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Testing for serial correlation, spatial autocorrelation and random effects using panel data

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Abstract

This paper considers a spatial panel data regression model with serial correlation on each spatial unit over time as well as spatial dependence between the spatial units at each point in time. In addition, the model allows for heterogeneity across the spatial units using random effects. The paper then derives several Lagrange multiplier tests for this panel data regression model including a joint test for serial correlation, spatial autocorrelation and random effects. These tests draw upon two strands of earlier work. The first is the LM tests for the spatial error correlation model discussed in Anselin and Bera [1998. Spatial dependence in linear regression models with an introduction to spatial econometrics. In: Ullah, A., Giles, D.E.A. (Eds.), Handbook of Applied Economic Statistics. Marcel Dekker, New York] and in the panel data context by Baltagi et al. [2003. Testing panel data regression models with spatial error correlation. Journal of Econometrics 117, 123–150]. The second is the LM tests for the error component panel data model with serial correlation derived by Baltagi and Li [1995. Testing AR(1) against MA(1) disturbances in an error component model. Journal of Econometrics 68, 133–151]. Hence, the joint LM test derived in this paper encompasses those derived in both strands of earlier works. In fact, in the context of our general model, the earlier LM tests become marginal LM tests that ignore either serial correlation over time or spatial error correlation. The paper then derives conditional LM and LR tests that do not ignore these correlations and contrast them with their marginal LM and LR counterparts. The small sample performance of these

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tests is investigated using Monte Carlo experiments. As expected, ignoring any correlation when it is significant can lead to misleading inference.

JEL classification: C23; C12

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1. Introduction

Spatial models deal with correlation across spatial units usually in a cross-section setting, see Anselin (1988). Panel data models allow the researcher to control for heterogeneity across these units, see Baltagi (2005). Spatial panel models can control for both heterogeneity and spatial correlation, see Baltagi et al. (2003). Recent spatial panel data applications in economics include household level survey data from villages observed over time to study nutrition, see Case (1991); per-capita expenditures on police to study their effect on reducing crime across counties, see Kelejian and Robinson (1992); the productivity of public capital like roads and highways in the private sector across U.S. states, see Holtz-Eakin (1994); hedonic housing equations using residential sales, see Bell and Bockstael (2000); unemployment clustering with respect to different social and economic metrics, see Conley and Topa (2002); and spatial price competition in the wholesale gasoline markets, see Pinkse et al. (2002). This paper adds another dimension to the correlation in the error structure. Namely, serial correlation in the remainder error term. The spatial error component model assumes that the only correlation over time is due to the presence of the same region effect across the panel. This may be a restrictive assumption in the analysis of panel data, such as investment across regions, where an unobserved shock in this period will affect the behavioral relationship for at least the next few periods. Ignoring the serial correlation in the error results in consistent, but inefficient estimates of the regression coefficients and biased standards errors, see Baltagi (2005). This paper considers a spatial panel data regression model with serial correlation on each spatial unit over time as well as spatial dependence between the spatial units at each point in time.

For the panel data model with no spatial effects, Baltagi and Li (1995) addressed the problem of jointly testing for serial correlation and individual effects. Testing for spatial dependence has been extensively studied by Anselin (1988, 2001) and Anselin and Bera (1998), to mention a few. Baltagi et al. (2003) considered the problem of jointly testing for random region effects in the panel as well as spatial correlation across these regions. However, the last study did not consider the added problem of serial correlation in the remainder error term. This paper generalizes the previous studies by deriving test statistics for the spatial panel data model with serial correlation. In particular, this paper derives joint and conditional LM and LR tests and studies their small sample properties using Monte Carlo experiments. One directional tests that test for spatial error correlation, for e.g., ignoring the presence of serial correlation over time and random effects among the spatial units could yield misleading inference when one or both of the left out components are significant. Conditional LM tests are proposed and their performance is contrasted with the corresponding marginal counterparts. Our Monte Carlo results show that these conditional tests guard against possible misspecification.

2. The model

Consider the following panel data regression model:

$$y_{ti} = X'_{ti}\beta + u_{ti}, \quad i = 1, \ldots, N, \;\; t = 1, \ldots, T, \tag{2.1}$$

where y_{ti} is the observation on the ith region for the tth time period, X_{ti} denotes the $k \times 1$ vector of observations on the nonstochastic regressors and u_{ti} is the regression disturbance. In vector form, the disturbance vector of (2.1) is assumed to have random region effects, spatially autocorrelated residual disturbances and a first-order autoregressive remainder disturbance term:

$$u_t = \mu + \varepsilon_t, \tag{2.2}$$

with

$$\varepsilon_t = \lambda W \varepsilon_t + v_t \quad \text{and} \quad v_t = \rho v_{t-1} + e_t, \tag{2.3}$$

where $u'_t = (u_{t1}, \ldots, u_{tN})$ and ε_t, v_t and e_t are similarly defined. $\mu' = (\mu_1, \mu_2, \ldots, \mu_N)$ denote the vector of random region effects which are assumed to be $II\mathrm{N}(0, \sigma^2_\mu)$. λ is the scalar spatial autoregressive coefficient with $|\lambda| < 1$, while ρ is the time-wise serial correlation coefficient satisfying $|\rho| < 1$. W is a known $N \times N$ spatial weight matrix whose diagonal elements are zero. W also satisfies the condition that $I_N - \lambda W$ is nonsingular, where I_N is an identity matrix of dimension N. $e_{ti} \sim II\mathrm{N}(0, \sigma^2_e)$ and $v_{i,0} \sim \mathrm{N}((0, \sigma^2_e)/(1 - \rho^2))$. We assume that μ and ε are independent. One can rewrite (2.3) as

$$\varepsilon_t = (I_N - \lambda W)^{-1} v_t = B^{-1} v_t, \tag{2.4}$$

where $B = I_N - \lambda W$. The model (2.1) can be rewritten in matrix notation as

$$y = X\beta + u, \tag{2.5}$$

where y is of dimension $NT \times 1$, X is $NT \times k$, β is $k \times 1$ and u is a $NT \times 1$. X is assumed to be of full column rank and its elements are assumed to be bounded in absolute value. The disturbance term can be written in vector form as

$$u = (\iota_T \otimes I_N)\mu + (I_T \otimes B^{-1})v, \tag{2.6}$$

where $v' = (v'_1, v'_2, \ldots, v'_T)$ and u is similarly defined. ι_T is a vector of ones of dimension T, I_T is an identity matrix of dimension T and $\otimes$ denotes the Kronecker product. Under these assumptions, the variance–covariance matrix of u can be written as

$$\Omega = \sigma^2_\mu (J_T \otimes I_N) + (V \otimes (B'B)^{-1}), \tag{2.7}$$

where J_T is a matrix of ones of dimension T, and $\mathrm{E}(vv') = V \otimes I_N$, where V is the familiar AR(1) variance–covariance matrix of dimension T,

$$V = \sigma^2_e \left(\frac{1}{1 - \rho^2} \right) V_1 = \sigma^2_e V_\rho, \tag{2.8}$$

where

$$V_1 = \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{T-1} \\ \rho & 1 & \rho & \cdots & \rho^{T-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \cdots & 1 \end{bmatrix} \quad \text{and} \quad V_\rho = \left(\frac{1}{1-\rho^2}\right) V_1.$$

It is well established that the Prais–Winsten transformation

$$C = \begin{bmatrix} (1-\rho^2)^{1/2} & 0 & 0 & \cdots & 0 & 0 & 0 \\ -\rho & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -\rho & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -\rho & 1 \end{bmatrix} \tag{2.9}$$

transforms the usual AR(1) model into serially uncorrelated classical disturbances with $CVC' = \sigma_e^2 I_T$. For panel data, this C transformation has to be applied repeatedly for N individuals. From (2.5), the transformed spatial panel data regression disturbances are given by

$$\begin{aligned} u^* &= (C \otimes I_N)u = (C\iota_T \otimes I_N)\mu + (C \otimes B^{-1})v \\ &= (1-\rho)(\iota_T^\alpha \otimes I_N)\mu + (C \otimes B^{-1})v, \end{aligned} \tag{2.10}$$

where $C\iota_T = (1-\rho)\iota_T^\alpha$ with $\iota_T^\alpha = (\alpha, \iota'_{T-1})$ and $\alpha = \sqrt{(1+\rho)/(1-\rho)}$.

Therefore, the variance–covariance matrix of the Prais–Winsten-transformed spatial panel data model is given by

$$\Omega^* = \mathrm{E}(u^* u^{*\prime}) = (1-\rho)^2 \sigma_\mu^2 (\iota_T^\alpha \iota_T^{\alpha\prime} \otimes I_N) + \sigma_e^2 (I_T \otimes (B'B)^{-1}) \tag{2.11}$$

since $(C \otimes B^{-1})\mathrm{E}(vv')(C \otimes B^{-1})' = \sigma_e^2 (I_T \otimes (B'B)^{-1})$. Replace $\iota_T^\alpha \iota_T^{\alpha\prime}$ by its idempotent counterpart $d^2 \bar{J}_T^\alpha$, where $\bar{J}_T^\alpha = \iota_T^\alpha \iota_T^{\alpha\prime}/d^2$ and $d^2 = \iota_T^{\alpha\prime} \iota_T^\alpha = \alpha^2 + (T-1)$. Replace I_T by $\mathrm{E}_T^\alpha + \bar{J}_T^\alpha$, where $\mathrm{E}_T^\alpha = I_T - \bar{J}_T^\alpha$ and collect like terms, see Baltagi and Li (1995), we get

$$\Omega^* = \bar{J}_T^\alpha \otimes [d^2(1-\rho)^2 \sigma_\mu^2 I_N + \sigma_e^2 (B'B)^{-1}] + \mathrm{E}_T^\alpha \otimes [\sigma_e^2 (B'B)^{-1}]. \tag{2.12}$$

One can easily verify that

$$\Omega^{*-1} = \bar{J}_T^\alpha \otimes Z + \mathrm{E}_T^\alpha \otimes [(\sigma_e^2)^{-1}(B'B)], \tag{2.13}$$

where $Z = [d^2(1-\rho)^2 \sigma_\mu^2 I_N + \sigma_e^2 (B'B)^{-1}]^{-1}$.

Note that $|\Omega^*| = |d^2(1-\rho)^2 \sigma_\mu^2 I_N + \sigma_e^2 (B'B)^{-1}| |\sigma_e^2 (B'B)^{-1}|^{(T-1)}$, see Magnus (1982). Also, Ω in (2.7) is related to Ω^* in (2.11) by $\Omega^* = (C \otimes I_N)\Omega(C' \otimes I_N)$ with $|C| = \sqrt{1-\rho^2}$ and $|I_N \otimes C| = |C|^N$. Under the assumption of normality, the log-likelihood function for this model can be written as

$$\begin{aligned} L(\beta, \sigma_e^2, \rho, \lambda) = {} & Const. + \frac{1}{2} N \ln(1-\rho^2) - \frac{1}{2} \ln |d^2(1-\rho)^2 \sigma_\mu^2 I_N + \sigma_e^2 (B'B)^{-1}| \\ & - \frac{N(T-1)}{2} \ln(\sigma_e^2) + (T-1) \ln |B| - \frac{1}{2} u^{*\prime} \Omega^{*-1} u^*, \end{aligned} \tag{2.14}$$

where u^* is given by (2.10) and Ω^{*-1} is given by (2.13).

3. Test statistics

The hypotheses under the consideration in this model are the following:

(J) H_0^a: $\lambda = \rho = \sigma_\mu^2 = 0$, this is the joint hypothesis that there is no spatial or serial error correlation and no random region effects. The alternative H_1^a is that at least one component is not zero, so that there may be serial or spatial error correlation or random region effects.

(M.1) H_0^b: $\lambda = 0$ (assuming $\rho = \sigma_\mu^2 = 0$), and the alternative is H_1^b: $\lambda \neq 0$ (assuming $\rho = \sigma_\mu^2 = 0$). This is a one-dimensional marginal test for no spatial error correlation ignoring the presence of serial correlation and random region effects.

(M.2) H_0^c: $\rho = 0$ (assuming $\lambda = \sigma_\mu^2 = 0$), and the alternative is H_1^c: $\rho \neq 0$ (assuming $\lambda = \sigma_\mu^2 = 0$). This is a one-dimensional marginal test for no serial correlation ignoring the presence of spatial error correlation or random region effects.

(M.3) H_0^d: $\sigma_\mu^2 = 0$ (assuming $\rho = \lambda = 0$), and the alternative is H_1^d: $\sigma_\mu^2 > 0$ (assuming $\rho = \lambda = 0$). This is a one-dimensional marginal test for no random region effects ignoring the presence of serial or spatial error correlation.

(M.4) H_0^e: $\lambda = \rho = 0$ (assuming $\sigma_\mu^2 = 0$), and the alternative H_1^e is that at least one component of λ or ρ is not zero (assuming $\sigma_\mu^2 = 0$). This is a two-dimensional marginal test for no spatial or serial error correlation ignoring the presence of random region effects.

(M.5) H_0^f: $\lambda = \sigma_\mu^2 = 0$ (assuming $\rho = 0$), and the alternative H_1^f is that at least one component of λ or σ_μ^2 is not zero (assuming $\rho = 0$). This is a two-dimensional marginal test for no spatial error correlation or random region effects ignoring the presence of serial correlation.

(M.6) H_0^g: $\sigma_\mu^2 = \rho = 0$ (assuming $\lambda = 0$), and the alternative H_1^g is that at least one component of σ_μ^2 or ρ is not zero (assuming $\lambda = 0$). This is a two-dimensional marginal test for no serial correlation or random region effects ignoring the presence of spatial error correlation.

(C.1) H_0^h: $\lambda = 0$ (allowing $\rho \neq 0$ and $\sigma_\mu^2 > 0$), and the alternative is H_1^h: $\lambda \neq 0$ (allowing $\rho \neq 0$ and $\sigma_\mu^2 > 0$). This is a one-dimensional conditional test for no spatial error correlation allowing the presence of both serial correlation and random region effects.

(C.2) H_0^i: $\rho = 0$ (allowing $\lambda \neq 0$ and $\sigma_\mu^2 > 0$), and the alternative is H_1^i: $\rho \neq 0$ (allowing $\lambda \neq 0$ and $\sigma_\mu^2 > 0$). This is a one-dimensional conditional test for no serial correlation allowing the presence of both spatial error correlation and random region effects.

(C.3) H_0^j: $\sigma_\mu^2 = 0$ (allowing $\rho \neq 0$ and $\lambda \neq 0$), and the alternative is H_1^j: $\sigma_\mu^2 > 0$ (allowing $\rho \neq 0$ and $\lambda \neq 0$). This is a one-dimensional conditional test for zero random region effects allowing the presence of both serial and spatial error correlation.

(C.4) H_0^k: $\lambda = \rho = 0$ (allowing $\sigma_\mu^2 > 0$), and the alternative H_1^k is that at least one component of λ or ρ is not zero (allowing $\sigma_\mu^2 > 0$). This is a two-dimensional conditional test for no serial or spatial error correlation allowing the presence of random region effects.

(C.5) H_0^l: $\lambda = \sigma_\mu^2 = 0$ (allowing $\rho \neq 0$), and the alternative H_1^l is that at least one component of λ or σ_μ^2 is not zero (allowing $\rho \neq 0$). This is a two-dimensional conditional test for no spatial error correlation or random region effects allowing the presence of serial error correlation.

(C.6) H_0^m: $\sigma_\mu^2 = \rho = 0$ (allowing $\lambda \neq 0$), and the alternative H_1^m is that at least one component of σ_μ^2 or ρ is not zero (allowing $\lambda \neq 0$). This is a two-dimensional conditional test for no random region effects or serial error correlation allowing the presence of spatial error correlation.

In the next subsections, we derive the corresponding LM tests for these hypotheses and we compare their performance with the corresponding LR tests using Monte Carlo experiments.

3.1. Joint tests for $\rho = \lambda = \sigma_\mu^2 = 0$

The joint LM test statistic for testing H_0^a: $\sigma_\mu^2 = \lambda = \rho = 0$ is given by

$$LM_J = \frac{NT^2}{2(T-1)(T-2)}[A^2 - 4AF + 2TF^2] + \frac{N^2T}{b}H^2, \tag{3.1}$$

where $A = \tilde{u}'(J_T \otimes I_N)\tilde{u}/\tilde{u}'\tilde{u} - 1$, $F = \frac{1}{2}(\tilde{u}'(G \otimes I_N)\tilde{u}/\tilde{u}'\tilde{u})$ and $H = \frac{1}{2}(\tilde{u}'(I_T \otimes (W' + W))\tilde{u}/\tilde{u}'\tilde{u})$ with $b = \mathrm{tr}(W + W')^2/2 = \mathrm{tr}(W^2 + W'W)$ and $\tilde{u}$ denoting the OLS residuals. G is the bidiagonal matrix with bidiagonal elements all equal to one. The derivation of this LM test statistic is given in Appendix A.1.Under H_0^a, LM_J is expected to be asymptotically distributed as χ_3^2. It is important to note that the large sample distribution of the LM test statistics derived in this paper are not formally established, but are likely to hold under similar sets of primitive assumptions developed in Kelejian and Prucha (2001) for the Moran I-test statistic and its close cousins the LM tests for spatial error correlation. See also Pinkse (1998, 2004) for general conditions under which Moran flavored tests for spatial correlation have a limiting normal distribution in the presence of nuisance parameters in six frequently encountered spatial models.

We also derive the joint LR test for H_0^a: $\sigma_\mu^2 = \lambda = \rho = 0$. This is given by

$$LR_J = 2(L_\mathrm{U} - L_\mathrm{R}), \tag{3.2}$$

where

$$L_\mathrm{U} = Const. + \frac{N}{2}\ln(1-\rho^2) - \frac{1}{2}\ln|d^2(1-\rho)^2\phi I_N + (B'B)^{-1}| - \frac{NT}{2}\ln(\sigma_e^2) + (T-1)\ln|B| - \frac{1}{2}u'\Omega^{-1}u, \tag{3.3}$$

see Appendix A.2. Here $\phi = \sigma_\mu^2/\sigma_e^2$, $d^2 = \alpha^2 + (T-1)$ and $\alpha = \sqrt{(1+\rho)/(1-\rho)}$. The restricted likelihood function under H_0^a is given by

$$L_\mathrm{R} = Const. - \frac{NT}{2}\ln\tilde{\sigma}_e^2 - \frac{1}{2\tilde{\sigma}_e^2}\tilde{u}'\tilde{u}. \tag{3.4}$$

Parameters of the unrestricted log-likelihood are estimated using the scoring method. This estimation procedure is described in Appendix A.2. Under the null hypothesis, the variance–covariance matrix reduces to $\Omega^* = \Omega = \sigma_e^2 I_{TN}$ and the restricted MLE of β is $\tilde{\beta}_\mathrm{OLS}$, so that $\tilde{u} = y - X\tilde{\beta}_\mathrm{OLS}$ are the OLS residuals and $\tilde{\sigma}_e^2 = \tilde{u}'\tilde{u}/NT$. This LR_J test is asymptotically distributed as χ^2 with three degrees of freedom.

3.2. One-dimensional marginal tests

Under H_0^b: $\lambda = 0$ (assuming $\rho = \sigma_\mu^2 = 0$), the Lagrange multiplier test, call it $LM_\lambda = (N^2T/b)H^2$ is the second term of (3.1). This is the marginal LM test for no spatial error correlation assuming no serial correlation or random region effects. This is in fact the LM test for spatial error correlation derived by Anselin (1988). Similarly, the marginal LM test for H_0^c: $\rho = 0$ (assuming $\lambda = \sigma_\mu^2 = 0$), call it $LM_\rho = (NT^2/(T-1))F^2$, is identical for large T to the third term in brackets of (3.1). This is the marginal LM test for no serial correlation assuming no spatial error correlation or random region effects. This is in fact the LM test for serial correlation derived by Breusch and Godfrey (1981) in time-series analysis. Finally, the marginal LM test for H_0^d: $\sigma_\mu^2 = 0$ (assuming $\rho = \lambda = 0$), call it $LM_\mu = (NT/2(T-1))A^2$ is identical for large T to the first term in brackets of (3.1). This is the marginal LM test for no random region effects assuming no spatial or serial error correlation. This is in fact the LM test for zero random effects derived by Breusch and Pagan (1980) for the error component model.

