.008	.009	.009
	.2	.046	.052	.047	.048	.032	.033	.031	.032	.023	.024	.023	.024
	.2	.042	.047	.042	.044	.031	.031	.030	.032	.022	.021	.022	.023
3	1	.025	.028	.025	.024	.018	.018	.017	.017	.012	.013	.012	.012
	1	.137	.060	.081	.126	.096	.039	.058	.092	.068	.026	.041	.066
	.5	.018	.020	.018	.019	.012	.012	.012	.013	.009	.008	.009	.009
	.2	.048	.050	.046	.048	.031	.033	.032	.032	.023	.023	.023	.024
	.2	.042	.045	.042	.044	.030	.030	.030	.032	.022	.021	.022	.023
1	1	.026	.029	.027	.027	.019	.020	.019	.019	.014	.014	.013	.013
	1	.082	.088	.081	.080	.056	.060	.058	.057	.040	.042	.041	.041
	0	.026	.028	.025	.025	.018	.018	.017	.017	.013	.013	.013	.013
	.2	.049	.052	.050	.064	.034	.034	.034	.035	.025	.025	.025	.025
	.2	.056	.053	.055	.067	.034	.031	.033	.037	.027	.024	.027	.030
2	1	.027	.031	.027	.026	.019	.021	.019	.019	.014	.014	.013	.013
	1	.185	.046	.081	.171	.130	.029	.058	.127	.094	.020	.041	.091
	0	.025	.029	.025	.025	.017	.019	.017	.017	.012	.013	.013	.013
	.2	.049	.056	.049	.049	.034	.036	.034	.035	.025	.025	.025	.025
	.2	.057	.056	.054	.060	.034	.033	.033	.037	.027	.025	.027	.030
3	1	.027	.030	.027	.026	.019	.020	.019	.019	.013	.014	.013	.013
	1	.134	.059	.081	.127	.093	.039	.058	.092	.067	.027	.041	.066
	0	.026	.028	.025	.025	.018	.018	.017	.017	.012	.013	.013	.013
	.2	.049	.053	.049	.051	.034	.035	.034	.035	.025	.025	.025	.025
	.2	.055	.054	.055	.061	.034	.032	.034	.037	.027	.025	.027	.030
1	1	.028	.030	.027	.027	.020	.020	.020	.020	.014	.014	.014	.014
	1	.080	.088	.081	.079	.058	.060	.058	.057	.041	.042	.041	.041
	-.5	.028	.031	.028	.028	.020	.021	.020	.020	.015	.015	.015	.015
	.2	.050	.053	.051	.055	.035	.036	.036	.038	.025	.026	.026	.027
	.2	.054	.054	.055	.060	.036	.035	.037	.040	.028	.026	.028	.030
2	1	.028	.032	.027	.027	.019	.021	.020	.019	.014	.015	.014	.014
	1	.183	.046	.081	.174	.131	.029	.058	.126	.093	.020	.041	.091
	-.5	.028	.034	.028	.028	.020	.022	.020	.020	.015	.015	.015	.015
	.2	.050	.057	.051	.054	.035	.037	.036	.038	.024	.026	.026	.026
	.2	.055	.058	.055	.060	.037	.037	.037	.039	.028	.027	.028	.030
3	1	.027	.030	.027	.027	.019	.021	.020	.019	.014	.014	.014	.014
	1	.136	.059	.081	.126	.094	.039	.058	.092	.066	.026	.041	.066
	-.5	.028	.032	.028	.028	.021	.021	.020	.020	.014	.015	.015	.015
	.2	.049	.054	.051	.055	.037	.037	.036	.038	.026	.026	.026	.026
	.2	.056	.055	.054	.060	.037	.036	.037	.040	.028	.027	.028	.030

Table 5a. Empirical Mean(sd) of CQMLE and M-Estimator, STLE Model, $T = 3, m = 5$

		$n = 50$		$n = 100$		$n = 200$	
err	ψ	CQMLE	M-Est	CQMLE	M-Est	CQMLE	M-Est
1	1	1.0076(.031)	.9995(.031)	1.0092(.024)	.9999(.025)	1.0057(.016)	.9999(.016)
	1	.9287(.133)	.9437(.138)	.9622(.099)	.9746(.102)	.9723(.070)	.9841(.071)
	.3	.2578(.033)	.3000(.035)	.2663(.021)	.2992(.022)	.2685(.014)	.2999(.015)
	.2	.1966(.073)	.1957(.075)	.1877(.059)	.1967(.060)	.2030(.037)	.1987(.038)
	.2	.2209(.084)	.2055(.091)	.2240(.047)	.2037(.049)	.2019(.037)	.2005(.039)
	.2	.1463(.185)	.1459(.191)	.1846(.132)	.1745(.135)	.1785(.092)	.1838(.094)
2	1	1.0084(.031)	1.0003(.032)	1.0091(.025)	.9998(.025)	1.0055(.016)	.9997(.016)
	1	.9288(.264)	.9448(.273)	.9591(.195)	.9717(.201)	.9713(.140)	.9832(.144)
	.3	.2564(.035)	.2986(.036)	.2661(.022)	.2988(.022)	.2685(.015)	.2998(.015)
	.2	.1989(.073)	.1983(.076)	.1895(.060)	.1985(.060)	.2043(.036)	.1999(.037)
	.2	.2168(.082)	.2008(.089)	.2211(.047)	.2008(.050)	.2015(.036)	.2003(.039)
	.2	.1367(.184)	.1358(.190)	.1805(.131)	.1702(.134)	.1799(.090)	.1855(.092)
3	1	1.0066(.031)	.9985(.031)	1.0084(.025)	.9991(.025)	1.0054(.016)	.9995(.016)
	1	.9318(.201)	.9476(.208)	.9644(.148)	.9769(.152)	.9727(.102)	.9846(.105)
	.3	.2590(.034)	.3014(.036)	.2679(.022)	.3008(.022)	.2689(.015)	.3003(.015)
	.2	.1983(.071)	.1978(.073)	.1879(.060)	.1969(.060)	.2046(.037)	.2003(.038)
	.2	.2193(.081)	.2035(.087)	.2207(.047)	.2003(.049)	.2009(.037)	.1998(.040)
	.2	.1412(.184)	.1403(.190)	.1852(.133)	.1750(.135)	.1794(.093)	.1849(.095)
1	1	1.0129(.033)	.9999(.034)	1.0128(.025)	1.0004(.025)	1.0103(.017)	.9991(.017)
	1	.9273(.134)	.9404(.138)	.9635(.098)	.9753(.100)	.9764(.069)	.9881(.071)
	0	-.0407(.035)	.0017(.038)	-.0365(.023)	-.0001(.024)	-.0352(.016)	.0004(.017)
	.2	.1939(.075)	.1971(.077)	.1921(.059)	.2006(.060)	.2094(.040)	.1973(.042)
	.2	.2168(.076)	.2006(.081)	.2116(.049)	.2020(.051)	.1919(.040)	.2008(.043)
	.2	.1508(.187)	.1464(.193)	.1808(.132)	.1722(.136)	.1713(.095)	.1866(.098)
2	1	1.0141(.033)	1.0010(.033)	1.0119(.026)	.9995(.026)	1.0107(.017)	.9996(.017)
	1	.9316(.271)	.9456(.279)	.9591(.203)	.9713(.208)	.9729(.140)	.9847(.143)
	0	-.0413(.037)	.0013(.038)	-.0357(.024)	.0005(.024)	-.0355(.017)	-.0001(.017)
	.2	.1939(.074)	.1972(.076)	.1896(.059)	.1981(.060)	.2122(.040)	.2004(.042)
	.2	.2183(.076)	.2018(.082)	.2101(.048)	.2004(.051)	.1920(.039)	.2007(.043)
	.2	.1495(.185)	.1446(.191)	.1769(.135)	.1683(.138)	.1703(.093)	.1852(.096)
3	1	1.0128(.032)	.9996(.033)	1.0119(.026)	.9994(.026)	1.0106(.017)	.9994(.017)
	1	.9404(.206)	.9540(.212)	.9622(.150)	.9742(.154)	.9752(.104)	.9870(.106)
	0	-.0416(.037)	.0014(.039)	-.0360(.024)	.0003(.025)	-.0346(.017)	.0009(.017)
	.2	.1964(.072)	.1996(.075)	.1907(.059)	.1994(.059)	.2118(.040)	.1999(.042)
	.2	.2155(.074)	.1983(.079)	.2089(.048)	.1991(.050)	.1916(.039)	.2003(.042)
	.2	.1399(.187)	.1347(.195)	.1790(.130)	.1703(.133)	.1711(.092)	.1862(.096)
1	1	1.0145(.034)	.9999(.035)	1.0133(.026)	.9998(.027)	1.0143(.017)	1.0005(.017)
	1	.9356(.138)	.9458(.141)	.9637(.099)	.9737(.101)	.9748(.069)	.9848(.071)
	-.3	-.3382(.036)	-.2990(.039)	-.3376(.025)	-.3011(.026)	-.3361(.017)	-.3004(.018)
	.2	.1935(.075)	.1965(.077)	.1942(.059)	.1995(.060)	.2140(.043)	.1996(.045)
	.2	.2129(.073)	.2003(.076)	.2036(.052)	.2003(.054)	.1880(.040)	.1985(.043)