3.3. Two-dimensional marginal tests

Consider the joint hypothesis H_0^e: $\lambda = \rho = 0$ (assuming $\sigma_\mu^2 = 0$). It is easy to show that the corresponding LM test is given by $LM_{\lambda\rho} = LM_\lambda + LM_\rho$. This is the joint LM test for no spatial or serial error correlation assuming no random region effects. Similarly, for the joint hypothesis H_0^f: $\lambda = \sigma_\mu^2 = 0$ (assuming $\rho = 0$), the corresponding LM test is given by $LM_{\lambda\mu} = LM_\lambda + LM_\mu$. This is the joint LM test for no spatial error correlation or random region effects assuming no serial correlation. This is identical to the joint LM test derived by Baltagi et al. (2003) for the spatial error component model.

Finally, for the joint hypothesis H_0^g: $\sigma_\mu^2 = \rho = 0$ (assuming $\lambda = 0$), the corresponding LM test is given by $LM_{\mu\rho} = (NT^2/2(T-1)(T-2))[A^2 - 4AF + 2TF^2]$. This is the joint LM test for no random region effects or serial error correlation assuming no spatial error correlation. This is identical to the joint LM test derived by Baltagi and Li (1995) for the error component model with serial correlation. The derivations of these LM statistics, $LM_{\lambda\rho}, LM_{\lambda\mu}$ and $LM_{\mu\rho}$ are not given here to save space but are available upon request from the authors.

3.4. One-dimensional conditional tests

Consider the null hypothesis H_0^h: $\lambda = 0$ (allowing $\rho \neq 0$ and $\sigma_\mu^2 > 0$). The corresponding conditional LM test, call it $LM_{\lambda/\rho\mu}$, tests for zero spatial error correlation allowing the existence of serial error correlation and random region effects. Under the null hypothesis H_0^h, the variance–covariance matrix in (2.7) reduces to $\Omega_0 = (J_T \otimes I_N)\sigma_\mu^2 + V \otimes I_N$, where V was defined in (2.8). In this case, $\Omega_0^{-1} = (V^{-1} - cV^{-1}J_TV^{-1}) \otimes I_N$, where $c = \sigma_e^2\sigma_\mu^2/d^2(1-\rho)^2\sigma_\mu^2 + \sigma_e^2$. The score under the null hypothesis, derived in Appendix A.3, is given by

$$\left.\frac{\partial L}{\partial \lambda}\right|_{H_0^h} = \hat{D}(\lambda) = \frac{1}{2}\hat{u}'\left[V^{-1} - 2cV^{-1}J_TV^{-1} + c^2[V^{-1}J_T]^2V^{-1}\right] \otimes (W' + W)\hat{u}, \tag{3.5}$$

where $\hat{u}$ denote the restricted maximum likelihood residuals under H_0^h, i.e., under a serially correlated error component model. The resulting LM statistic is given by

$$LM_{\lambda/\rho\mu} = \frac{\hat{D}(\lambda)^2}{b(T - 2cg + c^2g^2)}, \tag{3.6}$$

where b was defined below (3.1) and $g = \mathrm{tr}(V^{-1}J_T) = (1/\sigma_e^2)(1-\rho)\{2 + (T-2)(1-\rho)\}$. Under the null hypothesis, the LM statistic is asymptotically distributed as χ_1^2.

We can also get the LR test under H_0^h. The restricted likelihood function under H_0^h is given by

$$L_R = Const. + \frac{N}{2}\ln(1-\tilde{\rho}^2) - \frac{N}{2}\{d^2(1-\tilde{\rho})^2\tilde{\phi} + 1\} - \frac{NT}{2}\ln\tilde{\sigma}_e^2 - \frac{1}{2}\tilde{u}'\Omega^{-1}\tilde{u} \tag{3.7}$$

and the unrestricted likelihood L_U is the same as given in (3.3).

Next, we consider the null hypothesis $H_0^i\colon \rho = 0$ (allowing $\lambda \neq 0$ and $\sigma_\mu^2 > 0$). The corresponding conditional LM test, call it $LM_{\rho/\lambda\mu}$ tests for zero serial error correlation allowing the existence of spatial error correlation and random region effects. Under the null hypothesis H_0^i, the variance–covariance matrix in (2.7) reduces to $\Omega_0 = \sigma_\mu^2 J_T \otimes I_N + \sigma_e^2 I_T \otimes (B'B)^{-1}$, where B is defined in (2.4). In this case, $\Omega_0^{-1} = (\sigma_e^2)^{-1}\mathrm{E}_T \otimes (B'B) + \bar{J}_T \otimes Z_0$, where $Z_0 = [T\sigma_\mu^2 I_N + \sigma_e^2(B'B)^{-1}]^{-1}$ is a special case of the matrix Z defined in (2.13) when $\rho = 0$. The score under the null hypothesis, derived in Appendix A.4, is given by

$$\begin{aligned}\left.\frac{\partial L}{\partial \rho}\right|_{H_0^i} = \hat{D}(\rho) = & -\frac{T-1}{T}(\hat{\sigma}_e^2\,\mathrm{tr}(Z_0(B'B)^{-1}) - N) \\ & + \frac{\hat{\sigma}_e^2}{2}\hat{u}'\left(\frac{1}{\hat{\sigma}_e^4}(\mathrm{E}_T G \mathrm{E}_T) \otimes (B'B) + \frac{1}{\hat{\sigma}_e^2}(\bar{J}_T G \mathrm{E}_T) \otimes Z_0\right. \\ & \left. + \frac{1}{\hat{\sigma}_e^2}(\mathrm{E}_T G \bar{J}_T) \otimes Z_0 + (\bar{J}_T G \bar{J}_T) \otimes Z_0(B'B)^{-1}Z_0\right)\hat{u},\end{aligned} \tag{3.8}$$

where $\hat{u}$ denote the restricted maximum likelihood residuals under the null hypothesis H_0^i, i.e., under the one-way spatial error component model. The resulting LM statistic is given by

$$LM_{\rho/\lambda\mu} = \hat{D}^2(\rho)J_{33}^{-1}, \tag{3.9}$$

where J_{33}^{-1} is the (3.3) element of the inverse of the information matrix $\hat{J}_\theta$ evaluated under H_0^i. The latter is given by

$$\hat{J}_\theta = \begin{bmatrix} \frac{1}{2}\left(\frac{N(T-1)}{\hat{\sigma}_e^4} + d_1\right) & \frac{T}{2}d_2 & \frac{(T-1)}{T}\left(\hat{\sigma}_e^2 d_1 - \frac{N}{\hat{\sigma}_e^2}\right) & \frac{1}{2}\left[\frac{(T-1)}{\hat{\sigma}_e^2}d_3 + \hat{\sigma}_e^2 d_4\right] \\ \frac{T}{2}d_2 & \frac{T^2}{2}\mathrm{tr}[Z_0]^2 & (T-1)\hat{\sigma}_e^2 d_2 & \frac{T\hat{\sigma}_e^2}{2}d_5 \\ \frac{(T-1)}{T}\left(\hat{\sigma}_e^2 d_1 - \frac{N}{\hat{\sigma}_e^2}\right) & (T-1)\hat{\sigma}_e^2 d_2 & J_{\rho\rho} & \frac{T-1}{T}(\sigma_e^4 d_4 - d_3) \\ \frac{1}{2}\left[\frac{(T-1)}{\hat{\sigma}_e^2}d_3 + \hat{\sigma}_e^2 d_4\right] & \frac{T\hat{\sigma}_e^2}{2}d_5 & \frac{T-1}{T}(\hat{\sigma}_e^4 d_4 - d_3) & \frac{1}{2}[(T-1)d_6 + \hat{\sigma}_e^4 d_7] \end{bmatrix}, \tag{3.10}$$

where $\hat{\sigma}_\mu^2$ and $\hat{\sigma}_e^2$ are the restricted maximum likelihood estimates of σ_μ^2 and σ_e^2 and

$$\hat{J}_{\rho\rho} = \frac{N}{T^2}(T^3 - 3T^2 + 2T + 2) + \frac{2(T-1)^2\hat{\sigma}_e^4}{T^2} d_1,$$

$$d_1 = \mathrm{tr}[Z_0(B'B)^{-1}]^2,$$

$$d_2 = \mathrm{tr}[Z_0(B'B)^{-1}Z_0],$$

$$d_3 = \mathrm{tr}[(W'B + B'W)(B'B)^{-1}],$$

$$d_4 = \mathrm{tr}[Z_0(B'B)^{-1}(W'B + B'W)(B'B)^{-1}Z_0(B'B)^{-1}],$$

$$d_5 = \mathrm{tr}[Z_0(B'B)^{-1}(W'B + B'W)(B'B)^{-1}Z_0],$$

$$d_6 = \mathrm{tr}[(W'B + B'W)(B'B)^{-1}]^2,$$

$$d_7 = \mathrm{tr}[Z_0(B'B)^{-1}(W'B + B'W)(B'B)^{-1}]^2.$$

Under the null hypothesis, the LM statistic is asymptotically distributed as χ_1^2.

We can also get the LR test under H_0^i. The restricted likelihood function under H_0^i is given by

$$L_\mathrm{R} = Const. - \frac{NT}{2}\ln\tilde{\sigma}_e^2 - \frac{1}{2}\ln[|T\tilde{\phi}I_N + (B'B)^{-1}|] + (T-1)\ln|B| - \frac{1}{2}\tilde{u}'\Omega^{-1}\tilde{u} \quad (3.11)$$

and L_U is the same as (3.3).

Finally, we consider the null hypothesis $\mathrm{H}_0^j\colon \sigma_\mu^2 = 0$ (allowing $\rho \neq 0$ and $\lambda \neq 0$). The corresponding conditional LM test, call it $LM_{\mu/\rho\lambda}$, tests for zero random region effects allowing the existence of spatial and serial error correlation. Under the null hypothesis H_0^j, the variance–covariance matrix in (2.7) reduces to $\Omega_0 = \sigma_e^2 V_\rho \otimes (B'B)^{-1}$, where V_ρ is defined in (2.8). In this case, $\Omega_0^{-1} = (1/\sigma_e^2)V_\rho^{-1} \otimes (B'B)$. The score under the null hypothesis, derived in Appendix A.5, is given by

$$\left.\frac{\partial L}{\partial \sigma_\mu^2}\right|_{\mathrm{H}_0^j} = \hat{D}(\sigma_\mu^2) = -\frac{g}{2}\,\mathrm{tr}(B'B) + \frac{1}{2\sigma_e^4}\hat{u}'[V_\rho^{-1}J_T V_\rho^{-1} \otimes (B'B)^2]\hat{u}, \quad (3.12)$$

where $g = \mathrm{tr}(V^{-1}J_T)$ was defined below (3.6) and $\hat{u}$ denote the restricted maximum likelihood residuals under H_0^j, i.e., under a spatial error component model with serially correlated remainder error. The resulting LM statistic is given by

$$LM_{\mu/\lambda\rho} = \hat{D}^2(\sigma_\mu^2)J_{22}^{-1}, \quad (3.13)$$

where J_{22}^{-1} is the (2.2) element of the inverse of the information matrix $\hat{J}_\theta$ evaluated under H_0^j. The latter is given by

$$\hat{J}_\theta = \begin{bmatrix} \frac{NT}{2\sigma_e^4} & \frac{g\,\mathrm{tr}[B'B]}{2\sigma_e^2} & \frac{N\rho}{\sigma_e^2(1-\rho^2)} & \frac{Td_3}{2\sigma_e^2} \\ \frac{g\,\mathrm{tr}[B'B]}{2\sigma_e^2} & \frac{g^2\,\mathrm{tr}[B'B]^2}{2} & \frac{\mathrm{tr}(B'B)}{\sigma_e^2(1+\rho)}[(2-T)\rho^2+(T-1)+\rho] & \frac{g}{2}\mathrm{tr}[W'B+B'W] \\ \frac{N\rho}{\sigma_e^2(1-\rho^2)} & \frac{\mathrm{tr}(B'B)}{\sigma_e^2(1+\rho)}[(2-T)\rho^2+(T-1)+\rho] & \frac{N}{(1-\rho^2)^2}(3\rho^2-\rho^2T+T-1) & \frac{\rho d_3}{1-\rho^2} \\ \frac{Td_3}{2\sigma_e^2} & \frac{g}{2}\mathrm{tr}[W'B+B'W] & \frac{\rho d_3}{1-\rho^2} & \frac{Td_6}{2} \end{bmatrix}, \tag{3.14}$$

where $d_3 = \mathrm{tr}[(W'B+B'W)(B'B)^{-1}]$ and $d_6 = \mathrm{tr}[(W'B+B'W)(B'B)^{-1}]^2$ were defined below (3.10). Under the null hypothesis, the LM statistic is asymptotically distributed as χ_1^2.

We can get the LR test under H_0^j. The restricted likelihood function under H_0^j is given by

$$L_\mathrm{R} = Const. + \frac{N}{2}\ln(1-\rho^2) - \frac{NT}{2}\ln\tilde{\sigma}_e^2 + T\ln|B| - \frac{1}{2}\tilde{u}'\Omega^{-1}\tilde{u} \tag{3.15}$$

and L_U is the same as (3.3).

3.5. Two-dimensional conditional tests

Consider the joint hypothesis H_0^k: $\lambda=\rho=0$ (allowing $\sigma_\mu^2>0$). The corresponding conditional LM test, call it $LM_{\lambda\rho/\mu}$ tests for zero spatial and serial error correlation allowing the existence of random region effects. Under the null hypothesis H_0^k, the variance–covariance matrix in (2.7) reduces to $\Omega_0 = \sigma_\mu^2 J_T\otimes I_N + \sigma_e^2 I_{NT}$. It is the familiar form of the one-way error component model with $\Omega_0^{-1} = (\sigma_1^2)^{-1}(\bar{J}_T\otimes I_N) + (\sigma_e^2)^{-1}(\mathrm{E}_T\otimes I_N)$, where $\sigma_1^2 = T\sigma_\mu^2+\sigma_e^2$. The scores under the null hypothesis, derived in Appendix A.6, are given by

$$\left.\frac{\partial L}{\partial\rho}\right|_{\mathrm{H}_0^k} = \hat{D}(\rho) = \frac{N(T-1)}{T}\left(\frac{\hat{\sigma}_1^2-\hat{\sigma}_e^2}{\hat{\sigma}_1^2}\right) + \frac{\hat{\sigma}_e^2}{2}\hat{u}'[(\bar{J}_T/\hat{\sigma}_1^2+\mathrm{E}_T/\hat{\sigma}_e^2)G(\bar{J}_T/\hat{\sigma}_1^2+\mathrm{E}_T/\hat{\sigma}_e^2)\otimes I_N]\hat{u}, \tag{3.16}$$

$$\left.\frac{\partial L}{\partial\lambda}\right|_{\mathrm{H}_0^k} = \hat{D}(\lambda) = \frac{1}{2}\hat{u}'\left[\frac{\hat{\sigma}_e^2}{\hat{\sigma}_1^4}(\bar{J}_T\otimes(W'+W)) + \frac{1}{\hat{\sigma}_e^2}(\mathrm{E}_T\otimes(W'+W))\right]\hat{u} \tag{3.17}$$

and the information matrix is given by

$$\hat{J}_\theta = \begin{bmatrix} \frac{N}{2}\left(\frac{1}{\hat{\sigma}_1^4}+\frac{T-1}{\hat{\sigma}_e^4}\right) & \frac{NT}{2\hat{\sigma}_1^4} & \frac{N(T-1)}{T}\hat{\sigma}_e^2\left(\frac{1}{\hat{\sigma}_1^4}-\frac{1}{\hat{\sigma}_e^4}\right) & 0 \\ \frac{NT}{2\hat{\sigma}_1^4} & \frac{NT^2}{2\hat{\sigma}_1^4} & \frac{N(T-1)\hat{\sigma}_e^2}{\hat{\sigma}_1^4} & 0 \\ \frac{N(T-1)}{T}\hat{\sigma}_e^2\left(\frac{1}{\hat{\sigma}_1^4}-\frac{1}{\hat{\sigma}_e^4}\right) & \frac{N(T-1)\hat{\sigma}_e^2}{\hat{\sigma}_1^4} & \hat{J}_{\rho\rho} & 0 \\ 0 & 0 & 0 & (T-1)b+\frac{\hat{\sigma}_e^4}{\hat{\sigma}_1^4}b \end{bmatrix}, \tag{3.18}$$

where $\hat{\sigma}_1^2 = \hat{u}'(J_T \otimes I_N)\hat{u}/NT$ and $\hat{\sigma}_e^2 = \hat{u}'(\mathrm{E}_T \otimes I_N)\hat{u}/N(T-1)$ are the solutions of $(\partial L/\partial\sigma_\mu^2)|_{\mathrm{H}_0^k} = 0$ and $(\partial L/\partial\sigma_e^2)|_{\mathrm{H}_0^k} = 0$, respectively. $\hat{u}$ denote the restricted maximum likelihood residuals under H_0^k, i.e., under a one-way error component model. $\hat{J}_{\rho\rho} = N[2a^2(T-1)^2 + 2a(2T-3) + T - 1]$, $a = (\hat{\sigma}_e^2 - \hat{\sigma}_1^2)/T\sigma_1^2$, and $b = \mathrm{tr}(W^2 + W'W)$.

Since $\hat{D}_\theta' = (0, 0, \hat{D}(\rho), \hat{D}(\lambda))$, and $\hat{J}(\theta)$ is a block diagonal matrix with respect to $\theta_1 = (\sigma_e^2, \sigma_\mu^2, \rho)$ and λ, the resulting LM statistic for H_0^k is given by

$$LM_{\lambda\rho/\mu} = \hat{D}_\theta' \hat{J}_\theta^{-1} \hat{D}_\theta = \frac{\hat{D}(\rho)^2 N^2 T^2 (T-1)}{4\hat{\sigma}_1^4 \hat{\sigma}_e^4 \, det[J(\theta_1)]} + \frac{\hat{D}(\lambda)^2}{[(T-1) + \hat{\sigma}_e^4/\hat{\sigma}_1^4]b}, \tag{3.19}$$

where *det* denotes the determinants, $J(\theta_1)$ is the block diagonal information matrix corresponding to the parameters $(\sigma_e^2, \sigma_\mu^2, \rho)$, and $\hat{D}(\rho)$ and $\hat{D}(\lambda)$ are given by (3.16) and (3.17). The first term of (3.19) is the familiar term used in testing for serial correlation, see Baltagi (2005) and the second term of (3.10) is the familiar term used in testing the spatial error correlation. Under the null hypothesis, the LM statistic of (3.19) is asymptotically distributed as χ_2^2.

We can get the LR test for H_0^k. The restricted likelihood function under H_0^k is given by

$$L_{\mathrm{R}}^c = Const. - \frac{NT}{2}\ln\tilde{\sigma}_e^2 - \frac{N}{2}\ln(T\tilde{\phi} + 1) - \frac{1}{2}\tilde{u}'\tilde{\Omega}^{-1}\tilde{u}, \tag{3.20}$$

where $\phi = \sigma_\mu^2/\sigma_e^2$ and the unrestricted likelihood L_{U} is the same as (3.3).