	.2	.1516(.186)	.1477(.192)	.1735(.133)	.1686(.137)	.1671(.093)	.1854(.096)
2	1	1.0148(.033)	1.0001(.033)	1.0127(.025)	.9994(.025)	1.0150(.017)	1.0010(.017)
	1	.9470(.272)	.9579(.278)	.9547(.192)	.9648(.196)	.9785(.140)	.9887(.143)
	-.3	-.3394(.038)	-.2999(.039)	-.3363(.026)	-.3001(.026)	-.3359(.019)	-.3001(.019)
	.2	.1947(.077)	.1980(.080)	.1931(.059)	.1984(.060)	.2140(.042)	.1992(.044)
	.2	.2119(.076)	.1987(.079)	.2020(.051)	.1990(.053)	.1905(.040)	.2012(.042)
	.2	.1460(.191)	.1415(.197)	.1778(.134)	.1730(.138)	.1668(.094)	.1855(.097)
3	1	1.0125(.033)	.9979(.034)	1.0134(.026)	.9999(.026)	1.0137(.017)	.9999(.017)
	1	.9356(.201)	.9461(.205)	.9683(.150)	.9786(.153)	.9721(.104)	.9822(.106)
	-.3	-.3391(.038)	-.2999(.040)	-.3364(.026)	-.2997(.028)	-.3362(.017)	-.3006(.018)
	.2	.1939(.077)	.1970(.079)	.1935(.058)	.1990(.059)	.2140(.042)	.1996(.044)
	.2	.2149(.077)	.2015(.080)	.2025(.052)	.1993(.053)	.1905(.039)	.2008(.042)
	.2	.1429(.188)	.1389(.194)	.1760(.132)	.1709(.136)	.1648(.094)	.1831(.097)

Note: Par $= \psi = (\beta, \sigma_v^2, \rho, \lambda_1, \lambda_2, \lambda_3)'$; err = 1 (normal), 2 (normal mixture), and 3 (chi-square). X_t values are generated with $\theta_x = (g, \phi_1, \phi_2, \sigma_1, \sigma_2) = (.01, .5, .5, 3, 1)$, as in Footnote 13. W_1, W_2 and W_3 are all from Queen Contiguity, and equal.

Table 5a. Cont'd, $T = 7$

		$n = 50$		$n = 100$		$n = 200$	
err	ψ	CQMLE	M-Est	CQMLE	M-Est	CQMLE	M-Est
1	1	1.0048(.017)	1.0002(.017)	1.0054(.012)	1.0005(.012)	1.0053(.008)	.9999(.008)
	1	.9791(.079)	.9809(.080)	.9898(.058)	.9916(.058)	.9944(.042)	.9963(.042)
	.3	.2898(.013)	.2993(.013)	.2901(.010)	.2999(.010)	.2899(.007)	.3001(.007)
	.2	.1988(.033)	.2000(.033)	.1995(.029)	.2001(.029)	.1991(.019)	.1998(.019)
	.2	.2009(.035)	.1996(.036)	.2025(.026)	.1998(.026)	.2001(.019)	.2000(.019)
	.2	.1831(.099)	.1823(.100)	.1909(.073)	.1906(.074)	.1974(.051)	.1972(.052)
2	1	1.0045(.017)	.9999(.017)	1.0046(.012)	.9998(.012)	1.0053(.008)	.9999(.008)
	1	.9769(.187)	.9787(.187)	.9883(.132)	.9902(.132)	.9933(.092)	.9953(.092)
	.3	.2904(.014)	.2999(.014)	.2900(.010)	.2998(.010)	.2899(.007)	.3000(.007)
	.2	.1979(.034)	.1991(.034)	.1991(.029)	.1996(.029)	.1997(.020)	.2004(.020)
	.2	.2022(.036)	.2007(.037)	.2027(.026)	.2000(.026)	.1989(.019)	.1988(.019)
	.2	.1792(.096)	.1784(.097)	.1896(.070)	.1892(.071)	.1954(.051)	.1952(.051)
3	1	1.0052(.017)	1.0007(.017)	1.0053(.012)	1.0004(.012)	1.0054(.008)	1.0001(.008)
	1	.9786(.136)	.9804(.136)	.9886(.095)	.9905(.095)	.9899(.066)	.9918(.066)
	.3	.2897(.013)	.2992(.013)	.2901(.009)	.2998(.009)	.2896(.007)	.2998(.007)
	.2	.1989(.033)	.2000(.033)	.1996(.029)	.2002(.029)	.1992(.019)	.1999(.020)
	.2	.2023(.034)	.2010(.035)	.2028(.025)	.2001(.026)	.2003(.019)	.2002(.019)
	.2	.1851(.100)	.1843(.101)	.1902(.072)	.1899(.073)	.1951(.051)	.1950(.051)
1	1	1.0068(.018)	1.0004(.018)	1.0063(.013)	.9999(.013)	1.0069(.009)	1.0000(.009)