Next, we consider the joint hypothesis H_0^l: $\lambda = \sigma_\mu^2 = 0$ (allowing $\rho \neq 0$). The corresponding conditional LM test, call it $LM_{\lambda\mu/\rho}$, tests for zero spatial error correlation and random region effects allowing the existence of serial correlation. Under the null hypothesis H_0^l, the variance–covariance matrix in (2.7) reduces to $\Omega_0 = \sigma_e^2 V_\rho \otimes I_N$ and $\Omega_0^{-1} = (1/\sigma_e^2)V_\rho^{-1} \otimes I_N$. The scores under the null hypothesis derived in Appendix A.7, are given by

$$\left.\frac{\partial L}{\partial\sigma_\mu^2}\right|_{\mathrm{H}_0^l} = D(\sigma_\mu^2) = -\frac{Ng}{2} + \frac{1}{2\sigma_e^4}\hat{u}'[V_\rho^{-1}J_T V_\rho^{-1} \otimes I_N]\hat{u}, \tag{3.21}$$

$$\left.\frac{\partial L}{\partial\lambda}\right|_{\mathrm{H}_0^l} = \hat{D}(\lambda) = \frac{1}{2\sigma_e^2}\hat{u}'[V_\rho^{-1} \otimes (W' + W)]\hat{u} \tag{3.22}$$

and the information matrix is given by

$$\hat{J}_\theta = \begin{bmatrix} \frac{NT}{2\sigma_e^4} & \frac{Ng}{2\sigma_e^2} & \frac{N\rho}{\sigma_e^2(1-\rho^2)} & 0 \\ \frac{Ng}{2\sigma_e^2} & \frac{Ng^2}{2} & \frac{N}{\sigma_e^2(1+\rho)}[(2-T)\rho^2 + \rho + (T-1)] & 0 \\ \frac{N\rho}{\sigma_e^2(1-\rho^2)} & \frac{N}{\sigma_e^2(1+\rho)}[(2-T)\rho^2 + \rho + (T-1)] & \frac{N}{(1-\rho^2)^2}(3\rho^2 - \rho^2 T + T - 1) & 0 \\ 0 & 0 & 0 & T_b \end{bmatrix}, \tag{3.23}$$

where $\hat{u}$ denote the restricted MLE residuals under H_0^l, i.e., under a serially correlated regression model. Since $\hat{D}_\theta' = (0, \hat{D}(\sigma_\mu^2), 0, \hat{D}(\lambda))$, and $\hat{J}(\theta)$ is a block diagonal matrix with respect to $\theta_1 = (\sigma_e^2, \sigma_\mu^2, \rho)$ and λ, the resulting LM statistic of H_0^l is given by

$$LM_{\lambda\mu/\rho} = \hat{D}_\theta' \hat{J}_\theta^{-1} \hat{D}_\theta = \frac{\hat{D}^2(\sigma_\mu^2)}{det[J(\theta_1)]} \frac{N^2}{\sigma_e^4(1-\rho^2)} \left\{ \frac{T}{2}(3\rho^2 - \rho^2 T + T - 1) - \rho^2 \right\} + \frac{\hat{D}^2(\lambda)}{Tb}, \tag{3.24}$$

where $J(\theta_1)$ is the block diagonal information matrix corresponding to the parameters $(\sigma_e^2, \sigma_\mu^2, \rho)$...... $\hat{D}(\sigma_\mu^2)$ and $\hat{D}(\lambda)$ are given by (3.21) and (3.22). The first term of (3.24) is the familiar term used in testing for serial correlation, see Baltagi (2005) and the second term of (3.24) is the familiar term used in testing for spatial error correlation. Under the null hypothesis, the LM statistic in (3.24) is asymptotically distributed as χ_2^2.

We can also get the LR test for H_0^l. The restricted likelihood function under H_0^l is given by

$$L_R = Const. - \frac{NT}{2} \ln \tilde{\sigma}_e^2 + \frac{N}{2} \ln(1-\rho^2) - \frac{1}{2} \tilde{u}' \tilde{\Omega}^{-1} \tilde{u} \tag{3.25}$$

and L_U is the same as (3.3).

Finally, we consider the null hypothesis H_0^m: $\sigma_\mu^2 = \rho = 0$ (allowing $\lambda \neq 0$). The corresponding conditional LM test, call it $LM_{\mu\rho/\lambda}$, tests for zero serial error correlation and random region effects allowing the existence of spatial error correlation. Under the null hypothesis H_0^m, the variance–covariance matrix in (2.7) reduces to $\Omega_0 = \sigma_e^2 I_T \otimes (B'B)^{-1}$ and $\Omega_0^{-1} = (1/\sigma_e^2) I_T \otimes (B'B)$. The scores under the null hypothesis, derived in Appendix A.8, are given by

$$\left.\frac{\partial L}{\partial \sigma_\mu^2}\right|_{H_0^m} = D(\sigma_\mu^2) = -\frac{T}{2\sigma_e^2} \operatorname{tr}[B'B] + \frac{1}{2\sigma_e^4} \hat{u}'[J_T \otimes (B'B)^2]\hat{u}, \tag{3.26}$$

$$\left.\frac{\partial L}{\partial \rho}\right|_{H_0^m} = \hat{D}(\rho) = \frac{1}{2\sigma_e^2} \hat{u}'[G \otimes (B'B)]\hat{u} \tag{3.27}$$

and the information matrix is given by

$$\hat{J}_\theta = \begin{bmatrix} \frac{NT}{2\sigma_e^4} & \frac{T}{2\sigma_e^4}\operatorname{tr}[B'B] & 0 & \frac{T}{2\sigma_e^4} d_3 \\ \frac{T}{2\sigma_e^4}\operatorname{tr}[B'B] & \frac{T^2}{2\sigma_e^4}\operatorname{tr}[(B'B)^2] & \frac{T-1}{\sigma_e^2}\operatorname{tr}[B'B] & \frac{T}{2\sigma_e^4}\operatorname{tr}[W'B + B'W] \\ 0 & \frac{T-1}{\sigma_e^2}\operatorname{tr}[B'B] & N(T-1) & 0 \\ \frac{T}{2\sigma_e^4} d_3 & \frac{T}{2\sigma_e^4}\operatorname{tr}[W'B + B'W] & 0 & \frac{T}{2\sigma_e^4} d_6 \end{bmatrix}, \tag{3.28}$$

where d_3 and d_6 are defined below (3.10) and $\hat{u}$ denote the restricted MLE residuals under H_0^m, i.e., under a spatial error correlation model. Using $\hat{D}_\theta' = (0, \hat{D}(\sigma_\mu^2), \hat{D}(\rho), 0)$, the resulting LM statistic for H_0^m is given by

$$LM_{\mu\rho/\lambda} = \hat{D}_\theta' \hat{J}_\theta^{-1} \hat{D}_\theta. \tag{3.29}$$

Under the null hypothesis, this LM statistic is asymptotically distributed as χ_2^2.

We can get the LR test under H_0^m. The restricted likelihood function under H_0^m is given by

$$L_R = Const. - \frac{NT}{2} \ln \tilde{\sigma}_e^2 + T \ln |B| - \frac{1}{2} \tilde{u}' \tilde{\Omega}^{-1} \tilde{u} \tag{3.30}$$

and L_U is the same as (3.3).

4. Monte Carlo results

The experimental design for the Monte Carlo simulations is based on the format which was extensively used in earlier studies in the spatial regression model by Anselin and Rey (1991) and Anselin and Florax (1995) and in the panel data model by Nerlove (1971).

The model is set as follows:

$$y_{it} = \alpha + x_{it}'\beta + u_{it}, \quad i = 1, \ldots, N, \ t = 1, \ldots, T, \tag{4.1}$$

where $\alpha = 5$ and $\beta = 0.5$. x_{it} is generated by a similar method of Nerlove (1971). In fact, $x_{it} = 0.1t + 0.5x_{i,t-1} + z_{it}$, where z_{it} is uniformly distributed over the interval $[-0.5, 0.5]$. The initial values x_{i0} are chosen as $(5 + 10z_{i0})$. For the disturbances, $u_{it} = \mu_i + \varepsilon_{it}$, $\varepsilon_{it} = \lambda \sum_{j=1}^N w_{ij}\varepsilon_{it} + v_{it}$, $v_{it} = \rho v_{i,t-1} + e_{it}$, with $\mu_i \sim IIN(0, \sigma_\mu^2)$ and $e_{it} \sim IIN(0, \sigma_e^2)$, where the initial values v_{i0} are generated from $N(0, \sigma_e/(1-\rho^2))$. The matrix W is a Rook-type weight matrix, and the rows of this matrix are standardized so that they sum to one. We fix $\sigma_\mu^2 + \sigma_e^2 = 20$ and let $\eta = \sigma_\mu^2/(\sigma_\mu^2 + \sigma_e^2)$ vary over the set (0, 0.2, 0.5, 0.8). The spatial autocorrelation factor λ is varied over a positive range from 0 to 0.8 by increments of 0.2 and ρ takes six different values (0.0, 0.2, 0.4, 0.6, 0.8). Two values for $N = 25$ and 49, and two values for $T = 7$ and 12 are chosen. In total, this amounts to 400 experiments. For each experiment, the joint, conditional and marginal LM and LR tests are computed and 1000 replications are performed. Not all the Monte Carlo results are presented to save space. Here we focus on the joint and conditional tests since these are new contributions to the literature.

4.1. Joint tests for H_0^a: $\lambda = \rho = \sigma_\mu^2 = 0$

Table 1 gives the frequency of rejections at the 5% level for the joint LR and LM tests for H_0^a: $\lambda = \rho = \sigma_\mu^2 = 0$. For 1000 replications, counts between 37 and 63 are not significantly different from 50 at the 0.05 level. The results are reported for $N = 25$, 49 and $T = 7$, 12 for the Rook weight matrix. Table 1 shows that at the 5% level, the size of the joint LR test is typically less than 0.05 and varies between 2.3% and 4% depending on N and T. In contrast, the size of the joint LM test is not significantly different from 0.05 varying between 3.9% and 4.9% depending on N and T. The power of the joint LM and LR tests is reasonably high as long as λ or ρ or η are larger than 0.2. In fact, if λ or ρ or η $\geqslant 0.4$, this power is almost one in all cases. For a fixed λ, ρ or η, this power improves as N or T increase.

Table 1
Joint tests for H_0^a; $\sigma_\mu^2 = \lambda = \rho = 0$

N, T	λ	ρ	$\eta = 0.0$		$\eta = 0.2$		$\eta = 0.5$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
25 7	0.0	0.0	0.039	0.023	0.846	0.788	1.000	1.000
25 7	0.0	0.2	0.500	0.416	0.973	0.963	1.000	1.000
25 7	0.0	0.4	0.983	0.980	1.000	1.000	1.000	1.000
25 7	0.0	0.6	1.000	1.000	1.000	1.000	1.000	1.000
25 7	0.0	0.8	1.000	1.000	1.000	1.000	1.000	1.000
25 7	0.2	0.0	0.325	0.283	1.000	1.000	1.000	1.000
25 7	0.2	0.2	0.718	0.672	1.000	1.000	1.000	1.000
25 7	0.2	0.4	0.996	0.992	1.000	1.000	1.000	1.000
25 7	0.2	0.6	1.000	1.000	1.000	1.000	1.000	1.000
25 7	0.2	0.8	1.000	1.000	1.000	1.000	1.000	1.000
25 7	0.4	0.0	0.946	0.943	1.000	1.000	1.000	1.000
25 7	0.4	0.2	0.987	0.987	1.000	1.000	1.000	1.000
25 7	0.4	0.4	0.999	1.000	1.000	1.000	1.000	1.000
25 7	0.4	0.6	1.000	1.000	1.000	1.000	1.000	1.000
25 7	0.4	0.8	1.000	1.000	1.000	1.000	1.000	1.000
25 7	0.6	0.0	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.0	0.0	0.049	0.034	0.984	0.970	1.000	1.000
25 12	0.0	0.2	0.792	0.752	1.000	1.000	1.000	1.000
25 12	0.0	0.4	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.0	0.6	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.0	0.8	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.2	0.0	0.582	0.537	1.000	1.000	1.000	1.000
25 12	0.2	0.2	0.943	0.939	1.000	1.000	1.000	1.000
25 12	0.2	0.4	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.2	0.6	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.2	0.8	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.0	0.0	0.046	0.040	0.990	0.970	1.000	1.000
49 7	0.0	0.2	0.836	0.799	1.000	1.000	1.000	1.000
49 7	0.0	0.4	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.0	0.6	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.0	0.8	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.2	0.0	0.642	0.592	1.000	1.000	1.000	1.000
49 7	0.2	0.2	0.956	0.950	1.000	1.000	1.000	1.000
49 7	0.2	0.4	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.2	0.6	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.2	0.8	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.0	0.0	0.045	0.030	1.000	1.000	1.000	1.000
49 12	0.0	0.2	0.987	0.980	1.000	1.000	1.000	1.000
49 12	0.0	0.4	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.0	0.6	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.0	0.8	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.2	0.0	0.886	0.870	1.000	1.000	1.000	1.000
49 12	0.2	0.2	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.2	0.4	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.2	0.6	1.000	1.000	1.000	1.000	1.000	1.000

Table 1 (*continued*)

N, T	λ	ρ	$\eta = 0.0$		$\eta = 0.2$		$\eta = 0.5$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
49 12	0.2	0.8	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000

4.2. One-dimensional conditional tests

Table 2 gives the frequency of rejections at the 5% level for the one-dimensional conditional LR and LM tests for H_0^h: $\lambda = 0$ (allowing $\rho \neq 0$ and $\sigma_\mu^2 > 0$). The size of these conditional tests is not significantly different from 0.05 except in two cases. For $N = 25, T = 7$, this varies between 3.1% and 5.9% for the LM test and 3.3% to 6.2% for the LR test. The power of these conditional LM and LR tests is reasonably high as long as λ is larger than 0.2% .... In fact, if $\lambda \geqslant 0.4$, this power is almost one in all cases. For a small $\lambda = 0.2$, this power improves as N or T increase.

Table 3 gives the frequency of rejections at the 5% level for the one-dimensional conditional LR and LM tests for H_0^i: $\rho = 0$ (allowing $\lambda \neq 0$ and $\sigma_\mu^2 > 0$). The size of these conditional tests is not significantly different from 0.05 except in a few cases, like when $\eta = 0$, where the LM test is oversized ranging from 6.5% to 8% for $N = 25$ and $T = 7$, and 5.5% to 9.3% for $N = 49$ and $T = 7$. Things improve as T increases from 7 to 12 as expected. Note that the few cases where the conditional LM tests suffer significant size distortions tend also to be cases where there is a violation of one or more of the auxiliary assumptions. In this case, the violation of $\sigma_\mu^2 > 0$, since in this case $\sigma_\mu^2 = 0$. The LR test is better sized ranging from 4.0% to 6.7% for $\eta = 0$ and all values of N and T% .... The power of these conditional LM and LR tests is close to one as long as ρ is larger than 0.2. For a small $\rho = 0.2$, this power improves as N or T increase.

Table 4 gives the frequency of rejections at the 5% level for the one-dimensional conditional LR and LM tests for H_0^j: $\sigma_\mu^2 = 0$ (allowing $\rho \neq 0$ and $\lambda \neq 0$). The LR test is undersized ranging from 1.6% to 3.4% for $\lambda = 0$ and all values of N and T. In contrast, the LM test is not significantly different from 0.05 for $\lambda = 0$ and all values of N and T. The size of this LM test varies between 3.7% and 5.6%. The power of these conditional LM and LR tests increase with η, N and T. However, for a given η and λ, there is a drop in the power as ρ becomes larger than 0.6, yielding low power for $\rho = 0.8$. Things improve as N or T increase. This may be due to the interaction effect between the serial correlation over time due to the AR(1) process on the remainder disturbances and the constant serial correlation over time due to the same region effect.

4.3. Two-dimensional conditional tests

Table 5 gives the frequency of rejections at the 5% level for the two-dimensional conditional LR and LM tests for H_0^k: $\lambda = \rho = 0$ (allowing $\sigma_\mu^2 > 0$). The size of these conditional tests is not significantly different from 0.05 except for the LM test when $\eta = 0$ and $T = 7$. This varies between 3.9% to 7.7% for the LM test and 3.9% to 6.3% for the LR test. Once again, the few cases where there is significant size distortion tend also to be

Table 2
One-dimensional conditional tests for (C.1) H_0^h; $\lambda = 0$ (allowing $\rho \neq 0$ and $\sigma_\mu^2 > 0$)

N, T	λ	ρ	$\eta = 0.0$		$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
25 7	0.0	0.0	0.056	0.060	0.044	0.053	0.053	0.062	0.045	0.050
25 7	0.0	0.2	0.038	0.043	0.048	0.052	0.042	0.047	0.049	0.053
25 7	0.0	0.4	0.053	0.058	0.059	0.058	0.042	0.046	0.042	0.048
25 7	0.0	0.6	0.058	0.054	0.046	0.045	0.035	0.048	0.049	0.051
25 7	0.0	0.8	0.047	0.049	0.048	0.051	0.048	0.047	0.031	0.033
25 7	0.2	0.0	0.460	0.482	0.426	0.439	0.465	0.486	0.448	0.458
25 7	0.2	0.2	0.466	0.486	0.460	0.478	0.433	0.452	0.413	0.443
25 7	0.2	0.4	0.437	0.458	0.441	0.450	0.434	0.442	0.419	0.435
25 7	0.2	0.6	0.437	0.444	0.420	0.424	0.432	0.444	0.438	0.453
25 7	0.2	0.8	0.486	0.470	0.438	0.425	0.423	0.423	0.440	0.462
25 7	0.4	0.0	0.978	0.983	0.974	0.976	0.974	0.975	0.970	0.977
25 7	0.4	0.2	0.988	0.988	0.960	0.969	0.965	0.967	0.975	0.978
25 7	0.4	0.4	0.982	0.985	0.971	0.972	0.966	0.970	0.964	0.965
25 7	0.4	0.6	0.983	0.982	0.966	0.968	0.979	0.984	0.968	0.969
25 7	0.4	0.8	0.981	0.974	0.972	0.966	0.976	0.974	0.970	0.969
25 7	0.6	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.0	0.0	0.046	0.051	0.055	0.059	0.058	0.061	0.051	0.053
25 12	0.0	0.2	0.051	0.052	0.056	0.059	0.046	0.050	0.045	0.048
25 12	0.0	0.4	0.055	0.051	0.046	0.051	0.051	0.053	0.057	0.059
25 12	0.0	0.6	0.044	0.042	0.050	0.053	0.046	0.051	0.040	0.041
25 12	0.0	0.8	0.056	0.049	0.060	0.048	0.037	0.040	0.045	0.047
25 12	0.2	0.0	0.760	0.768	0.710	0.721	0.733	0.747	0.743	0.747
25 12	0.2	0.2	0.754	0.754	0.735	0.741	0.735	0.743	0.730	0.734
25 12	0.2	0.4	0.741	0.747	0.715	0.723	0.720	0.727	0.704	0.712
25 12	0.2	0.6	0.734	0.726	0.724	0.727	0.722	0.726	0.735	0.744
25 12	0.2	0.8	0.735	0.698	0.737	0.712	0.721	0.724	0.728	0.737
25 12	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.0	0.0	0.066	0.068	0.053	0.053	0.038	0.040	0.042	0.045
49 7	0.0	0.2	0.053	0.053	0.041	0.043	0.058	0.058	0.050	0.051
49 7	0.0	0.4	0.040	0.042	0.048	0.048	0.059	0.062	0.058	0.053
49 7	0.0	0.6	0.047	0.046	0.050	0.057	0.038	0.044	0.042	0.043
49 7	0.0	0.8	0.046	0.034	0.043	0.035	0.036	0.043	0.053	0.051
49 7	0.2	0.0	0.771	0.786	0.732	0.737	0.722	0.727	0.737	0.738
49 7	0.2	0.2	0.756	0.768	0.764	0.769	0.694	0.703	0.726	0.732
49 7	0.2	0.4	0.799	0.806	0.746	0.747	0.715	0.724	0.738	0.739
49 7	0.2	0.6	0.770	0.774	0.727	0.739	0.732	0.740	0.723	0.729
49 7	0.2	0.8	0.738	0.741	0.721	0.728	0.724	0.718	0.717	0.718
49 7	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.0	0.0	0.043	0.046	0.045	0.047	0.062	0.065	0.040	0.041
49 12	0.0	0.2	0.052	0.053	0.058	0.058	0.058	0.058	0.048	0.049
49 12	0.0	0.4	0.053	0.055	0.037	0.040	0.072	0.074	0.040	0.040
49 12	0.0	0.6	0.051	0.050	0.036	0.037	0.045	0.045	0.039	0.039
49 12	0.0	0.8	0.051	0.044	0.038	0.033	0.050	0.051	0.057	0.053
49 12	0.2	0.0	0.965	0.966	0.940	0.942	0.942	0.944	0.941	0.943
49 12	0.2	0.2	0.954	0.952	0.934	0.938	0.948	0.949	0.942	0.940
49 12	0.2	0.4	0.955	0.956	0.947	0.949	0.922	0.926	0.945	0.945
49 12	0.2	0.6	0.959	0.955	0.946	0.948	0.950	0.955	0.925	0.947

Table 2 (*continued*)

N,T	λ	ρ	$\eta=0.0$		$\eta=0.2$		$\eta=0.5$		$\eta=0.8$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
49 12	0.2	0.8	0.953	0.934	0.941	0.937	0.954	0.957	0.949	0.953
49 12	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 3
One-dimensional conditional tests for (C.2) H_0^i; $\rho=0$ (allowing $\lambda\neq 0$ and $\sigma_\mu^2>0$)

N,T	ρ	λ	$\eta=0.0$		$\eta=0.2$		$\eta=0.5$		$\eta=0.8$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
25 7	0.0	0.0	0.073	0.053	0.043	0.045	0.055	0.060	0.027	0.032
25 7	0.0	0.2	0.080	0.053	0.038	0.035	0.053	0.058	0.058	0.062
25 7	0.0	0.4	0.073	0.053	0.052	0.053	0.042	0.055	0.045	0.045
25 7	0.0	0.6	0.072	0.067	0.070	0.068	0.048	0.052	0.033	0.038
25 7	0.0	0.8	0.065	0.045	0.053	0.062	0.043	0.052	0.047	0.052
25 7	0.2	0.0	0.447	0.415	0.497	0.505	0.522	0.530	0.447	0.455
25 7	0.2	0.2	0.485	0.463	0.465	0.482	0.450	0.455	0.463	0.487
25 7	0.2	0.4	0.480	0.470	0.465	0.460	0.477	0.482	0.458	0.468
25 7	0.2	0.6	0.478	0.455	0.427	0.450	0.473	0.475	0.440	0.453
25 7	0.2	0.8	0.478	0.470	0.467	0.500	0.492	0.502	0.457	0.460
25 7	0.4	0.0	0.867	0.862	0.962	0.958	0.950	0.950	0.948	0.948
25 7	0.4	0.2	0.965	0.968	0.958	0.970	0.963	0.968	0.952	0.958
25 7	0.4	0.4	0.957	0.960	0.957	0.958	0.962	0.957	0.958	0.957
25 7	0.4	0.6	0.973	0.977	0.955	0.955	0.950	0.957	0.942	0.948
25 7	0.4	0.8	0.972	0.977	0.962	0.973	0.973	0.977	0.950	0.955
25 7	0.6	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.0	0.0	0.062	0.051	0.051	0.053	0.066	0.070	0.051	0.053
25 12	0.0	0.2	0.071	0.061	0.056	0.061	0.045	0.045	0.043	0.044
25 12	0.0	0.4	0.055	0.047	0.051	0.053	0.034	0.035	0.042	0.043
25 12	0.0	0.6	0.062	0.051	0.051	0.056	0.040	0.042	0.047	0.050
25 12	0.0	0.8	0.051	0.041	0.048	0.046	0.030	0.031	0.042	0.044
25 12	0.2	0.0	0.815	0.803	0.816	0.817	0.848	0.848	0.836	0.834
25 12	0.2	0.2	0.793	0.785	0.819	0.827	0.813	0.817	0.818	0.821
25 12	0.2	0.4	0.849	0.842	0.813	0.812	0.812	0.810	0.845	0.843
25 12	0.2	0.6	0.843	0.826	0.809	0.816	0.810	0.810	0.851	0.852
25 12	0.2	0.8	0.837	0.835	0.813	0.814	0.813	0.810	0.814	0.815
25 12	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.0	0.0	0.070	0.040	0.053	0.057	0.040	0.043	0.052	0.057
49 7	0.0	0.2	0.093	0.060	0.040	0.043	0.035	0.035	0.067	0.075
49 7	0.0	0.4	0.090	0.060	0.053	0.053	0.040	0.040	0.050	0.050
49 7	0.0	0.6	0.085	0.057	0.047	0.043	0.040	0.050	0.048	0.052
49 7	0.0	0.8	0.055	0.040	0.038	0.042	0.043	0.048	0.048	0.052
49 7	0.2	0.0	0.757	0.733	0.750	0.755	0.780	0.777	0.743	0.753
49 7	0.2	0.2	0.813	0.807	0.750	0.758	0.783	0.792	0.785	0.793
49 7	0.2	0.4	0.793	0.783	0.778	0.780	0.753	0.755	0.773	0.777
49 7	0.2	0.6	0.790	0.792	0.783	0.785	0.753	0.753	0.767	0.770
49 7	0.2	0.8	0.818	0.817	0.798	0.800	0.757	0.765	0.770	0.773
49 7	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.998

Table 3 (*continued*)

N, T	ρ	λ	$\eta = 0.0$		$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
49 12	0.0	0.0	0.068	0.054	0.055	0.055	0.054	0.052	0.052	0.052
49 12	0.0	0.2	0.058	0.050	0.041	0.041	0.051	0.051	0.056	0.056
49 12	0.0	0.4	0.061	0.057	0.049	0.049	0.047	0.047	0.051	0.055
49 12	0.0	0.6	0.068	0.065	0.053	0.058	0.052	0.056	0.054	0.054
49 12	0.0	0.8	0.061	0.064	0.051	0.051	0.045	0.049	0.045	0.044
49 12	0.2	0.0	0.982	0.982	0.987	0.989	0.967	0.968	0.978	0.979
49 12	0.2	0.2	0.976	0.975	1.000	1.000	1.000	1.000	1.000	0.000
49 12	0.2	0.4	0.991	0.983	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.2	0.6	0.993	0.986	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.2	0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 4
One-dimensional conditional tests for (C.3) H_0^j; $\sigma_\mu^2 = 0$ (allowing $\rho \neq 0$ and $\lambda \neq 0$)

N, T	η	ρ	$\lambda = 0.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
25 7	0.0	0.0	0.044	0.019	0.045	0.020	0.052	0.026	0.037	0.014	0.036	0.014
25 7	0.0	0.2	0.048	0.022	0.048	0.012	0.038	0.025	0.025	0.014	0.048	0.026
25 7	0.0	0.4	0.038	0.018	0.047	0.024	0.045	0.025	0.025	0.020	0.042	0.029
25 7	0.0	0.6	0.053	0.022	0.051	0.029	0.057	0.036	0.038	0.025	0.038	0.021
25 7	0.0	0.8	0.042	0.024	0.045	0.029	0.042	0.028	0.040	0.036	0.046	0.030
25 7	0.2	0.0	0.713	0.730	0.710	0.717	0.793	0.812	0.830	0.843	0.873	0.903
25 7	0.2	0.2	0.407	0.436	0.487	0.503	0.570	0.549	0.612	0.601	0.637	0.676
25 7	0.2	0.4	0.150	0.197	0.207	0.232	0.264	0.257	0.393	0.383	0.467	0.477
25 7	0.2	0.6	0.070	0.113	0.113	0.132	0.122	0.124	0.147	0.133	0.237	0.187
25 7	0.2	0.8	0.056	0.042	0.038	0.053	0.037	0.044	0.060	0.042	0.078	0.071
25 7	0.5	0.0	0.997	1.000	1.000	1.000	0.993	1.000	0.984	0.993	0.997	0.997
25 7	0.5	0.2	0.952	0.976	0.937	0.978	0.964	1.000	0.947	0.981	0.963	0.994
25 7	0.5	0.4	0.622	0.752	0.711	0.780	0.734	0.833	0.793	0.867	0.843	0.927
25 7	0.5	0.6	0.293	0.343	0.297	0.343	0.447	0.467	0.536	0.553	0.633	0.687
25 7	0.5	0.8	0.066	0.093	0.098	0.136	0.132	0.143	0.191	0.173	0.250	0.257
25 7	0.8	0.0	0.997	1.000	0.993	1.000	1.000	1.000	0.983	1.000	0.983	1.000
25 7	0.8	0.2	0.973	1.000	0.998	1.000	0.967	1.000	0.943	1.000	0.953	1.000
25 7	0.8	0.4	0.873	0.977	0.872	0.987	0.853	0.993	0.852	0.992	0.917	0.997
25 7	0.8	0.6	0.473	0.728	0.543	0.753	0.613	0.850	0.773	0.927	0.827	0.967
25 7	0.8	0.8	0.191	0.187	0.237	0.268	0.297	0.383	0.456	0.512	0.537	0.697
25 12	0.0	0.0	0.055	0.017	0.053	0.026	0.032	0.016	0.042	0.017	0.045	0.021
25 12	0.0	0.2	0.037	0.016	0.034	0.012	0.052	0.022	0.032	0.022	0.037	0.022
25 12	0.0	0.4	0.052	0.021	0.039	0.022	0.048	0.024	0.040	0.018	0.038	0.018
25 12	0.0	0.6	0.050	0.022	0.045	0.022	0.036	0.022	0.048	0.027	0.058	0.034
25 12	0.0	0.8	0.041	0.021	0.042	0.029	0.044	0.027	0.059	0.029	0.042	0.024
25 12	0.2	0.0	0.987	0.983	0.986	0.985	0.999	0.988	0.995	0.985	0.999	1.000
25 12	0.2	0.2	0.923	0.896	0.920	0.915	0.867	0.864	0.895	0.900	0.932	0.925
25 12	0.2	0.4	0.575	0.535	0.558	0.552	0.625	0.634	0.694	0.685	0.735	0.741

Table 4 (*continued*)

N, T	η	ρ	$\lambda = 0.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
25 12	0.2	0.6	0.158	0.162	0.235	0.225	0.265	0.268	0.375	0.355	0.435	0.355
25 12	0.2	0.8	0.062	0.048	0.055	0.062	0.115	0.075	0.075	0.065	0.165	0.128
25 12	0.5	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.5	0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.5	0.4	0.999	0.996	0.999	1.000	0.998	1.000	0.975	0.995	1.000	1.000
25 12	0.5	0.6	0.726	0.785	0.721	0.774	0.745	0.805	0.805	0.852	0.845	0.905
25 12	0.5	0.8	0.116	0.153	0.184	0.204	0.211	0.218	0.264	0.271	0.475	0.475
25 12	0.8	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.8	0.2	1.000	1.000	1.000	1.000	1.000	1.000	0.998	1.000	1.000	1.000
25 12	0.8	0.4	1.000	1.000	1.000	1.000	0.996	1.000	0.997	1.000	0.999	1.000
25 12	0.8	0.6	0.978	1.000	0.924	1.000	0.918	0.997	0.939	1.000	0.986	0.993
25 12	0.8	0.8	0.465	0.537	0.582	0.663	0.501	0.614	0.624	0.769	0.774	0.842
49 7	0.0	0.0	0.047	0.020	0.047	0.015	0.051	0.023	0.050	0.020	0.040	0.022
49 7	0.0	0.2	0.047	0.018	0.059	0.029	0.054	0.026	0.034	0.019	0.048	0.019
49 7	0.0	0.4	0.047	0.017	0.061	0.028	0.056	0.030	0.051	0.027	0.048	0.022
49 7	0.0	0.6	0.054	0.026	0.047	0.032	0.044	0.013	0.043	0.021	0.043	0.024
49 7	0.0	0.8	0.056	0.034	0.062	0.035	0.046	0.033	0.042	0.017	0.051	0.027
49 7	0.2	0.0	0.974	0.972	0.962	0.964	0.981	0.981	0.990	0.982	0.963	0.964
49 7	0.2	0.2	0.744	0.775	0.852	0.857	0.856	0.884	0.791	0.831	0.894	0.914
49 7	0.2	0.4	0.352	0.376	0.434	0.498	0.446	0.421	0.662	0.634	0.731	0.731
49 7	0.2	0.6	0.117	0.117	0.114	0.148	0.196	0.172	0.213	0.189	0.312	0.283
49 7	0.2	0.8	0.045	0.041	0.135	0.114	0.051	0.081	0.132	0.084	0.095	0.072
49 7	0.5	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.5	0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.5	0.4	0.950	0.972	0.972	0.991	0.965	0.985	0.984	0.998	0.992	1.000
49 7	0.5	0.6	0.491	0.577	0.554	0.631	0.618	0.720	0.898	0.917	0.852	0.890
49 7	0.5	0.8	0.126	0.125	0.121	0.159	0.156	0.153	0.322	0.323	0.417	0.450
49 7	0.8	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.8	0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.8	0.4	0.999	1.000	0.998	1.000	1.000	1.000	0.999	1.000	1.000	1.000
49 7	0.8	0.6	0.898	0.947	0.863	0.946	0.877	1.000	0.955	0.971	0.997	1.000
49 7	0.8	0.8	0.339	0.398	0.374	0.392	0.536	0.615	0.696	0.766	0.926	0.928
49 12	0.0	0.0	0.042	0.021	0.041	0.019	0.050	0.023	0.060	0.028	0.047	0.018
49 12	0.0	0.2	0.050	0.016	0.047	0.023	0.051	0.024	0.049	0.019	0.037	0.017
49 12	0.0	0.4	0.050	0.023	0.053	0.023	0.047	0.021	0.043	0.023	0.045	0.017
49 12	0.0	0.6	0.053	0.022	0.046	0.022	0.061	0.032	0.046	0.016	0.052	0.029
49 12	0.0	0.8	0.049	0.025	0.053	0.023	0.051	0.030	0.050	0.028	0.052	0.024
49 12	0.2	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.2	0.2	0.979	0.972	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.2	0.4	0.824	0.839	0.853	0.841	0.849	0.836	0.842	0.849	0.833	0.827
49 12	0.2	0.6	0.358	0.354	0.365	0.382	0.402	0.413	0.452	0.465	0.499	0.488
49 12	0.2	0.8	0.075	0.053	0.127	0.118	0.136	0.109	0.245	0.176	0.208	0.193
49 12	0.5	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.5	0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.5	0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.5	0.6	0.967	0.972	0.976	0.993	0.913	0.966	0.943	0.964	0.924	0.947
49 12	0.5	0.8	0.298	0.314	0.373	0.315	0.425	0.419	0.591	0.623	0.721	0.728
49 12	0.8	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.8	0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 4 (*continued*)

N, T	η	ρ	$\lambda = 0.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
49 12	0.8	0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.8	0.6	1.000	1.000	1.000	1.000	0.985	1.000	0.987	1.000	1.000	1.000
49 12	0.8	0.8	0.647	0.725	0.829	0.987	0.820	0.947	0.943	0.989	0.998	0.997

Table 5
Two-dimensional conditional tests for (C.4) H_0^k; $\lambda = \rho = 0$ (allowing $\sigma_\mu^2 > 0$)

N, T	ρ	λ	$\eta = 0.0$		$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
25 7	0.0	0.0	0.077	0.055	0.060	0.057	0.040	0.048	0.055	0.060
25 7	0.0	0.2	0.410	0.373	0.377	0.373	0.382	0.390	0.383	0.382
25 7	0.0	0.4	0.927	0.930	0.913	0.918	0.912	0.923	0.925	0.930
25 7	0.0	0.6	1.000	1.000	1.000	1.000	0.833	0.833	0.998	0.998
25 7	0.0	0.8	1.000	1.000	1.000	1.000	0.833	0.833	1.000	1.000
25 7	0.2	0.0	0.387	0.413	0.348	0.373	0.408	0.410	0.397	0.397
25 7	0.2	0.2	0.655	0.653	0.620	0.638	0.638	0.647	0.633	0.658
25 7	0.2	0.4	0.967	0.975	0.978	0.982	0.948	0.955	0.962	0.972
25 7	0.2	0.6	1.000	1.000	1.000	1.000	0.833	0.833	1.000	1.000
25 7	0.2	0.8	1.000	1.000	1.000	1.000	0.833	0.833	1.000	1.000
25 7	0.4	0.0	0.953	0.958	0.948	0.950	0.955	0.955	0.943	0.948
25 7	0.4	0.2	0.982	0.988	0.972	0.978	0.978	0.980	0.982	0.988
25 7	0.4	0.4	1.000	1.000	0.997	0.998	0.998	0.998	1.000	1.000
25 7	0.4	0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 7	0.4	0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 7	0.6	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.0	0.0	0.052	0.049	0.047	0.043	0.057	0.063	0.047	0.050
25 12	0.0	0.2	0.735	0.717	0.728	0.734	0.768	0.768	0.738	0.742
25 12	0.0	0.4	1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.999
25 12	0.0	0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.0	0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.2	0.0	0.644	0.665	0.635	0.643	0.643	0.652	0.600	0.613
25 12	0.2	0.2	0.948	0.949	0.951	0.954	0.936	0.945	0.946	0.946
25 12	0.2	0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.2	0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.2	0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.0	0.0	0.073	0.060	0.042	0.040	0.052	0.057	0.050	0.053
49 7	0.0	0.2	0.705	0.678	0.705	0.708	0.663	0.665	0.642	0.638
49 7	0.0	0.4	0.997	0.997	0.997	0.998	1.000	1.000	0.995	0.995
49 7	0.0	0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.0	0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.2	0.0	0.848	0.867	0.675	0.672	0.640	0.632	0.612	0.610
49 7	0.2	0.2	0.937	0.935	0.930	0.943	0.912	0.917	0.920	0.922
49 7	0.2	0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998
49 7	0.2	0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.2	0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5 (*continued*)

N, T	ρ	λ	$\eta = 0.0$		$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
49 7	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.0	0.0	0.060	0.057	0.039	0.039	0.049	0.053	0.048	0.047
49 12	0.0	0.2	0.980	0.958	0.964	0.963	0.967	0.968	0.970	0.969
49 12	0.0	0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.0	0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.0	0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.2	0.0	0.904	0.907	0.898	0.899	0.897	0.898	0.900	0.908
49 12	0.2	0.2	0.998	0.998	0.999	0.998	1.000	1.000	1.000	1.000
49 12	0.2	0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.2	0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.2	0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 6
Two-dimensional conditional tests for (C.5) H_0^l; $\lambda = \sigma_\mu^2 = 0$ (allowing $\rho \neq 0$)

N, T	λ	ρ	$\eta = 0.0$		$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
25 7	0.0	0.0	0.051	0.026	0.638	0.655	0.994	0.990	1.000	1.000
25 7	0.0	0.2	0.046	0.036	0.311	0.373	0.909	0.946	0.945	1.000
25 7	0.0	0.4	0.049	0.025	0.147	0.154	0.553	0.691	0.833	0.981
25 7	0.0	0.6	0.045	0.036	0.072	0.077	0.185	0.233	0.448	0.672
25 7	0.0	0.8	0.034	0.020	0.042	0.041	0.061	0.058	0.088	0.146
25 7	0.2	0.0	0.364	0.344	0.768	0.795	0.996	1.000	1.000	1.000
25 7	0.2	0.2	0.376	0.362	0.547	0.646	0.928	0.984	0.997	1.000
25 7	0.2	0.4	0.399	0.323	0.436	0.453	0.774	0.821	0.920	0.981
25 7	0.2	0.6	0.414	0.304	0.409	0.367	0.463	0.525	0.665	0.844
25 7	0.2	0.8	0.349	0.283	0.348	0.337	0.351	0.362	0.368	0.442
25 7	0.4	0.0	0.962	0.960	0.984	0.977	1.000	1.000	1.000	1.000
25 7	0.4	0.2	0.936	0.956	0.947	0.973	0.984	1.000	0.998	1.000
25 7	0.4	0.4	0.932	0.964	0.943	0.957	0.977	0.995	0.997	1.000
25 7	0.4	0.6	0.942	0.943	0.958	0.960	0.952	0.980	0.982	0.983
25 7	0.4	0.8	0.935	0.947	0.951	0.933	0.950	0.943	0.949	0.970
25 7	0.6	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.0	0.0	0.046	0.028	0.968	0.955	1.000	1.000	1.000	1.000
25 12	0.0	0.2	0.051	0.029	0.837	0.795	1.000	1.000	1.000	1.000
25 12	0.0	0.4	0.050	0.025	0.465	0.400	0.983	0.982	1.000	1.000
25 12	0.0	0.6	0.058	0.044	0.138	0.146	0.598	0.668	0.929	0.995
25 12	0.0	0.8	0.047	0.027	0.036	0.046	0.107	0.144	0.248	0.434
25 12	0.2	0.0	0.657	0.571	0.985	0.994	1.000	1.000	1.000	1.000
25 12	0.2	0.2	0.614	0.580	0.941	0.952	1.000	1.000	1.000	1.000
25 12	0.2	0.4	0.652	0.643	0.806	0.873	0.972	0.985	1.000	1.000
25 12	0.2	0.6	0.655	0.571	0.643	0.696	0.943	0.951	1.000	1.000
25 12	0.2	0.8	0.649	0.663	0.627	0.620	0.638	0.676	0.751	0.842
25 12	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 6 (*continued*)