	1	.9835(.084)	.9851(.085)	.9896(.057)	.9913(.057)	.9949(.042)	.9967(.042)
	0	-.0121(.017)	-.0005(.017)	-.0118(.012)	-.0001(.012)	-.0124(.008)	.0000(.009)
	.2	.1992(.037)	.1991(.037)	.2021(.031)	.2008(.031)	.1997(.021)	.1987(.021)
	.2	.2027(.036)	.2011(.036)	.2014(.027)	.1996(.027)	.2002(.020)	.2010(.020)
	.2	.1817(.100)	.1820(.101)	.1879(.074)	.1893(.075)	.1937(.053)	.1951(.054)
2	1	1.0073(.018)	1.0009(.018)	1.0064(.013)	.9999(.013)	1.0070(.009)	1.0001(.009)
	1	.9802(.183)	.9818(.184)	.9948(.130)	.9964(.130)	.9919(.095)	.9936(.095)
	0	-.0126(.016)	-.0011(.016)	-.0116(.012)	.0002(.012)	-.0125(.008)	-.0002(.009)
	.2	.1995(.037)	.1994(.037)	.2007(.031)	.1995(.031)	.2008(.021)	.1997(.022)
	.2	.2020(.037)	.2005(.038)	.2019(.026)	.2001(.026)	.1990(.020)	.1997(.020)
	.2	.1808(.099)	.1811(.100)	.1884(.072)	.1898(.073)	.1935(.052)	.1949(.052)
3	1	1.0063(.018)	.9999(.018)	1.0069(.013)	1.0004(.013)	1.0071(.009)	1.0002(.009)
	1	.9825(.133)	.9842(.133)	.9927(.094)	.9943(.094)	.9936(.067)	.9954(.068)
	0	-.0119(.016)	-.0003(.016)	-.0119(.012)	-.0001(.012)	-.0121(.009)	.0002(.009)
	.2	.2015(.037)	.2013(.037)	.2017(.030)	.2005(.031)	.2001(.021)	.1991(.021)
	.2	.2003(.036)	.1988(.037)	.2010(.027)	.1992(.027)	.1996(.020)	.2004(.020)
	.2	.1798(.098)	.1800(.099)	.1881(.074)	.1895(.075)	.1944(.052)	.1958(.053)
1	1	1.0076(.019)	1.0001(.019)	1.0072(.014)	1.0000(.014)	1.0073(.009)	.9999(.009)
	1	.9815(.081)	.9830(.081)	.9896(.058)	.9911(.058)	.9938(.040)	.9954(.040)
	-.3	-.3134(.019)	-.3002(.020)	-.3133(.014)	-.3000(.014)	-.3140(.010)	-.3002(.010)
	.2	.2020(.039)	.2009(.040)	.2019(.033)	.1996(.033)	.2011(.022)	.1993(.023)
	.2	.1991(.038)	.1982(.038)	.2015(.027)	.2005(.028)	.1991(.021)	.2000(.021)
	.2	.1784(.101)	.1797(.102)	.1882(.075)	.1908(.076)	.1944(.053)	.1965(.054)
2	1	1.0079(.019)	1.0005(.019)	1.0075(.013)	1.0003(.014)	1.0076(.009)	1.0001(.009)
	1	.9868(.181)	.9883(.182)	.9910(.129)	.9925(.129)	.9940(.092)	.9956(.093)
	-.3	-.3138(.019)	-.3006(.019)	-.3136(.014)	-.3002(.014)	-.3141(.010)	-.3002(.010)
	.2	.2017(.040)	.2005(.041)	.2011(.033)	.1988(.033)	.2020(.022)	.2003(.022)
	.2	.2009(.038)	.1999(.039)	.2015(.027)	.2006(.028)	.1998(.021)	.2007(.021)
	.2	.1780(.101)	.1793(.102)	.1887(.074)	.1912(.075)	.1925(.052)	.1946(.052)
3	1	1.0083(.020)	1.0008(.020)	1.0071(.013)	.9999(.013)	1.0072(.009)	.9997(.009)
	1	.9855(.133)	.9869(.133)	.9886(.094)	.9901(.094)	.9910(.066)	.9925(.066)
	-.3	-.3141(.020)	-.3009(.020)	-.3136(.014)	-.3002(.014)	-.3136(.010)	-.2997(.010)
	.2	.2017(.038)	.2006(.039)	.2032(.032)	.2009(.033)	.2010(.022)	.1992(.022)
	.2	.2001(.037)	.1992(.038)	.2003(.029)	.1993(.029)	.1993(.021)	.2003(.021)
	.2	.1781(.098)	.1794(.100)	.1873(.073)	.1898(.074)	.1949(.052)	.1970(.052)

Table 5b. Empirical sd and average of standard errors of M-Estimator
STLE Model, $T = 3, m = 5$, Parameter configurations as in Table 5a.