N, T	λ	ρ	$\eta = 0.0$		$\eta = 0.2$		$\eta = 0.5$		$\eta = 0.8$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
49 7	0.0	0.0	0.047	0.029	0.924	0.918	1.000	1.000	1.000	1.000
49 7	0.0	0.2	0.046	0.032	0.658	0.675	0.998	1.000	1.000	1.000
49 7	0.0	0.4	0.039	0.034	0.259	0.295	0.903	0.914	0.997	1.000
49 7	0.0	0.6	0.042	0.033	0.094	0.103	0.415	0.436	0.815	0.965
49 7	0.0	0.8	0.046	0.032	0.063	0.037	0.096	0.048	0.193	0.268
49 7	0.2	0.0	0.669	0.619	0.971	0.986	1.000	1.000	1.000	1.000
49 7	0.2	0.2	0.693	0.616	0.884	0.911	1.000	1.000	1.000	1.000
49 7	0.2	0.4	0.620	0.519	0.759	0.691	0.978	0.987	1.000	1.000
49 7	0.2	0.6	0.684	0.648	0.657	0.653	0.817	0.854	0.948	0.988
49 7	0.2	0.8	0.679	0.593	0.614	0.667	0.615	0.743	0.717	0.883
49 7	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.0	0.0	0.051	0.024	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.0	0.2	0.048	0.027	0.987	0.983	1.000	1.000	1.000	1.000
49 12	0.0	0.4	0.059	0.040	0.756	0.729	1.000	1.000	0.927	1.000
49 12	0.0	0.6	0.036	0.033	0.228	0.240	0.916	0.928	0.264	1.000
49 12	0.0	0.8	0.043	0.026	0.047	0.059	0.143	0.217	0.586	0.801
49 12	0.2	0.0	0.911	0.907	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.2	0.2	0.905	0.909	0.998	0.999	1.000	1.000	1.000	1.000
49 12	0.2	0.4	0.916	0.843	0.993	0.938	1.000	1.000	1.000	1.000
49 12	0.2	0.6	0.906	0.915	0.936	0.906	0.994	1.000	0.967	1.000
49 12	0.2	0.8	0.927	0.978	0.925	0.941	0.917	0.943	0.986	0.987
49 12	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

cases where there is violation of the auxiliary assumption $\sigma_\mu^2 > 0$, since in this case $\sigma_\mu^2 = 0$. The power of these conditional LM and LR tests is close to one as long as λ or ρ is larger than 0.2. For small λ (or ρ) $= 0.2$, this power improves as N, T or ρ (λ) increase.

Table 6 gives the frequency of rejections at the 5% level for the two-dimensional conditional LR and LM tests for H_0^l: $\lambda = \sigma_\mu^2 = 0$ (allowing $\rho \neq 0$). The LR test is undersized with size ranging from 2% to 4.4%, while the LM test has size between 3.4% and 5.9%. This is not significantly different from 5% except in two cases. The power of these conditional LM and LR tests is close to one as long as λ is larger than 0.2. For small $\lambda = 0.2$, this power improves as N or T or η increase. However, this increase in power with η is slow for $\rho = 0.8$, and yields low power for $T = 7$. Things improve as T increases from 7 to 12. Again this may be due to the interaction between the serial correlation due to ρ and that due to η.

Table 7 gives the frequency of rejections at the 5% level for the two-dimensional conditional LR and LM tests for H_0^m: $\sigma_\mu^2 = \rho = 0$ (allowing $\lambda \neq 0$). The LR test is undersized for only three cases when $N = 25$ and $T = 7$ and $\lambda = 0, 0.2$, and 0.4. However, the size of the LR is not significantly different from 5% for larger N or T. The LM test is properly sized in all cases but one. This is for $N = 25$, $T = 2$ and $\lambda = 0$. In all other cases, it is not significantly different from 5%. The power of these conditional LM and LR tests is close to one as long as λ or ρ is larger than 0.2. For small ρ (or η) $= 0.2$, this power improves as N or T or η (or ρ) increase.

Table 7
Two-dimensional conditional tests for (C.6) H_0^m; $\sigma_\mu^2 = \rho = 0$ (allowing $\lambda \neq 0$)

N, T	ρ	λ	$\eta = 0.0$		$\eta = 0.2$		$\eta = 0.5$	
			LM_J	LR_J	LM_J	LR_J	LM_J	LR_J
25 7	0.0	0.0	0.034	0.032	0.859	0.813	1.000	1.000
25 7	0.0	0.2	0.045	0.035	0.874	0.864	1.000	1.000
25 7	0.0	0.4	0.046	0.035	0.882	0.860	1.000	1.000
25 7	0.0	0.6	0.039	0.037	0.858	0.869	1.000	1.000
25 7	0.0	0.8	0.053	0.051	0.943	0.901	1.000	1.000
25 7	0.2	0.0	0.564	0.524	0.985	0.986	1.000	1.000
25 7	0.2	0.2	0.547	0.521	0.999	0.998	1.000	1.000
25 7	0.2	0.4	0.552	0.508	1.000	1.000	1.000	1.000
25 7	0.2	0.6	0.570	0.526	1.000	1.000	1.000	1.000
25 7	0.2	0.8	0.565	0.523	1.000	1.000	1.000	1.000
25 7	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000
25 12	0.0	0.0	0.041	0.043	0.985	0.971	1.000	1.000
25 12	0.0	0.2	0.045	0.039	0.985	0.990	1.000	1.000
25 12	0.0	0.4	0.051	0.047	0.989	0.986	1.000	1.000
25 12	0.0	0.6	0.039	0.041	0.994	0.967	1.000	1.000
25 12	0.0	0.8	0.040	0.042	0.994	0.995	1.000	1.000
25 12	0.2	0.0	0.846	0.812	0.999	0.999	1.000	1.000
25 12	0.2	0.2	0.841	0.823	0.998	1.000	1.000	1.000
25 12	0.2	0.4	0.850	0.817	1.000	1.000	1.000	1.000
25 12	0.2	0.6	0.847	0.810	1.000	1.000	1.000	1.000
25 12	0.2	0.8	0.852	0.817	1.000	1.000	1.000	1.000
25 12	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000
49 7	0.0	0.0	0.042	0.037	0.981	0.980	1.000	1.000
49 7	0.0	0.2	0.060	0.052	0.997	1.000	1.000	1.000
49 7	0.0	0.4	0.061	0.052	1.000	0.999	1.000	1.000
49 7	0.0	0.6	0.053	0.042	1.000	1.000	1.000	1.000
49 7	0.0	0.8	0.052	0.048	1.000	1.000	1.000	1.000
49 7	0.2	0.0	0.910	0.873	1.000	1.000	1.000	1.000
49 7	0.2	0.2	0.878	0.857	1.000	1.000	1.000	1.000
49 7	0.2	0.4	0.872	0.873	1.000	1.000	1.000	1.000
49 7	0.2	0.6	0.895	0.875	1.000	1.000	1.000	1.000
49 7	0.2	0.8	0.901	0.890	1.000	1.000	1.000	1.000
49 7	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.0	0.0	0.050	0.037	1.000	1.000	1.000	1.000
49 12	0.0	0.2	0.051	0.050	1.000	1.000	1.000	1.000
49 12	0.0	0.4	0.063	0.051	1.000	1.000	1.000	1.000
49 12	0.0	0.6	0.053	0.049	1.000	1.000	1.000	1.000
49 12	0.0	0.8	0.048	0.045	1.000	1.000	1.000	1.000
49 12	0.2	0.0	0.959	0.961	1.000	1.000	1.000	1.000
49 12	0.2	0.2	1.000	0.997	1.000	1.000	1.000	1.000
49 12	0.2	0.4	0.996	0.998	1.000	1.000	1.000	1.000
49 12	0.2	0.6	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.2	0.8	1.000	1.000	1.000	1.000	1.000	1.000
49 12	0.4	0.0	1.000	1.000	1.000	1.000	1.000	1.000

5. Conclusion

This paper considered a spatial panel regression model with serial correlation over time for each spatial unit and spatial dependence across these units at a particular point in time. In addition, the model allowed for heterogeneity across the spatial units through random effects. Testing for any one of these symptoms ignoring the other two is shown to lead to misleading results. The paper derived joint, conditional and marginal LM and LR tests for these symptoms and studied their performance using Monte Carlo experiments. This paper generalized the Baltagi and Li (1995) paper by allowing for spatial error correlation. It also generalized the Baltagi et al. (2003) paper by allowing for serial correlation over time. In effect, the tests derived in this paper encompass the earlier ones. Ignoring these correlations whether spatial at a point in time or serial correlation for a spatial unit over time may result in misleading inference. The paper does not consider alternative forms of spatial lag dependence and this should be the subject of future research. In addition, it is important to point out that the asymptotic distribution of our test statistics were not explicitly derived in the paper but that they are likely to hold under a similar set of primitive assumptions developed by Kelejian and Prucha (2001). The results in the paper should be tempered by the fact that the $N = 25, 49$ used in our Monte Carlo experiments may be small for a typical micro panel. Larger N will probably improve the performance of these tests whose critical values are based on their large sample distributions. However, it is well known that maximum likelihood and quasi-maximum likelihood estimation of the spatial autocorrelation coefficient λ can be computationally difficult, particularly when N is large. Hence, for the null hypotheses, H_0^i, H_0^j and H_0^m, where restricted maximum likelihood estimation of λ is required, it might be interesting to consider LM statistics derived from a generalized moment estimation framework such as that of Kapoor et al. (2006), which are computationally easier. In addition, it would be of interest to compare the finite sample performance of the LM statistics derived from a generalized moment estimation with those derived using a maximum likelihood framework via Monte Carlo experiments. We leave this for future research.

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Appendix A

A.1. Joint LM test for $\rho = \lambda = \sigma_\mu^2 = 0$

This appendix derives the joint LM test for spatial error correlation, random region effects and first-order serial correlation in the remainder error term. The null hypothesis is given by H_0^a: $\sigma_\mu^2 = \rho = \lambda = 0$. Let $\theta' = (\sigma_e^2, \sigma_\mu^2, \rho, \lambda)$. Note that the part of the information matrix corresponding to β will be ignored in computing the LM statistic, since the information matrix is block diagonal between the θ and β parameters and the first derivative with respect to β evaluated at the restricted MLE is zero. The LM statistic is given by

$$LM = \tilde{D}'_{\theta}\tilde{J}_{\theta}^{-1}\tilde{D}_{\theta}, \tag{A.1}$$

where $\tilde{D}_{\theta} = (\partial L/\partial\theta)(\tilde{\theta})$ is a 4×1 vector of partial derivatives of the likelihood function with respect to each element of θ, evaluated at the restricted MLE $\tilde{\theta}$. Also, $J_{\theta} = \mathrm{E}[-\partial^2 L/\partial\theta\partial\theta']$ is the part of the information matrix corresponding to θ, and $\tilde{J}_{\theta}$ is J_{θ} evaluated at the restricted MLE $\tilde{\theta}$. Under the null hypothesis H_0^a, the variance–covariance matrix given in (2.7) reduces to $\Omega_0 = \sigma_e^2 I_T \otimes I_N$ and the restricted MLE of β is $\tilde{\beta}_{\mathrm{OLS}}$, so that $\tilde{u} = y - X\tilde{\beta}_{\mathrm{OLS}}$ are the OLS residuals and $\tilde{\sigma}_e^2 = \tilde{u}'\tilde{u}/NT$. Hartley and Rao (1967) and Hemmerle and Hartley (1973) give a general useful formula that helps in obtaining $\tilde{D}_{\theta}$:

$$\frac{\partial L}{\partial\theta_r} = -\frac{1}{2}\,\mathrm{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\theta_r}\right] + \frac{1}{2}u'\left(\Omega^{-1}\frac{\partial\Omega}{\partial\theta_r}\Omega^{-1}\right)u \tag{A.2}$$

for $r = 1,2,3,4$. It is easy to show from (2.7) that $\partial\Omega/\partial\sigma_e^2 = V_\rho \otimes (B'B)^{-1}$, $\partial\Omega/\partial\sigma_\mu^2 = J_T \otimes I_N$ and $\partial\Omega/\partial\lambda = V \otimes (B'B)^{-1}(W'B + B'W)(B'B)^{-1}$ using the fact that $\partial(B'B)^{-1}/\partial\lambda = (B'B)^{-1}(W'B + B'W)(B'B)^{-1}$, see Anselin (1988, p. 164). $\partial V_1/\partial\rho|_{\mathrm{H}_o^a} = G$, where G is a bidiagonal matrix with bidiagonal elements all equal to one:

$$\Omega^{-1}|_{\mathrm{H}_0^a} = \frac{1}{\sigma_e^2} I_T \otimes I_N, \tag{A.3}$$

$$(\partial\Omega/\partial\sigma_e^2)|_{\mathrm{H}_0^a} = I_T \otimes I_N, \tag{A.4}$$

$$(\partial\Omega/\partial\sigma_\mu^2)|_{\mathrm{H}_0^a} = J_T \otimes I_N, \tag{A.5}$$

$$(\partial\Omega/\partial\rho)|_{\mathrm{H}_0^a} = \sigma_e^2(G \otimes I_N), \tag{A.6}$$

$$(\partial\Omega/\partial\lambda)|_{\mathrm{H}_0^a} = \sigma_e^2 I_T \otimes (W' + W). \tag{A.7}$$

This uses the fact that, under H_0^a, $B = I_N$ and $V_1 = I_T$. Using (A.2), the score with respect to each element of θ, evaluated at the restricted MLE is given by

$$\tilde{D}_1 = \begin{bmatrix} D(\tilde{\sigma}_e^2) \\ D(\tilde{\sigma}_\mu^2) \\ D(\tilde{\rho}) \\ D(\tilde{\lambda}) \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{NT}{2\tilde{\sigma}_e^2}\left(\dfrac{\tilde{u}'(J_T \otimes I_N)\tilde{u}}{\tilde{u}'\tilde{u}} - 1\right) \\ \dfrac{NT\tilde{u}'(G \otimes I_N)\tilde{u}}{2\quad\tilde{u}'\tilde{u}} \\ \dfrac{NT\tilde{u}'(I_T \otimes (W' + W))\tilde{u}}{2\quad\tilde{u}'\tilde{u}} \end{bmatrix}. \tag{A.8}$$

Using the following matrix differentiation formula given in Harville (1977):

$$J_{rs} = E\left[-\frac{\partial^2 L}{\partial\theta_r\partial\theta_s}\right] = \frac{1}{2}\,\mathrm{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\theta_r}\Omega^{-1}\frac{\partial\Omega}{\partial\theta_s}\right] \quad \text{for } r,s = 1,2,3,4. \tag{A.9}$$

One gets the information matrix under H_0^a:

$$\tilde{J}_\theta = \frac{NT}{2\tilde{\sigma}_e^4}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & T & \dfrac{2(T-1)\tilde{\sigma}_e^2}{T} & 0 \\ 0 & \dfrac{2(T-1)\tilde{\sigma}_e^2}{T} & \dfrac{2(T-1)\tilde{\sigma}_e^4}{T} & 0 \\ 0 & 0 & 0 & \dfrac{2b\tilde{\sigma}_e^4}{N} \end{bmatrix}, \tag{A.10}$$

where $b = \mathrm{tr}(W^2 + W'W)$. Note that $\tilde{J}_\theta$ is a block diagonal matrix with respect to $(\sigma_e^2, \sigma_\mu^2, \rho)$ and λ. Substituting (A.8) and (A.10) in (A.1), the resulting LM statistic is given by

$$LM_J = \frac{NT^2}{2(T-1)(T-2)}[A^2 - 4AF + 2TF^2] + \frac{N^2T}{b}H^2, \tag{A.11}$$

where $A = \tilde{u}'(J_T \otimes I_N)\tilde{u}/\tilde{u}'\tilde{u} - 1$, $F = \frac{1}{2}(\tilde{u}'(G \otimes I_N)\tilde{u}/\tilde{u}'\tilde{u})$ and $H = \frac{1}{2}(\tilde{u}'(I_T \otimes (W' + W))\tilde{u}/\tilde{u}'\tilde{u})$. Under the null hypothesis H_0^a, this LM statistic is asymptotically distributed as χ_3^2.

A.2. Joint LR test for $\rho = \lambda = \sigma_\mu^2 = 0$

This appendix derives the LR test for the joint significance of spatial error correlation, random region effects and first-order serial correlation. Using (2.7), the variance–covariance matrix can be rewritten as

$$\Omega = \sigma_e^2[(J_T \otimes I_N)\phi + V_\rho \otimes (B'B)^{-1}] = \sigma_e^2\Sigma,$$

where $\phi = \sigma_\mu^2/\sigma_e^2$, $V_\rho = (1/(1-\rho^2))V_1$, and V_1 is defined in (2.8). In this case, $\Sigma^{-1} = \Omega^{-1}/\sigma_e^2$ with $\Sigma^{*-1} = \Omega^{*-1}/\sigma_e^2$ similarly defined. Ω^* is given by (2.12). In fact,

$$\Sigma^{*-1} = \bar{J}_T^\alpha \otimes Z_0 + E_T^\alpha \otimes (B'B), \tag{A.12}$$

where $Z_0 = [d^2(1-\rho)^2\phi I_N + (B'B)^{-1}]^{-1} = \sigma_e^2 Z$. Using $\Omega^* = (C \otimes I_N)\Omega(C' \otimes I_N)$, we get

$$\begin{aligned}\Sigma^{-1} &= [C' \otimes I_N]\Sigma^{*-1}[C \otimes I_N] \\ &= V_\rho^{-1} \otimes (B'B) + \frac{1}{d^2(1-\rho)^2}(V_\rho^{-1}J_TV_\rho^{-1}) \otimes [Z_0 - (B'B)],\end{aligned} \tag{A.13}$$

where $d^2 = \alpha^2 + (T-1)$ and $\alpha = \sqrt{(1+\rho)/(1-\rho)}$. This uses the fact that $C\iota_T = (1-\rho)\iota_T^\alpha$ and $C'C = V_\rho^{-1}$. Also,

$$|\Sigma^*| = |d^2(1-\rho)^2\phi I_N + (B'B)^{-1}| \cdot |(B'B)^{-1}|^{T-1}, \tag{A.14}$$

and using

$$\Sigma = [C \otimes I_N]\Sigma^*[C' \otimes I_N] \tag{A.15}$$

we get

$$|\Sigma| = |\Sigma^*|/(1-\rho^2)^N. \tag{A.16}$$

Therefore, under the normality assumption of the disturbances, the log-likelihood function can be written as

$$\begin{aligned}L &= Const. - \frac{1}{2}\ln|\Omega^*| + \frac{1}{2}u^{*\prime}\Omega^{*-1}u^* \\ &= Const. - \frac{NT}{2}\ln\sigma_e^2 - \frac{1}{2}\ln|\Sigma^*| - \frac{1}{2\sigma_e^2}u^{*\prime}\Sigma^{*-1}u^* \\ &= Const. + \frac{N}{2}\ln(1-\rho^2) - \frac{1}{2}\ln|d^2(1-\rho)^2\phi I_N + (B'B)^{-1}| - \frac{NT}{2}\ln\sigma_e^2 \\ &\quad + (T-1)\ln|B| - \frac{1}{2\sigma_e^2}u'\Sigma^{-1}u. \end{aligned} \tag{A.17}$$

The first-order conditions give closed form solutions for $\hat{\beta}$ and $\hat{\sigma}_e^2$ conditional on $\hat{\lambda}$, $\hat{\phi}$ and $\hat{\rho}$:

$$\hat{\beta} = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}y, \tag{A.18}$$

$$\hat{\sigma}_e^2 = (y - X\beta)'\Sigma^{-1}(y - X\beta)/NT. \tag{A.19}$$

Following Hemmerle and Hartley (1973), we get from (A.11) that

$$\begin{aligned}\frac{\partial\Sigma}{\partial\rho} &= \frac{\partial}{\partial\rho}\left(\frac{1}{1-\rho^2}V_1\right)\otimes(B'B)^{-1} \\ &= \left(\frac{2\rho}{(1-\rho^2)^2}V_1 + \frac{1}{1-\rho^2}F_\rho\right)\otimes(B'B)^{-1} \\ &= \frac{1}{1-\rho^2}\left(2\rho V_\rho + F_\rho\right)\otimes(B'B)^{-1},\end{aligned} \tag{A.20}$$

where

$$F_\rho = \frac{\partial V_1}{\partial\rho} = \begin{bmatrix} 0 & 1 & 2\rho & \cdots & (T-1)\rho^{T-2} \\ 1 & 0 & 1 & \cdots & (T-2)\rho^{T-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ (T-1)\rho^{T-2} & (T-2)\rho^{T-3} & (T-3)\rho^{T-4} & \cdots & 0 \end{bmatrix}. \tag{A.21}$$

Therefore, using (A.13) and (A.20), we get

$$\begin{aligned}\frac{\partial L}{\partial\rho} &= -\frac{1}{2}\operatorname{tr}\left[\Sigma^{-1}\frac{\partial\Sigma}{\partial\rho}\right] + \frac{1}{2\sigma_e^2}u'\left(\Sigma^{-1}\frac{\partial\Sigma}{\partial\rho}\Sigma^{-1}\right)u \\ &= -N\rho/(1-\rho^2) \\ &\quad - \frac{1}{2d^2(1-\rho)^2(1-\rho^2)}[4\rho(1-\rho) + 2\rho(T-2)(1-\rho)^2 + 2(1-\rho^2)(T-1)] \\ &\quad \times\{\operatorname{tr}(d^2(1-\rho)^2\phi(B'B) + I_N)^{-1} - N\} \\ &\quad + \frac{1}{2(1-\rho^2)\sigma_e^2}\hat{u}'(\Sigma^{-1}[(2\rho V_\rho + F_\rho)\otimes(B'B)^{-1}]\Sigma^{-1})\hat{u},\end{aligned} \tag{A.22}$$

where

$$\Sigma^{-1}\frac{\partial\Sigma}{\partial\rho} = \frac{1}{1-\rho^2}(2\rho I_T + V_\rho^{-1}F_\rho)\otimes I_N + \frac{1}{d^2(1-\rho)^2(1-\rho^2)}(2\rho V_\rho^{-1}J_T + V_\rho^{-1}J_T V_\rho^{-1}F_\rho) \otimes \{(d^2(1-\rho)^2\phi(B'B)+I_N)^{-1} - I_N\}, \tag{A.23}$$

$$\mathrm{tr}\left[\Sigma^{-1}\frac{\partial\Sigma}{\partial\rho}\right] = \frac{2\rho N}{(1-\rho^2)} + \frac{1}{d^2(1-\rho)^2(1-\rho^2)} \times[4\rho(1-\rho)+2\rho(T-2)(1-\rho)^2+2(1-\rho^2)(T-1)] \times\{\mathrm{tr}(d^2(1-\rho)^2\phi(B'B)+I_N)^{-1} - N\} \tag{A.24}$$

using the fact that $\mathrm{tr}[V_\rho^{-1}J_T] = (1-\rho)\{2+(T-2)(1-\rho)\}$, $\mathrm{tr}[V_\rho^{-1}F_\rho] = -2\rho(T-1)$ and $\mathrm{tr}[V_\rho^{-1}J_T V_\rho^{-1}F_\rho] = 2(1-\rho)^2(T-1)$.

$$\Sigma^{-1}\frac{\partial\Sigma}{\partial\rho}\Sigma^{-1} = \Sigma^{-1}[(2\rho V_\rho + F_\rho)\otimes(B'B)^{-1}]\Sigma^{-1}/(1-\rho^2) = \frac{1}{1-\rho^2}\Sigma^{-1}[(2\rho V_\rho + F_\rho)\otimes(B'B)^{-1}]\Sigma^{-1}.$$

Similarly,

$$\frac{\partial L}{\partial\phi} = -\frac{1}{2}\mathrm{tr}[\Sigma^{-1}\frac{\partial\Sigma}{\partial\phi}] + \frac{1}{2\sigma_e^2}u'\Sigma^{-1}\frac{\partial\Sigma}{\partial\phi}\Sigma^{-1}u = -\frac{T}{2}\mathrm{tr}[(T\phi I_N + (B'B)^{-1})^{-1}] + \frac{1}{2\sigma_e^2}u'[J_T\otimes(T\phi I_N+(B'B)^{-1})^{-2}]u, \tag{A.25}$$

using $\partial\Sigma/\partial\phi = J_T\otimes I_N$

$$\Sigma^{-1}\frac{\partial\Sigma}{\partial\phi} = \left(V_\rho^{-1}\otimes(B'B) + \frac{1}{d^2(1-\rho)^2}(V_\rho^{-1}J_T V_\rho^{-1})\otimes[Z_0-(B'B)]\right)(J_T\otimes I_N) = V_\rho^{-1}J_T\otimes(B'B) + \frac{1}{d^2(1-\rho)^2}V_\rho^{-1}J_T V_\rho^{-1}J_T\otimes[Z_0-(B'B)],$$

$$\mathrm{tr}\left[\Sigma^{-1}\frac{\partial\Sigma}{\partial\phi}\right] = (1-\rho)\{2+(T-2)(1-\rho)\}\,\mathrm{tr}(B'B) + \frac{1}{d^2(1-\rho)^2}[(1-\rho)\{2+(T-2)(1-\rho)\}]^2\,\mathrm{tr}[Z_0-(B'B)] = \left[k_2 - \frac{k_2^2}{d^2(1-\rho)^2}\right]\mathrm{tr}(B'B) + \frac{k_2^2}{d^2(1-\rho)^2}\,\mathrm{tr}(Z_0), \tag{A.26}$$

where $k_2 = (1-\rho)\{2+(T-2)(1-\rho)\}$.

$$\Sigma^{-1}\frac{\partial\Sigma}{\partial\phi}\Sigma^{-1} = \left(V_\rho^{-1}J_T\otimes(B'B) + \frac{1}{d^2(1-\rho)^2}V_\rho^{-1}J_T V_\rho^{-1}J_T\otimes[Z_0-(B'B)]\right) \times\left(V_\rho^{-1}\otimes(B'B) + \frac{1}{d^2(1-\rho)^2}V_\rho^{-1}J_T V_\rho^{-1}\otimes[Z_0-(B'B)]\right). \tag{A.27}$$

Also,

$$\frac{\partial L}{\partial \lambda} = -\frac{1}{2}\operatorname{tr}[(T\phi I_N + (B'B)^{-1})^{-1}(B'B)^{-1}(W'B + B'W)(B'B)^{-1}] \\ -\frac{T-1}{2}\operatorname{tr}[(W'B + B'W)(B'B)^{-1}] + \frac{1}{2\sigma_e^2}u'\left[\Sigma^{-1}\frac{\partial \Sigma}{\partial \lambda}\Sigma^{-1}\right]u, \tag{A.28}$$

using

$$\frac{\partial \Sigma}{\partial \lambda} = V_\rho \otimes (B'B)^{-1}(W'B + B'W)(B'B)^{-1}, \tag{A.29}$$

$$\Sigma^{-1}\frac{\partial \Sigma}{\partial \lambda} = I_T \otimes (W'B + B'W)(B'B)^{-1} \\ + \frac{1}{d^2(1-\rho)^2}V_\rho^{-1}J_T \otimes [Z_0 - (B'B)](B'B)^{-1}(W'B + B'W)(B'B)^{-1}, \tag{A.30}$$

$$\operatorname{tr}\left(\Sigma^{-1}\frac{\partial \Sigma}{\partial \lambda}\right) = T\operatorname{tr}((W'B + B'W)(B'B)^{-1}) + \frac{1}{d^2(1-\rho)^2}(1-\rho)\{(1-\rho)(T-2)+2\} \\ \times\{\operatorname{tr}[Z_0(B'B)^{-1}(W'B + B'W)(B'B)^{-1}] \\ - \operatorname{tr}[(W'B + B'W)(B'B)^{-1}]\}. \tag{A.31}$$

The Fisher scoring procedure is used to estimate ϕ, λ and ρ. Using the formula in Harville (1977), the elements of the information matrix corresponding to ϕ, λ and ρ can be obtained from the expressions derived above. For example

$$\mathrm{E}\left[-\frac{\partial^2 L}{\partial \phi^2}\right] = \frac{1}{2}\operatorname{tr}\left[\Sigma^{-1}\frac{\partial \Sigma}{\partial \phi}\right]^2 = \frac{T^2}{2}\operatorname{tr}[\{T\phi I_N + (B'B)^{-1}\}^{-2}] \tag{A.32}$$

and

$$\mathrm{E}\left[-\frac{\partial^2 L}{\partial \rho^2}\right] = \frac{1}{2}\operatorname{tr}\left[\Sigma^{-1}\frac{\partial \Sigma}{\partial \rho}\right]^2 = \frac{T^2}{2}\operatorname{tr}[\{T\phi I_N + (B'B)^{-1}\}^{-2}]. \tag{A.33}$$

Starting with an initial value, the $(r+1)$th updated value of λ, ϕ and ρ are given by

$$\begin{bmatrix}\hat\lambda\\ \hat\phi\\ \hat\rho\end{bmatrix}_{r+1} = \begin{bmatrix}\hat\lambda\\ \hat\phi\\ \hat\rho\end{bmatrix}_{r} + \begin{bmatrix} \mathrm{E}\left[-\frac{\partial^2 L}{\partial\lambda^2}\right] & E\left[-\frac{\partial^2 L}{\partial\lambda\partial\phi}\right] & \mathrm{E}\left[-\frac{\partial^2 L}{\partial\lambda\partial\rho}\right]\\ \mathrm{E}\left[-\frac{\partial^2 L}{\partial\lambda\partial\phi}\right] & \mathrm{E}\left[-\frac{\partial^2 L}{\partial\phi^2}\right] & \mathrm{E}\left[-\frac{\partial^2 L}{\partial\phi\partial\rho}\right]\\ \mathrm{E}\left[-\frac{\partial^2 L}{\partial\lambda\partial\rho}\right] & \mathrm{E}\left[-\frac{\partial^2 L}{\partial\phi\partial\rho}\right] & \mathrm{E}\left[-\frac{\partial^2 L}{\partial\rho^2}\right] \end{bmatrix}_r^{-1} \begin{bmatrix}\frac{\partial L}{\partial\lambda}\\ \frac{\partial L}{\partial\phi}\\ \frac{\partial L}{\partial\rho}\end{bmatrix}, \tag{A.34}$$

where at each step, $\partial L/\partial\lambda$, $\partial L/\partial\phi$ and $\partial L/\partial\rho$ are obtained from Eqs. (A.28), (A.25) and (A.22). $\hat\beta$ and $\hat\sigma_e^2$ are obtained from (A.18) and (A.19), and the information matrix is obtained from equations like (A.32–A.33). The subscript r means that these terms are evaluated at the estimates of the rth iteration.

A.3. (C.1) LM test for $\mathrm{H}_0^h\colon \lambda = 0$ allowing $\rho \neq 0$ and $\sigma_\mu^2 > 0$

Under $\mathrm{H}_0^h\colon \lambda = 0$ allowing $\rho \neq 0$ and $\sigma_\mu^2 > 0$, Ω in (2.7) reduces to $\Omega_0 = \sigma_\mu^2 (J_T \otimes I_N) + \sigma_e^2 (V_\rho \otimes I_N)$ with

$$\Omega_0^{-1} = (V^{-1} - cV^{-1} J_T V^{-1}) \otimes I_N,$$

where $c = \sigma_e^2 \sigma_\mu^2 / (d^2 (1-\rho)^2 \sigma_\mu^2 + \sigma_e^2)$,

$$d^2 = \alpha^2 + (T-1), \quad \alpha = \sqrt{\frac{1+\rho}{1-\rho}},$$

$$\left.\frac{\partial \Omega}{\partial \sigma_e^2}\right|_{\mathrm{H}_0^h} = V_\rho \otimes I_N = \frac{1}{\sigma_e^2} V \otimes I_N,$$

$$\left.\Omega^{-1} \frac{\partial \Omega}{\partial \sigma_e^2}\right|_{\mathrm{H}_0^h} = \frac{1}{\sigma_e^2} (I_T - cV^{-1} J_T) \otimes I_N,$$

$$\left.\mathrm{tr}\left[\Omega^{-1} \frac{\partial \Omega}{\partial \sigma_e^2}\right]\right|_{\mathrm{H}_0^h} = N(T - cg)/\sigma_e^2,$$

where $g = \mathrm{tr}(V^{-1} J_T) = ((1-\rho)/\sigma_e^2)\{2 + (T-2)(1-\rho)\}$,

$$\begin{aligned} \left.\Omega^{-1} \frac{\partial \Omega}{\partial \sigma_e^2} \Omega^{-1}\right|_{\mathrm{H}_0^h} &= \left(\frac{1}{\sigma_e^2} (I_T - cV^{-1} J_T) \otimes I_N\right) (V^{-1} - cV^{-1} J_T V^{-1} \otimes I_N) \\ &= \frac{1}{\sigma_e^2} (V^{-1} - 2cV^{-1} J_T V^{-1} + [cV^{-1} J_T]^2 V^{-1}) \otimes I_N. \end{aligned}$$

Using (A.2), we get

$$\begin{aligned} \left.\frac{\partial L}{\partial \sigma_e^2}\right|_{\mathrm{H}_0^h} = &-\frac{N}{2\sigma_e^2} [T - cg] \\ &+ \frac{1}{2\sigma_e^2} \hat{u}' [(V^{-1} - 2cV^{-1} J_T V^{-1} + [cV^{-1} J_T]^2 V^{-1}) \otimes I_N] \hat{u}. \end{aligned}$$

Similarly,

$$\left.\frac{\partial \Omega}{\partial \sigma_\mu^2}\right|_{\mathrm{H}_0^h} = J_T \otimes I_N,$$

$$\left.\Omega^{-1} \frac{\partial \Omega}{\partial \sigma_\mu^2}\right|_{\mathrm{H}_0^h} = (V^{-1} J_T - c(V^{-1} J_T)^2) \otimes I_N,$$

$$\operatorname{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_\mu^2}\right]\Big|_{\mathrm{H}_0^h} = Ng(1-cg),$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_\mu^2}\Omega^{-1}\Big|_{\mathrm{H}_0^h} = (V^{-1}-cV^{-1}J_TV^{-1})J_T(V^{-1}-cV^{-1}J_TV^{-1})\otimes I_N.$$

Using (A.2), we get

$$\begin{aligned}\frac{\partial L}{\partial\sigma_\mu^2}\Big|_{\mathrm{H}_0^h} =& -\frac{N}{2}\left(\frac{(1-\rho)}{\sigma_e^2}\{2+(T-2)(1-\rho)\} - c\left[\frac{(1-\rho)}{\sigma_e^2}\{2+(T-2)(1-\rho)\}\right]^2\right)\\ &+\frac{1}{2}u'[(V^{-1}-cV^{-1}J_TV^{-1})J_T(V^{-1}-cV^{-1}J_TV^{-1})\otimes I_N]u,\end{aligned}$$

$$\frac{\partial\Omega}{\partial\rho}\Big|_{\mathrm{H}_0^h} = \sigma_e^2\left[\frac{2\rho}{(1-\rho^2)^2}V_1 + \frac{1}{1-\rho^2}F_\rho\right]\otimes I_N = \sigma_e^2H_\rho\otimes I_N,$$

where $H_\rho = [(2\rho/(1-\rho^2)^2)V_1 + (1/(1-\rho^2))F_\rho] = [(2\rho V_\rho + F_\rho)/(1-\rho^2)]$ and F_ρ is defined in (A.21).

$$\Omega^{-1}\frac{\partial\Omega}{\partial\rho}\Big|_{\mathrm{H}_0^h} = \sigma_e^2(V^{-1}-cV^{-1}J_TV^{-1})H_\rho\otimes I_N,$$

$$\operatorname{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\rho}\right]\Big|_{\mathrm{H}_0^h} = N\sigma_e^2\operatorname{tr}[(V^{-1}-cV^{-1}J_TV^{-1})H_\rho],$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\rho}\Omega^{-1}\Big|_{\mathrm{H}_0^h} = \sigma_e^2[(V^{-1}-cV^{-1}J_TV^{-1})H_\rho(V^{-1}-cV^{-1}J_TV^{-1})]\otimes I_N.$$

Using (A.2), we get

$$\begin{aligned}\frac{\partial L}{\partial\rho}\Big|_{\mathrm{H}_0^h} =& -\frac{N\sigma_e^2}{2}\operatorname{tr}[(V^{-1}-cV^{-1}J_TV^{-1})H_\rho]\\ &+\frac{\sigma_e^2}{2}u'[(V^{-1}-cV^{-1}J_TV^{-1})H_\rho(V^{-1}-cV^{-1}J_TV^{-1})\otimes I_N]u,\end{aligned}$$

$$\frac{\partial\Omega}{\partial\lambda}\Big|_{\mathrm{H}_0^h} = V\otimes(W'+W),$$

$$\begin{aligned}\Omega^{-1}\frac{\partial\Omega}{\partial\lambda}\Big|_{\mathrm{H}_0^h} &= (V^{-1}-cV^{-1}J_TV^{-1}\otimes I_N)(V\otimes(W'+W))\\ &= (I_T - cV^{-1}J_T)\otimes(W'+W),\end{aligned}$$

$$\operatorname{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\lambda}\right]\Big|_{\mathrm{H}_0^h} = 0,$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\lambda}\Omega^{-1}\bigg|_{\mathrm{H}_0^h} = (V^{-1} - cV^{-1}J_TV^{-1})V(V^{-1} - cV^{-1}J_TV^{-1}) \otimes (W' + W).$$

Using (A.2), we get

$$\frac{\partial L}{\partial\lambda}\bigg|_{\mathrm{H}_0^h} = \hat{D}(\lambda) = \frac{1}{2}u'[V^{-1} - 2cV^{-1}J_TV^{-1} + c^2[V^{-1}J_T]^2V^{-1}] \otimes (W' + W)u,$$

which is given in (3.5). Using (A.9), elements of the information matrix under H_0^h are given by

$$J_{11} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^2}(I_T - cV^{-1}J_T) \otimes I_N\right]^2 = \frac{N}{2\sigma_e^4}[T - 2cg + (cg)^2],$$

$$J_{12} = \frac{1}{2}\,\mathrm{tr}\left[\left(\frac{1}{\sigma_e^2}(I_T - cV^{-1}J_T) \otimes I_N\right)((V^{-1}J_T - cV^{-1}J_TV^{-1}J_T) \otimes I_N)\right] = \frac{N}{2\sigma_e^2}g(1 - cg)^2,$$

$$J_{13} = \frac{1}{2}\,\mathrm{tr}\left[\left(\frac{1}{\sigma_e^2}(I_T - cV^{-1}J_T) \otimes I_N\right)(\sigma_e^2(V^{-1} - cV^{-1}J_TV^{-1})H_\rho \otimes I_N)\right] = \frac{1}{2}\,\mathrm{tr}[V^{-1}H_\rho - 2cV^{-1}J_TV^{-1}H_\rho + c^2(V^{-1}J_T)^2V^{-1}H_\rho],$$

$$J_{14} = \frac{1}{2}\,\mathrm{tr}\left[\left(\frac{1}{\sigma_e^2}(I_T - cV^{-1}J_T) \otimes I_N\right)((I_T - cV^{-1}J_T) \otimes (W' + W))\right] = 0,$$

$$J_{22} = \frac{1}{2}\,\mathrm{tr}[(V^{-1}J_T - cV^{-1}J_TV^{-1}J_T) \otimes I_N]^2 = \frac{N}{2}g^2(1 - cg)^2,$$

$$J_{23} = \frac{1}{2}\,\mathrm{tr}[((V^{-1}J_T - cV^{-1}J_TV^{-1}J_T) \otimes I_N)(\sigma_e^2(V^{-1} - cV^{-1}J_TV^{-1})H_\rho \otimes I_N)] = \frac{N\sigma_e^2}{2}\,\mathrm{tr}[(V^{-1}J_TV^{-1} - 2c(V^{-1}J_T)^2V^{-1} + c^2(V^{-1}J_T)^3V^{-1})H_\rho],$$

$$J_{24} = \frac{1}{2}\,\mathrm{tr}[((V^{-1}J_T - c(V^{-1}J_T)^2) \otimes I_N)((I_T - cV^{-1}J_T) \otimes (W' + W)] = 0,$$

$$J_{33} = \frac{1}{2}\,\mathrm{tr}[\sigma_e^2(V^{-1} - cV^{-1}J_TV^{-1})H_\rho \otimes I_N]^2 = \frac{N\sigma_e^4}{2}\,\mathrm{tr}[(V^{-1} - cV^{-1}J_TV^{-1})H_\rho]^2,$$

$$J_{34} = \frac{1}{2}\,\mathrm{tr}[(\sigma_e^2(V^{-1} - cV^{-1}J_TV^{-1})H_\rho \otimes I_N)((I_T - cV^{-1}J_T) \otimes (W' + W))] = 0,$$

$$J_{44} = \frac{1}{2}\,\mathrm{tr}[(I_T - cV^{-1}J_T) \otimes (W' + W)]^2 = b(T - 2cg + c^2g^2),$$

where $\mathrm{tr}[V^{-1}H_\rho] = 2\rho/\sigma_e^2(1 - \rho^2)$.