		$n = 50$				$n = 100$				$n = 200$			
err	ψ	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$
1	1	.031	.035	.031	.030	.025	.026	.025	.025	.016	.017	.016	.016
	1	.138	.156	.136	.132	.102	.106	.099	.098	.071	.073	.071	.070
	.3	.035	.038	.035	.035	.022	.023	.021	.022	.015	.015	.015	.015
	.2	.075	.080	.071	.071	.060	.062	.059	.060	.038	.039	.038	.038
	.2	.091	.079	.085	.108	.049	.043	.049	.060	.039	.033	.039	.050
	.2	.191	.209	.184	.186	.135	.141	.132	.132	.094	.096	.093	.094
2	1	.032	.039	.031	.030	.025	.028	.025	.024	.016	.017	.016	.016
	1	.273	.109	.137	.239	.201	.065	.099	.187	.144	.042	.071	.139
	.3	.036	.042	.034	.035	.022	.024	.021	.022	.015	.016	.015	.015
	.2	.076	.089	.070	.069	.060	.067	.059	.059	.037	.041	.038	.038
	.2	.089	.088	.083	.104	.050	.047	.049	.059	.039	.034	.039	.049
	.2	.190	.243	.184	.177	.134	.154	.132	.130	.092	.102	.093	.092
3	1	.031	.037	.031	.030	.025	.027	.025	.025	.016	.017	.016	.016
	1	.208	.124	.137	.187	.152	.079	.099	.142	.105	.052	.071	.104
	.3	.036	.040	.035	.035	.022	.023	.021	.022	.015	.015	.015	.015
	.2	.073	.084	.070	.069	.060	.065	.059	.059	.038	.040	.038	.038
	.2	.087	.084	.084	.106	.049	.045	.049	.059	.040	.033	.039	.050
	.2	.190	.224	.183	.181	.135	.147	.131	.130	.095	.098	.093	.094
1	1	.034	.036	.032	.032	.025	.027	.025	.025	.017	.017	.017	.017
	1	.138	.155	.136	.132	.100	.106	.099	.098	.071	.074	.071	.070
	0	.038	.041	.037	.037	.024	.025	.024	.024	.017	.017	.017	.017
	.2	.077	.082	.073	.072	.060	.062	.059	.060	.042	.041	.041	.044
	.2	.081	.073	.079	.096	.051	.042	.050	.064	.043	.032	.042	.057
	.2	.193	.208	.183	.184	.136	.141	.133	.135	.098	.097	.096	.100
2	1	.033	.040	.032	.031	.026	.029	.025	.025	.017	.018	.017	.017
	1	.279	.108	.137	.242	.208	.066	.099	.185	.143	.042	.071	.137
	0	.038	.046	.037	.037	.024	.027	.024	.024	.017	.018	.017	.017
	.2	.076	.093	.072	.071	.060	.066	.058	.060	.042	.043	.041	.044
	.2	.082	.082	.079	.096	.051	.045	.050	.063	.043	.034	.042	.057
	.2	.191	.246	.184	.174	.138	.156	.133	.131	.096	.103	.096	.098
3	1	.033	.038	.032	.032	.026	.028	.025	.025	.017	.018	.017	.017
	1	.212	.125	.138	.187	.154	.079	.099	.141	.106	.052	.071	.104
	0	.039	.043	.037	.037	.025	.026	.024	.024	.017	.017	.017	.017
	.2	.075	.087	.072	.072	.059	.064	.059	.059	.042	.042	.041	.044
	.2	.079	.077	.079	.096	.050	.043	.050	.063	.042	.033	.042	.057
	.2	.195	.226	.185	.182	.133	.147	.132	.133	.096	.100	.096	.099
1	1	.035	.038	.033	.033	.027	.027	.026	.026	.017	.018	.017	.017
	1	.141	.156	.136	.132	.101	.106	.099	.098	.071	.073	.071	.070
	-.3	.039	.043	.038	.038	.026	.028	.026	.026	.018	.019	.018	.018