A.4. LM test for H_0^i: $\rho = 0$ *allowing* $\lambda \neq 0$ *and* $\sigma_\mu^2 > 0$

Under H_0^i: $\rho = 0$ allowing $\lambda \neq 0$ and $\sigma_\mu^2 > 0$, Ω in (2.7) reduces to

$$\Omega_0 = \sigma_\mu^2 J_T \otimes I_N + \sigma_e^2 I_T \otimes (B'B)^{-1}$$

replacing J_T by $T\bar{J}_T$ and I_T by $\mathrm{E}_T + \bar{J}_T$, we get

$$\Omega_0 = \bar{J}_T \otimes [T\sigma_\mu^2 I_N + \sigma_e^2 (B'B)^{-1}] + \sigma_e^2 \mathrm{E}_T \otimes (B'B)^{-1}.$$

Hence

$$\Omega_0^{-1} = \frac{1}{\sigma_e^2}\mathrm{E}_T \otimes (B'B) + \bar{J}_T \otimes Z_0,$$

where $Z_0 = [T\sigma_\mu^2 I_N + \sigma_e^2 (B'B)^{-1}]^{-1}$ is a special case of the matrix Z defined in (2.13) when $\rho = 0$.

$$\left.\frac{\partial \Omega}{\partial \sigma_e^2}\right|_{\mathrm{H}_0^i} = I_T \otimes (B'B)^{-1},$$

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \sigma_e^2}\right|_{\mathrm{H}_0^i} = \frac{1}{\sigma_e^2}(\mathrm{E}_T \otimes I_N) + \bar{J}_T \otimes Z_0 (B'B)^{-1},$$

$$\left.\mathrm{tr}\left[\Omega^{-1}\frac{\partial \Omega}{\partial \sigma_e^2}\right]\right|_{\mathrm{H}_0^i} = \frac{(T-1)N}{\sigma_e^2} + \mathrm{tr}[Z_0 (B'B)^{-1}],$$

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \sigma_e^2}\Omega^{-1}\right|_{\mathrm{H}_0^i} = \frac{1}{\sigma_e^4}\mathrm{E}_T \otimes (B'B) + \bar{J}_T \otimes Z_0 (B'B)^{-1} Z_0.$$

Using (A.2), we get

$$\begin{aligned}\left.\frac{\partial L}{\partial \sigma_e^2}\right|_{\mathrm{H}_0^i} = & -\frac{1}{2}\left[\frac{(T-1)N}{\sigma_e^2} + \mathrm{tr}[Z_0 (B'B)^{-1}]\right] \\ & + \frac{1}{2}u'\left[\frac{1}{\sigma_e^4}\mathrm{E}_T \otimes (B'B) + \bar{J}_T \otimes Z_0 (B'B)^{-1} Z_0\right]u,\end{aligned}$$

$$\left.\frac{\partial \Omega}{\partial \sigma_\mu^2}\right|_{\mathrm{H}_0^i} = T\bar{J}_T \otimes I_N,$$

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \sigma_\mu^2}\right|_{\mathrm{H}_0^i} = \left(\frac{1}{\sigma_e^2}\mathrm{E}_T \otimes (B'B) + \bar{J}_T \otimes Z_0\right)(T\bar{J}_T \otimes I_N) = T\bar{J}_T \otimes Z_0,$$

$$\left.\mathrm{tr}\left[\Omega^{-1}\frac{\partial \Omega}{\partial \sigma_\mu^2}\right]\right|_{\mathrm{H}_0^i} = T\,\mathrm{tr}[Z_0],$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_\mu^2}\Omega^{-1}\bigg|_{\mathrm{H}_0^i} = (T\bar{J}_T\otimes Z_0)\left(\frac{1}{\sigma_e^2}\mathrm{E}_T\otimes(B'B)+\bar{J}_T\otimes Z_0\right) = T\bar{J}_T\otimes Z_0^2.$$

Using (A.2), we get

$$\frac{\partial L}{\partial\sigma_\mu^2}\bigg|_{\mathrm{H}_0^i} = -\frac{T}{2}\,\mathrm{tr}[Z_0]+\frac{T}{2}u'(\bar{J}_T\otimes Z_0^2)u,$$

$$\frac{\partial\Omega}{\partial\lambda}\bigg|_{\mathrm{H}_0^i} = \sigma_e^2 I_T\otimes(B'B)^{-1}(W'B+B'W)(B'B)^{-1},$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\lambda}\bigg|_{\mathrm{H}_0^i} = \mathrm{E}_T\otimes(W'B+B'W)(B'B)^{-1}+\sigma_e^2\bar{J}_T\otimes Z_0(B'B)^{-1}(W'B+B'W)(B'B)^{-1},$$

$$\mathrm{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\lambda}\right]\bigg|_{\mathrm{H}_0^i} = (T-1)\,\mathrm{tr}[(W'B+B'W)(B'B)^{-1}] + \sigma_e^2\,\mathrm{tr}[Z_0(B'B)^{-1}(W'B+B'W)(B'B)^{-1}],$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\lambda}\Omega^{-1}\bigg|_{\mathrm{H}_0^i} = \frac{1}{\sigma_e^2}\mathrm{E}_T\otimes(W'B+B'W)+\sigma_e^2\bar{J}_T\otimes Z_0(B'B)^{-1}(W'B+B'W)(B'B)^{-1}Z_0.$$

Using (A.2), we get

$$\frac{\partial L}{\partial\lambda}\bigg|_{\mathrm{H}_0^i} = -\frac{1}{2}((T-1)\,\mathrm{tr}[(W'B+B'W)(B'B)^{-1}]+\sigma_e^2\,\mathrm{tr}[Z_0(B'B)^{-1}(W'B+B'W)(B'B)^{-1}]) + \frac{1}{2}u'\left(\frac{1}{\sigma_e^2}\mathrm{E}_T\otimes(W'B+B'W)+\sigma_e^2\bar{J}_T\otimes Z_0(B'B)^{-1}(W'B+B'W)(B'B)^{-1}Z_0\right)u,$$

$$\frac{\partial\Omega}{\partial\rho}\bigg|_{\mathrm{H}_0^i} = \sigma_e^2 G\otimes(B'B)^{-1},$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\rho}\bigg|_{\mathrm{H}_0^i} = \mathrm{E}_T G\otimes I_N+\sigma_e^2\bar{J}_T G\otimes Z_0(B'B)^{-1},$$

$$\mathrm{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\rho}\right]\bigg|_{\mathrm{H}_0^i} = \frac{2(T-1)}{T}(\sigma_e^2\,\mathrm{tr}[Z_0(B'B)^{-1}]-N),$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\rho}\Omega^{-1}\bigg|_{\mathrm{H}_0^i} = \mathrm{E}_T G\bar{J}_T\otimes Z_0+\sigma_e^2\bar{J}_T G\bar{J}_T\otimes Z_0(B'B)^{-1}Z_0 + \frac{1}{\sigma_e^2}\mathrm{E}_T G\mathrm{E}_T\otimes(B'B)+\bar{J}_T G\mathrm{E}_T\otimes Z_0.$$

Using (A.2), we get

$$\left.\frac{\partial L}{\partial \rho}\right|_{\mathrm{H}_0^i} = \hat{D}(\rho) = -\frac{(T-1)}{T}(\sigma_e^2\,\mathrm{tr}[Z_0(B'B)^{-1}] - N)$$
$$+\frac{1}{2}u'(\mathrm{E}_T G\bar{J}_T \otimes Z_0 + \sigma_e^2\bar{J}_T G\bar{J}_T \otimes Z_0(B'B)^{-1}Z_0$$
$$+\frac{1}{\sigma_e^2}\mathrm{E}_T G\mathrm{E}_T \otimes (B'B) + \bar{J}_T G\mathrm{E}_T \otimes Z_0)u,$$

which is given by (3.8) when we substitute the restricted MLE under H_0^i. Using (A.9), the information matrix has the following elements under H_0^i:

$$J_{11} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^2}\mathrm{E}_T \otimes I_N + \bar{J}_T \otimes Z_0(B'B)^{-1}\right]^2$$
$$= \frac{1}{2}\left(\frac{(T-1)N}{\sigma_e^4} + \mathrm{tr}[Z_0(B'B)^{-1}]^2\right),$$

$$J_{12} = \frac{1}{2}\,\mathrm{tr}\left[\left(\frac{1}{\sigma_e^2}\mathrm{E}_T \otimes I_N + \bar{J}_T \otimes Z_0(B'B)^{-1}\right)(T\bar{J}_T \otimes Z_0)\right]$$
$$= \frac{1}{2}\,\mathrm{tr}[T\bar{J}_T \otimes Z_0(B'B)^{-1}Z_0] = \frac{T}{2}\,\mathrm{tr}[Z_0(B'B)^{-1}Z_0],$$

$$J_{13} = \frac{1}{2}\,\mathrm{tr}\left[\left(\frac{1}{\sigma_e^2}\mathrm{E}_T \otimes I_N + \bar{J}_T \otimes Z_0(B'B)^{-1}\right)(\mathrm{E}_T G \otimes I_N + \sigma_e^2\bar{J}_T G \otimes Z_0(B'B)^{-1})\right]$$
$$= \frac{(T-1)}{T}\left(\sigma_e^2\,\mathrm{tr}[Z_0(B'B)^{-1}]^2 - \frac{N}{\sigma_e^2}\right),$$

$$J_{14} = \frac{1}{2}\,\mathrm{tr}\left[\left(\frac{1}{\sigma_e^2}\mathrm{E}_T \otimes I_N + \bar{J}_T \otimes Z_0(B'B)^{-1}\right)(\mathrm{E}_T \otimes (W'B + B'W)(B'B)^{-1}\right.$$
$$\left.+\,\sigma_e^2\bar{J}_T \otimes Z_0(B'B)^{-1}(W'B + B'W)(B'B)^{-1})\right]$$
$$= \frac{1}{2}\left(\frac{(T-1)}{\sigma_e^2}\,\mathrm{tr}[(W'B + B'W)(B'B)^{-1}]\right.$$
$$\left.+\,\sigma_e^2\,\mathrm{tr}[(Z_0(B'B)^{-1})^2(W'B + B'W)(B'B)^{-1}]\right),$$

$$J_{22} = \frac{1}{2}\,\mathrm{tr}[T\bar{J}_T \otimes Z_0]^2 = \frac{T^2}{2}\,\mathrm{tr}[Z_0]^2,$$

$$J_{23} = \frac{1}{2}\,\mathrm{tr}[(T\bar{J}_T \otimes Z_0)(\mathrm{E}_T G \otimes I_N + \sigma_e^2\bar{J}_T G \otimes Z_0(B'B)^{-1})] = (T-1)\sigma_e^2\,\mathrm{tr}[Z_0^2(B'B)^{-1}],$$

$$J_{24} = \frac{1}{2}\,\mathrm{tr}[(T\bar{J}_T \otimes Z_0)(\mathrm{E}_T \otimes (W'B + B'W)(B'B)^{-1}$$
$$+\,\sigma_e^2\bar{J}_T \otimes Z_0(B'B)^{-1}(W'B + B'W)(B'B)^{-1})]$$
$$= \frac{T\sigma_e^2}{2}\,\mathrm{tr}[Z_0^2(B'B)^{-1}(W'B + B'W)(B'B)^{-1}],$$

$$J_{33} = \frac{1}{2}\,\mathrm{tr}[(\mathrm{E}_T G \otimes I_N + \sigma_e^2 \bar{J}_T G \otimes Z_0(B'B)^{-1})]^2$$
$$= \frac{N}{T^2}\{T^3 - 3T^2 + 2T + 2\} + \frac{2(T-1)^2\sigma_e^4}{T^2}\,\mathrm{tr}[Z_0(B'B)^{-1}]^2,$$
$$J_{34} = \frac{1}{2}\,\mathrm{tr}[(\mathrm{E}_T G \otimes I_N + \sigma_e^2 \bar{J}_T G \otimes Z_0(B'B)^{-1})(\mathrm{E}_T \otimes (W'B + B'W)(B'B)^{-1}$$
$$+ \sigma_e^2 \bar{J}_T \otimes Z_0(B'B)^{-1}(W'B + B'W)(B'B)^{-1})]$$
$$= \frac{T-1}{T}(\mathrm{tr}[\sigma_e^4\{Z_0(B'B)^{-1}\}^2(W'B + B'W)(B'B)^{-1}] - \mathrm{tr}[(W'B + B'W)(B'B)^{-1}]),$$

$$J_{44} = \frac{1}{2}\,\mathrm{tr}[\mathrm{E}_T \otimes (W'B + B'W)(B'B)^{-1}$$
$$+ \sigma_e^2 \bar{J}_T \otimes Z_0(B'B)^{-1}(W'B + B'W)(B'B)^{-1}]^2$$
$$= \frac{T-1}{2}\,\mathrm{tr}[(W'B + B'W)(B'B)^{-1}]^2 + \frac{\sigma_e^4}{2}\,\mathrm{tr}[Z_0(B'B)^{-1}(W'B + B'W)(B'B)^{-1}]^2.$$

This yields the information matrix given by (3.10) when we substitute the restricted MLE under H_0^i.

A.5. (C.3) LM test for $\mathrm{H}_0^j\colon \sigma_\mu^2 = 0$ allowing $\lambda \neq 0$ and $\rho \neq 0$

Under $\mathrm{H}_0^j\colon \sigma_\mu^2 = 0$ allowing $\lambda \neq 0$ and $\rho \neq 0$, the variance–covariance matrix in (2.7) reduces to $\Omega_0 = \sigma_e^2 V_\rho \otimes (B'B)^{-1}$ with $\Omega_0^{-1} = (1/\sigma_e^2)V_\rho^{-1} \otimes (B'B)$:

$$\left.\frac{\partial\Omega}{\partial\sigma_e^2}\right|_{\mathrm{H}_0^j} = V_\rho \otimes (B'B)^{-1},$$

$$\left.\frac{\partial\Omega}{\partial\sigma_\mu^2}\right|_{\mathrm{H}_0^j} = J_T \otimes I_N,$$

$$\left.\frac{\partial\Omega}{\partial\rho}\right|_{\mathrm{H}_0^j} = \sigma_e^2 \frac{1}{1-\rho^2}(2\rho V_\rho + F_\rho) \otimes (B'B)^{-1} = \sigma_e^2 H_\rho \otimes (B'B)^{-1},$$

$$\left.\frac{\partial\Omega}{\partial\lambda}\right|_{\mathrm{H}_0^j} = \sigma_e^2 V_\rho \otimes (B'B)^{-1}(W'B + B'W)(B'B)^{-1},$$

$$\left.\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_e^2}\right|_{\mathrm{H}_0^j} = \left(\frac{1}{\sigma_e^2} I_T \otimes I_N\right),$$

$$\left.\mathrm{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_e^2}\right]\right|_{\mathrm{H}_0^j} = \frac{NT}{\sigma_e^2},$$

$$\left.\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_e^2}\Omega^{-1}\right|_{\mathrm{H}_0^j} = \frac{1}{\sigma_e^4} V_\rho^{-1} \otimes (B'B).$$

Using (A.2), we get

$$\left.\frac{\partial L}{\partial \sigma_e^2}\right|_{\mathrm{H}_0^j} = -\frac{NT}{2\sigma_e^2} + \frac{1}{2\sigma_e^4}\hat{u}'[V_\rho^{-1} \otimes (B'B)]\hat{u} = 0.$$

Also,

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \sigma_\mu^2}\right|_{\mathrm{H}_0^j} = \frac{1}{\sigma_e^2}V_\rho^{-1}J_T \otimes (B'B),$$

$$\left.\mathrm{tr}\left[\Omega^{-1}\frac{\partial \Omega}{\partial \sigma_\mu^2}\right]\right|_{\mathrm{H}_0^j} = \frac{1}{\sigma_e^2}(1-\rho)\{2+(T-2)(1-\rho)\}\,\mathrm{tr}(B'B) = g\,\mathrm{tr}(B'B),$$

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \sigma_\mu^2}\Omega^{-1}\right|_{\mathrm{H}_0^j} = \frac{1}{\sigma_e^4}V_\rho^{-1}J_T V_\rho^{-1} \otimes (B'B)^2.$$

Using (A.2), we get

$$\left.\frac{\partial L}{\partial \sigma_\mu^2}\right|_{\mathrm{H}_0^j} = -\frac{g}{2}\,\mathrm{tr}(B'B) + \frac{1}{2\sigma_e^4}\hat{u}'[V_\rho^{-1}J_T V_\rho^{-1} \otimes (B'B)^2]\hat{u}$$

which is given in (3.12). Similarly,

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \rho}\right|_{\mathrm{H}_0^j} = \frac{1}{1-\rho^2}(2\rho I_T + V_\rho^{-1}F_\rho) \otimes I_N,$$

$$\left.\mathrm{tr}\left[\Omega^{-1}\frac{\partial \Omega}{\partial \rho}\right]\right|_{\mathrm{H}_0^j} = \frac{N}{1-\rho^2}(2\rho T - 2\rho T + 2\rho) = \frac{2\rho N}{1-\rho^2},$$

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \rho}\Omega^{-1}\right|_{\mathrm{H}_0^j} = \frac{1}{\sigma_e^2(1-\rho^2)}(2\rho V_\rho^{-1} + V_\rho^{-1}F_\rho V_\rho^{-1}) \otimes (B'B).$$

Using (A.2), we get

$$\left.\frac{\partial L}{\partial \rho}\right|_{\mathrm{H}_0^j} = -\frac{N\rho}{1-\rho^2} + \frac{1}{2\sigma_e^2(1-\rho^2)}\hat{u}'[(2\rho V_\rho^{-1} + V_\rho^{-1}F_\rho V_\rho^{-1}) \otimes (B'B)]\hat{u} = 0.$$