	.2	.077	.085	.075	.076	.060	.061	.059	.061	.045	.043	.044	.048
	.2	.076	.072	.075	.089	.054	.048	.053	.063	.043	.034	.042	.055
	.2	.192	.211	.185	.185	.137	.141	.133	.136	.096	.098	.097	.102
2	1	.033	.042	.033	.033	.025	.030	.026	.025	.017	.019	.017	.017
	1	.278	.109	.138	.246	.196	.064	.098	.185	.143	.042	.071	.139
	-.3	.039	.049	.038	.038	.026	.029	.026	.026	.019	.019	.018	.019
	.2	.080	.096	.075	.075	.060	.066	.059	.061	.044	.045	.044	.048
	.2	.079	.081	.074	.087	.053	.052	.053	.062	.042	.035	.042	.055
	.2	.197	.246	.185	.178	.138	.156	.133	.132	.097	.105	.098	.101
3	1	.034	.039	.033	.033	.026	.028	.026	.026	.017	.018	.017	.017
	1	.205	.123	.136	.185	.153	.080	.099	.141	.106	.052	.070	.104
	-.3	.040	.045	.038	.038	.028	.028	.026	.026	.018	.019	.018	.018
	.2	.079	.089	.075	.075	.059	.064	.059	.061	.044	.043	.044	.048
	.2	.080	.076	.074	.088	.053	.050	.053	.063	.042	.034	.042	.054
	.2	.194	.225	.186	.183	.136	.148	.133	.134	.097	.101	.098	.102

Table 5b. Cont'd, $T = 7$

		$n = 50$				$n = 100$				$n = 200$			
err	ψ	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$	sd	$\widetilde{se}$	$\widehat{se}$	$\widehat{rse}$
1	1	.017	.019	.017	.016	.012	.013	.012	.012	.008	.009	.008	.008
	1	.080	.089	.081	.079	.058	.061	.058	.057	.042	.042	.041	.041
	.3	.013	.015	.013	.013	.010	.010	.009	.009	.007	.007	.007	.007
	.2	.033	.036	.033	.034	.029	.030	.029	.030	.019	.020	.020	.020
	.2	.036	.034	.035	.040	.026	.024	.026	.029	.019	.018	.019	.021
	.2	.100	.109	.098	.097	.074	.076	.073	.072	.052	.053	.051	.052
2	1	.017	.020	.017	.016	.012	.013	.012	.012	.008	.009	.008	.008
	1	.187	.047	.080	.170	.132	.030	.058	.126	.092	.020	.041	.091
	.3	.014	.015	.013	.013	.010	.010	.009	.009	.007	.007	.007	.007
	.2	.034	.038	.033	.033	.029	.031	.029	.029	.020	.020	.020	.020
	.2	.037	.036	.035	.040	.026	.025	.026	.029	.019	.018	.019	.021
	.2	.097	.120	.098	.094	.071	.081	.073	.071	.051	.055	.052	.051
3	1	.017	.019	.017	.016	.012	.013	.012	.012	.008	.009	.008	.008
	1	.136	.061	.081	.125	.095	.040	.058	.091	.066	.027	.041	.066
	.3	.013	.015	.013	.013	.009	.010	.009	.009	.007	.007	.007	.007
	.2	.033	.037	.033	.033	.029	.031	.029	.029	.020	.020	.020	.020
	.2	.035	.035	.035	.040	.026	.025	.026	.029	.019	.018	.019	.021
	.2	.101	.113	.098	.096	.073	.079	.073	.072	.051	.054	.051	.051
1	1	.018	.020	.018	.018	.013	.013	.013	.013	.009	.009	.009	.009
	1	.085	.089	.081	.079	.057	.061	.058	.057	.042	.042	.041	.041
	0	.017	.019	.017	.016	.012	.012	.012	.012	.009	.009	.009	.009
	.2	.037	.039	.036	.038	.031	.031	.031	.032	.021	.021	.021	.022
	.2	.036	.035	.037	.043	.027	.024	.027	.031	.020	.019	.020	.023
	.2	.101	.110	.100	.099	.075	.078	.074	.074	.054	.053	.052	.053
2	1	.018	.021	.018	.017	.013	.014	.013	.013	.009	.009	.009	.009
	1	.184	.048	.081	.170	.130	.030	.058	.127	.095	.020	.041	.091
	0	.016	.020	.016	.016	.012	.013	.012	.012	.009	.009	.009	.009
	.2	.037	.042	.036	.037	.031	.033	.031	.032	.022	.022	.021	.022
	.2	.038	.037	.037	.042	.026	.025	.027	.031	.020	.019	.020	.022
	.2	.100	.122	.100	.096	.073	.082	.074	.073	.052	.055	.052	.052
3	1	.018	.020	.018	.018	.013	.014	.013	.013	.009	.009	.009	.009
	1	.133	.060	.081	.127	.094	.040	.058	.092	.068	.027	.041	.066
	0	.016	.019	.017	.016	.012	.013	.012	.012	.009	.009	.009	.009
	.2	.037	.040	.036	.037	.031	.032	.031	.032	.021	.021	.021	.022
	.2	.037	.036	.037	.043	.027	.025	.027	.031	.020	.019	.020	.023
	.2	.099	.116	.100	.097	.075	.079	.074	.074	.053	.054	.052	.052
1	1	.019	.021	.019	.019	.014	.014	.013	.013	.009	.009	.009	.009
	1	.081	.089	.081	.079	.058	.061	.058	.057	.040	.042	.041	.041
	-.3	.020	.022	.020	.019	.014	.015	.014	.014	.010	.010	.010	.010
	.2	.040	.041	.039	.042	.033	.033	.033	.035	.023	.022	.022	.024
	.2	.038	.035	.038	.046	.028	.026	.028	.032	.021	.019	.021	.023
	.2	.102	.112	.101	.101	.076	.078	.075	.076	.054	.054	.053	.053
2	1	.019	.023	.019	.018	.014	.015	.013	.013	.009	.010	.009	.009
	1	.182	.047	.081	.174	.129	.030	.058	.126	.093	.020	.041	.091
	-.3	.019	.024	.020	.019	.014	.016	.014	.014	.010	.011	.010	.010
	.2	.041	.045	.040	.042	.033	.034	.033	.035	.022	.023	.023	.024
	.2	.039	.039	.038	.045	.028	.027	.028	.032	.021	.020	.021	.023
	.2	.102	.124	.102	.099	.075	.083	.075	.074	.052	.056	.053	.053
3	1	.020	.022	.019	.019	.013	.014	.013	.013	.009	.010	.009	.009
	1	.133	.061	.081	.126	.094	.040	.057	.091	.066	.027	.041	.065
	-.3	.020	.023	.020	.019	.014	.015	.014	.014	.010	.010	.010	.010
	.2	.039	.043	.040	.042	.033	.033	.033	.035	.022	.022	.022	.023
	.2	.038	.037	.038	.045	.029	.026	.028	.032	.021	.019	.021	.023
	.2	.100	.117	.101	.100	.074	.080	.075	.075	.052	.055	.053	.053

Table 6a. CQMLE, FQMLE, , M-Est and its t-Ratio based on Munnell Data: SE Model

	Full Data				Last 6 Years				First 6 Years			
	CQM	FQM	M-Est	t-ratio	CQM	FQM	M-Est	t-ratio	CQM	FQM	M-Est	t-ratio
β_1	-.0433	-.0234	-.0467	-1.877	-.1008	-.1124	-.0852	-2.440	-.0851	-.0922	-.0810	-1.136
β_2	-.0393	-.0309	-.0702	-2.796	-.0305	-.0336	-.0501	-1.373	.0644	.0106	-.0714	-.639
β_3	.2644	.2008	.1654	3.329	.7840	.6504	.5971	5.526	.4192	.3532	.3161	2.353
β_4	-.0024	-.0026	-.0028	-5.306	-.002	-.0018	-.0021	-3.590	-.0028	-.0031	-.0031	-4.389
σ_v^2	.0001	.0001	.0001	5.931	.0000	.0000	.0000	5.366	.0000	.0000	.0000	3.998
ρ	.7772	.8283	.9140	17.222	.4409	.5728	.6265	7.162	.4594	.5942	.6521	4.018
λ_3	.7592	.7550	.7697	20.665	.7133	.7460	.7638	14.021	.7114	.7120	.7155	13.842

Table 6b. CQMLE, M-Est and its t-Ratio based on Munnell Data: Other Models

	Full Data			Last 6 Years			First 6 Years		
	CQMLE	M-Est	t-ratio	CQMLE	M-Est	t-ratio	CQMLE	M-Est	t-ratio
`SL Model`									
β_1	-0.0620	-0.0598	-1.8194	-0.1850	-0.1692	-2.5069	-0.0165	-0.0079	-0.1005
β_2	0.0296	0.0105	0.3514	-0.0365	-0.0540	-1.1542	-0.1081	-0.2194	-2.7020
β_3	0.3045	0.2480	3.1542	0.9917	0.9012	10.4729	0.3916	0.2369	1.2416
β_4	-0.0025	-0.0027	-4.0988	-0.0016	-0.0019	-2.5384	-0.0018	-0.0018	-2.5330
σ_v^2	0.0001	0.0001	9.5094	0.0001	0.0001	8.6974	0.0001	0.0001	3.5254
ρ	0.5333	0.6132	7.0194	0.1625	0.2448	4.4754	0.2849	0.4801	2.8386
λ_1	0.2131	0.2046	4.3797	0.2077	0.1991	4.4475	0.3767	0.4134	4.0345
`SLE Model`									
β_1	-0.0412	-0.0454	-1.6237	-0.0888	-0.0755	-1.8749	-0.1023	-0.0829	-1.1113
β_2	-0.0364	-0.0675	-1.3981	-0.0197	-0.0373	-0.8777	0.4341	0.0429	0.1011
β_3	0.2649	0.1685	1.4418	0.7585	0.5904	5.3430	0.4201	0.3343	2.2261
β_4	-0.0024	-0.0027	-3.9247	-0.0021	-0.0023	-3.5416	-0.0025	-0.0031	-3.8491
σ_v^2	0.0001	0.0001	5.1623	0.0000	0.0000	4.8590	0.0000	0.0000	2.9107
ρ	0.7752	0.9092	5.9496	0.4515	0.6189	8.0173	0.3754	0.6123	2.9549
λ_1	-0.0235	-0.0123	-0.3143	-0.0804	-0.0789	-0.8565	-0.3615	-0.1289	-0.4139
λ_3	0.7753	0.7757	17.6446	0.7800	0.8015	10.7070	0.8878	0.7789	4.2353
`STL Model`									
β_1	-0.0383	-0.0343	-1.2882	-0.1367	-0.1072	-3.0105	-0.0791	-0.0727	-0.8560
β_2	0.0215	0.0040	0.1641	-0.0158	-0.0262	-0.6303	0.1456	0.0937	0.8758
β_3	0.2414	0.1844	2.9434	0.7215	0.5669	5.5058	0.4769	0.4040	4.3346
β_4	-0.0011	-0.0012	-3.4687	-0.0014	-0.0017	-2.8457	-0.0017	-0.0018	-3.1086
σ_v^2	0.0001	0.0001	6.1872	0.0000	0.0000	5.0666	0.0000	0.0000	4.9172
ρ	0.7547	0.8474	12.1490	0.4757	0.6365	7.2715	0.4258	0.5700	4.6003
λ_1	0.6662	0.681	15.2637	0.4890	0.5409	7.9038	0.5533	0.5565	10.9247
λ_2	-0.6350	-0.6747	-11.3723	-0.466	-0.5797	-6.4991	-0.5343	-0.5775	-4.5748
`STLE Model`									
β_1	-0.0399	-0.0432	-1.7639	-0.1255	-0.1071	-2.8461	-0.0657	-0.0322	-0.2979
β_2	-0.0370	-0.0617	-1.3938	-0.0180	-0.0264	-0.5836	0.1254	0.0584	0.5115
β_3	0.2146	0.1353	1.2129	0.7684	0.5690	3.7925	0.4517	0.3512	2.5418
β_4	-0.0023	-0.0026	-3.5825	-0.0017	-0.0017	-2.3548	-0.0015	-0.0012	-1.1755
σ_v^2	0.0000	0.0001	4.5221	0.0000	0.0000	5.0517	0.0000	0.0000	4.2264
ρ	0.7973	0.9164	6.2388	0.4484	0.6349	5.3390	0.4367	0.6001	3.8399
λ_1	-0.5538	-0.5566	-5.3667	0.4137	0.5381	3.6888	0.5976	0.6711	3.9109
λ_2	0.4985	0.5331	4.8853	-0.4138	-0.5770	-3.6064	-0.5514	-0.6536	-3.4999
λ_3	0.9074	0.9059	31.9162	0.2058	0.0078	0.0237	-0.1215	-0.3409	-0.6752