Finally,

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \lambda}\right|_{\mathrm{H}_0^j} = I_T \otimes (W'B + B'W)(B'B)^{-1},$$

$$\left.\mathrm{tr}\left[\Omega^{-1}\frac{\partial \Omega}{\partial \lambda}\right]\right|_{\mathrm{H}_0^j} = T\,\mathrm{tr}[(W'B + B'W)(B'B)^{-1}],$$

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \lambda}\Omega^{-1}\right|_{\mathrm{H}_0^j} = \frac{1}{\sigma_e^2}V_\rho^{-1} \otimes (W'B + B'W).$$

Using (A.2), we get

$$\left.\frac{\partial L}{\partial \lambda}\right|_{H_0^j} = -\frac{T}{2}\,\mathrm{tr}[(W'B + B'W)(B'B)^{-1}] + \frac{1}{2\sigma_e^2}\hat{u}'[V_\rho^{-1} \otimes (W'B + B'W)]\hat{u} = 0.$$

Using (A.9), elements of the information matrix under H_0^j are given by

$$J_{11} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^4} I_T \otimes I_N\right] = \frac{NT}{2\sigma_e^4},$$

$$J_{12} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^4} V_\rho^{-1} J_T \otimes (B'B)\right] = \frac{g}{2\sigma_e^2}\,\mathrm{tr}[B'B],$$

$$J_{13} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^2}\frac{1}{1-\rho^2}(2\rho I_T + V_\rho^{-1} F_\rho) \otimes I_N\right] = \frac{N\rho}{\sigma_e^2(1-\rho^2)},$$

$$J_{14} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^2} I_T \otimes (W'B + B'W)(B'B)^{-1}\right] = \frac{T}{2\sigma_e^2}\,\mathrm{tr}[(W'B + B'W)(B'B)^{-1}],$$

$$J_{22} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^4} V_\rho^{-1} J_T V_\rho^{-1} J_T \otimes (B'B)^2\right] = \frac{g^2}{2}\,\mathrm{tr}[(B'B)^2],$$

$$\begin{aligned} J_{23} &= \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^2}\frac{1}{1-\rho^2} V_\rho^{-1} J_T (2\rho I_T + V_\rho^{-1} F_\rho) \otimes (B'B)\right] \\ &= \frac{\mathrm{tr}[B'B]}{\sigma_e^2(1+\rho)}[(2-T)\rho^2 + (T-1) + \rho], \end{aligned}$$

$$\begin{aligned} J_{24} &= \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^2} V_\rho^{-1} J_T \otimes (B'B)(W'B + B'W)(B'B)^{-1}\right] \\ &= \frac{g}{2}\mathrm{tr}(W'B + B'W), \end{aligned}$$

$$\begin{aligned} J_{33} &= \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{(1-\rho^2)^2}(2\rho I_T + V_\rho^{-1} F_\rho)^2 \otimes I_N\right] \\ &= \frac{N}{(1-\rho^2)^2}(3\rho^2 - \rho^2 T + T - 1), \end{aligned}$$

$$\begin{aligned} J_{34} &= \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{1-\rho^2}(2\rho I_T + V_\rho^{-1} F_\rho) \otimes (W'B + B'W)(B'B)^{-1}\right] \\ &= \frac{\rho}{1-\rho^2}\,\mathrm{tr}[(W'B + B'W)(B'B)^{-1}], \end{aligned}$$

$$J_{44} = \frac{1}{2}\,\mathrm{tr}[I_T \otimes \{(W'B + B'W)(B'B)^{-1}\}^2] = \frac{T}{2}[\mathrm{tr}\{(W'B + B'W)(B'B)^{-1}\}^2].$$

This yields the information matrix given in (3.14).

A.6. Conditional LM test for $\mathrm{H}_0^k\colon \rho = \lambda = 0$ allowing $\sigma_\mu^2 > 0$

Under $\mathrm{H}_0^k\colon \rho = 0$ and $\lambda = 0$ allowing $\sigma_\mu^2 > 0$, Ω in (2.7) reduces to

$$\Omega_0 = \sigma_\mu^2 J_T \otimes I_N + \sigma_e^2 I_T \otimes I_N$$

which is the usual error component variance–covariance matrix with

$$\Omega_0^{-1} = \frac{1}{\sigma_1^2}(\bar{J}_T \otimes I_N) + \frac{1}{\sigma_e^2}(\mathrm{E}_T \otimes I_N),$$

where $\sigma_1^2 = T\sigma_\mu^2 + \sigma_e^2$. It is easy to check that under H_0^k,

$$\left.\frac{\partial \Omega}{\partial \sigma_e^2}\right|_{\mathrm{H}_0^k} = I_T \otimes I_N,$$

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \sigma_e^2}\right|_{\mathrm{H}_0^k} = \frac{1}{\sigma_e^2} I_T \otimes I_N + \bar{J}_T \otimes \left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_e^2}\right) I_N,$$

$$\left.\frac{\partial \Omega}{\partial \sigma_\mu^2}\right|_{\mathrm{H}_0^k} = T\bar{J}_T \otimes I_N,$$

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \sigma_\mu^2}\right|_{\mathrm{H}_0^k} = \frac{1}{\sigma_1^2} J_T \otimes I_N,$$

$$\left.\frac{\partial \Omega}{\partial \rho}\right|_{\mathrm{H}_0^k} = \sigma_e^2 G \otimes I_N,$$

where G is a bidiagonal matrix with bidiagonal elements all equal to one.

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \rho}\right|_{\mathrm{H}_0^k} = \sigma_e^2 \left(\frac{\bar{J}_T}{\sigma_1^2} + \frac{\mathrm{E}_T}{\sigma_e^2}\right) G \otimes I_N,$$

$$\left.\mathrm{tr}\left[\Omega^{-1}\frac{\partial \Omega}{\partial \rho}\right]\right|_{\mathrm{H}_0^k} = \frac{2(T-1)N}{T}\left(\frac{\sigma_e^2 - \sigma_1^2}{\sigma_1^2}\right)$$

since $\mathrm{tr}(\bar{J}_T G) = 2(T-1)/T$.

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \rho}\Omega^{-1}\right|_{\mathrm{H}_0^k} = \sigma_e^2 \left(\frac{\bar{J}_T}{\sigma_1^2} + \frac{\mathrm{E}_T}{\sigma_e^2}\right) G \left(\frac{\bar{J}_T}{\sigma_1^2} + \frac{\mathrm{E}_T}{\sigma_e^2}\right) \otimes I_N.$$

Using (A.2), we get

$$\left.\frac{\partial L}{\partial \rho}\right|_{\mathrm{H}_0^k} = \hat{D}(\rho) = \frac{(T-1)N}{T}\left(\frac{\sigma_1^2 - \sigma_e^2}{\sigma_1^2}\right) + \frac{\sigma_e^2}{2} u'\left[\left(\frac{\bar{J}_T}{\sigma_1^2} + \frac{\mathrm{E}_T}{\sigma_e^2}\right) G \left(\frac{\bar{J}_T}{\sigma_1^2} + \frac{\mathrm{E}_T}{\sigma_e^2}\right) \otimes I_N\right] u,$$

which is given by (3.16) when we substitute the restricted MLE under H_0^k. Similarly,

$$\left.\frac{\partial\Omega}{\partial\lambda}\right|_{H_0^k} = \sigma_e^2 I_T \otimes (W' + W),$$

$$\left.\Omega^{-1}\frac{\partial\Omega}{\partial\lambda}\right|_{H_0^k} = I_T \otimes (W' + W) + \left(\frac{\sigma_e^2 - \sigma_1^2}{\sigma_1^2}\right)\bar{J}_T \otimes (W' + W),$$

$$\operatorname{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\lambda}\right]\Big|_{H_0^k} = 0 \quad \text{since } \operatorname{tr}(W) = 0,$$

$$\left.\Omega^{-1}\frac{\partial\Omega}{\partial\lambda}\Omega^{-1}\right|_{H_0^k} = \left(\frac{\sigma_e^2}{\sigma_1^4}\bar{J}_T + \frac{1}{\sigma_e^2}\mathrm{E}_T\right) \otimes (W' + W).$$

Using (A.2), we get

$$\left.\frac{\partial L}{\partial\lambda}\right|_{H_0^k} = \hat{D}(\lambda) = \frac{1}{2}u'\left[\left(\frac{\sigma_e^2}{\sigma_1^4}\bar{J}_T + \frac{1}{\sigma_e^2}\mathrm{E}_T\right) \otimes (W' + W)\right]u$$

which is given by (3.17) when we substitute the restricted MLE under H_0^k. Using (A.9), the information matrix has the following elements under H_0^k:

$$J_{11} = \frac{1}{2}\operatorname{tr}\left[\frac{1}{\sigma_e^2}I_T \otimes I_N + \left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_e^2}\right)\bar{J}_T \otimes I_N\right]^2 = \frac{N}{2}\left(\frac{1}{\sigma_1^4} + \frac{T-1}{\sigma_e^4}\right),$$

$$J_{12} = \frac{1}{2}\operatorname{tr}\left[\left(\frac{1}{\sigma_e^2}I_T \otimes I_N + \left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_e^2}\right)\bar{J}_T \otimes I_N\right)\left(\frac{1}{\sigma_1^2}J_T \otimes I_N\right)\right] = \frac{NT}{2\sigma_1^4},$$

$$\begin{aligned} J_{13} &= \frac{1}{2}\operatorname{tr}\left[\left(\frac{1}{\sigma_e^2}I_T \otimes I_N + \left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_e^2}\right)\bar{J}_T \otimes I_N\right)\left(\frac{\sigma_e^2}{\sigma_1^2}\bar{J}_T G \otimes I_N + \mathrm{E}_T G \otimes I_N\right)\right] \\ &= \frac{(T-1)N}{T}\left(\frac{\sigma_e^2}{\sigma_1^4} - \frac{1}{\sigma_e^2}\right) \end{aligned}$$

using $\operatorname{tr}(G) = 0$ and $\operatorname{tr}(\bar{J}_T G) = 2(T-1)/T$.

$$\begin{aligned} J_{14} = \frac{1}{2}\operatorname{tr}\Bigg[&\left(\frac{1}{\sigma_e^2}I_T \otimes I_N + \left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_e^2}\right)\bar{J}_T \otimes I_N\right)(I_T \otimes (W' + W)) \\ &+ \sigma_e^2\left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_e^2}\right)\bar{J}_T \otimes (W' + W)\Bigg] = 0, \end{aligned}$$

$$J_{22} = \frac{1}{2}\operatorname{tr}\left[\frac{1}{\sigma_1^2}J_T \otimes I_N\right]^2 = \frac{NT^2}{2\sigma_1^4},$$

$$J_{23} = \frac{1}{2}\operatorname{tr}\left[\left(\frac{1}{\sigma_1^2}J_T \otimes I_N\right)\left(\frac{\sigma_e^2}{\sigma_1^2}\bar{J}_T G \otimes I_N + \mathrm{E}_T G \otimes I_N\right)\right] = \frac{N(T-1)\sigma_e^2}{\sigma_1^4},$$

$$J_{24} = \frac{1}{2}\,\mathrm{tr}\left[\left(\frac{1}{\sigma_1^2}J_T \otimes I_N\right)\left(I_T \otimes (W' + W) + \left(\frac{\sigma_e^2 - \sigma_1^2}{\sigma_1^2}\right)\bar{J}_T \otimes (W' + W)\right)\right] = 0,$$

$$\begin{aligned} J_{33} &= \frac{1}{2}\,\mathrm{tr}\left[\frac{\sigma_e^2}{\sigma_1^2}\bar{J}_T G \otimes I_N + \mathrm{E}_T G \otimes I_N\right]^2 \\ &= N[2a^2(T-1)^2 + 2a(2T-3) + T - 1], \end{aligned}$$

where $a = (\sigma_e^2 - \sigma_1^2)/T\sigma_1^2$, $\mathrm{tr}(G^2) = 2(T-1)$, $\mathrm{tr}(\bar{J}_T G)^2 = 4(T-1)^2/T^2$ and $\mathrm{tr}(\bar{J}_T G^2) = (4T-6)/T$.

$$\begin{aligned} J_{34} = \frac{1}{2}\,\mathrm{tr}\Bigg[&\left(\frac{\sigma_e^2}{\sigma_1^2}\bar{J}_T G \otimes I_N + \mathrm{E}_T G \otimes I_N\right) \\ &\times\left(I_T \otimes (W' + W) + \left(\frac{\sigma_e^2 - \sigma_1^2}{\sigma_1^2}\right)\bar{J}_T \otimes (W' + W)\right)\Bigg] = 0, \end{aligned}$$

$$\begin{aligned} J_{44} &= \frac{1}{2}\,\mathrm{tr}\left[I_T \otimes (W' + W) + \left(\frac{\sigma_e^2 - \sigma_1^2}{\sigma_1^2}\right)\bar{J}_T \otimes (W' + W)\right]^2 \\ &= \mathrm{tr}[W^2 + W'W]\left(\frac{\sigma_e^4}{\sigma_1^4} + (T-1)\right). \end{aligned}$$

This yields the information matrix given by (3.18) when we substitute the restricted MLE under H_0^k.

A.7. (C.5) LM test for $\mathrm{H}_0^l\colon \sigma_\mu^2 = \lambda = 0$ *allowing* $\rho \neq 0$

Under $\mathrm{H}_0^l\colon \sigma_\mu^2 = \lambda = 0$ allowing $\rho \neq 0$, Ω in (2.7) reduces to $\Omega_0 = \sigma_e^2 V_\rho \otimes I_N$ with $\Omega_0^{-1} = (1/\sigma_e^2)V_\rho^{-1} \otimes I_N$:

$$\left.\frac{\partial\Omega}{\partial\sigma_e^2}\right|_{\mathrm{H}_0^l} = V_\rho \otimes I_N,$$

$$\left.\frac{\partial\Omega}{\partial\sigma_\mu^2}\right|_{\mathrm{H}_0^l} = J_T \otimes I_N,$$

$$\left.\frac{\partial\Omega}{\partial\rho}\right|_{\mathrm{H}_0^l} = \sigma_e^2\frac{1}{1-\rho^2}(2\rho V_\rho + F_\rho) \otimes I_N,$$

$$\left.\frac{\partial\Omega}{\partial\lambda}\right|_{\mathrm{H}_0^l} = \sigma_e^2 V_\rho \otimes (W' + W),$$

$$\left.\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_e^2}\right|_{\mathrm{H}_0^l} = \left(\frac{1}{\sigma_e^2}I_T \otimes I_N\right),$$

$$\operatorname{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_e^2}\right]\Big|_{\mathrm{H}_0^l} = \frac{NT}{\sigma_e^2},$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_e^2}\Omega^{-1}\Big|_{\mathrm{H}_0^l} = \frac{1}{\sigma_e^4}V_\rho^{-1}\otimes I_N.$$

Using (A.2), we get

$$\frac{\partial L}{\partial\sigma_e^2}\Big|_{\mathrm{H}_0^l} = -\frac{NT}{2\sigma_e^2}+\frac{1}{2\sigma_e^4}\hat{u}'\left[V_\rho^{-1}\otimes I_N\right]\hat{u}=0.$$

Similarly

$$\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_\mu^2}\Big|_{\mathrm{H}_0^l} = \frac{1}{\sigma_e^2}V_\rho^{-1}J_T\otimes I_N,$$

$$\operatorname{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_\mu^2}\right]\Big|_{\mathrm{H}_0^l} = \frac{N}{\sigma_e^2}(1-\rho)\{2+(T-2)(1-\rho)\},$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_\mu^2}\Omega^{-1}\Big|_{\mathrm{H}_0^l} = \frac{1}{\sigma_e^4}V_\rho^{-1}J_T V_\rho^{-1}\otimes I_N.$$

Using (A.2), we get

$$\frac{\partial L}{\partial\sigma_\mu^2}\Big|_{\mathrm{H}_0^l} = -\frac{N}{2\sigma_e^2}(1-\rho)\{2+(T-2)(1-\rho)\}+\frac{1}{2\sigma_e^4}\hat{u}'[V_\rho^{-1}J_T V_\rho^{-1}\otimes I_N]\hat{u},$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\rho}\Big|_{\mathrm{H}_0^l} = \frac{1}{1-\rho^2}(2\rho I_T + V_\rho^{-1}F_\rho)\otimes I_N,$$

$$\operatorname{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\rho}\right]\Big|_{\mathrm{H}_0^l} = \frac{N}{1-\rho^2}(2\rho T-2\rho T+2\rho) = \frac{2\rho N}{1-\rho^2},$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\rho}\Omega^{-1}\Big|_{\mathrm{H}_0^l} = \frac{1}{\sigma_e^2(1-\rho^2)}(2\rho V_\rho^{-1}+V_\rho^{-1}F_\rho V_\rho^{-1})\otimes I_N.$$

Using (A.2), we get

$$\frac{\partial L}{\partial\rho}\Big|_{\mathrm{H}_0^l} = -\frac{N\rho}{1-\rho^2}+\frac{1}{2\sigma_e^2(1-\rho^2)}\hat{u}'[(2\rho V_\rho^{-1}+V_\rho^{-1}F_\rho V_\rho^{-1})\otimes I_N]\hat{u}=0.$$

Finally,

$$\Omega^{-1}\frac{\partial\Omega}{\partial\lambda}\Big|_{\mathrm{H}_0^l} = I_T\otimes(W'+W),$$

$$\operatorname{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\lambda}\right]\Big|_{\mathrm{H}_0^l} = 0,$$

$$\Omega^{-1}\frac{\partial\Omega}{\partial\lambda}\Omega^{-1}\Big|_{\mathrm{H}_0^l} = \frac{1}{\sigma_e^2}V_\rho^{-1}\otimes(W'+W).$$

Using (A.2), we get

$$\frac{\partial L}{\partial\lambda}\Big|_{\mathrm{H}_0^l} = \frac{1}{2\sigma_e^2}\hat{u}[V_\rho^{-1}\otimes(W'+W)]\hat{u}.$$

Using (A.9), the elements of the information matrix under H_0^l are given by

$$J_{11} = \frac{1}{2}\operatorname{tr}\left[\frac{1}{\sigma_e^4}I_T\otimes I_N\right] = \frac{NT}{2\sigma_e^4},$$

$$J_{12} = \frac{1}{2}\operatorname{tr}\left[\frac{1}{\sigma_e^4}V_\rho^{-1}J_T\otimes I_N\right] = \frac{N}{2\sigma_e^4}(1-\rho)\{2+(T-2)(1-\rho)\} = \frac{Ng}{2\sigma_e^2},$$

$$J_{13} = \frac{1}{2}\operatorname{tr}\left[\frac{1}{\sigma_e^2}\frac{1}{1-\rho^2}(2\rho I_T + V_\rho^{-1}F_\rho)\otimes I_N\right] = \frac{N\rho}{\sigma_e^2(1-\rho^2)},$$

$$J_{14} = \frac{1}{2}\operatorname{tr}\left[\frac{1}{\sigma_e^2}I_T\otimes(W'+W)\right] = 0,$$

$$J_{22} = \frac{1}{2}\operatorname{tr}\left[\frac{1}{\sigma_e^4}V_\rho^{-1}J_T V_\rho^{-1}J_T\otimes I_N\right] = \frac{Ng^2}{2},$$

$$J_{23} = \frac{1}{2}\operatorname{tr}\left[\frac{1}{\sigma_e^2}\frac{1}{1-\rho^2}V_\rho^{-1}J_T(2\rho I_T + V_\rho^{-1}F_\rho)\otimes I_N\right]$$
$$= \frac{N}{\sigma_e^2(1+\rho)}[(2-T)\rho^2+\rho+(T-1)],$$

$$J_{24} = \frac{1}{2}\operatorname{tr}\left[\frac{1}{\sigma_e^2}V_\rho^{-1}J_T\otimes(W'+W)\right] = 0,$$

$$J_{33} = \frac{1}{2}\operatorname{tr}\left[\frac{1}{(1-\rho^2)^2}(2\rho I_T + V_\rho^{-1}F_\rho)^2\otimes I_N\right]$$
$$= \frac{N}{(1-\rho^2)^2}(3\rho^2-\rho^2T+T-1),$$

$$J_{34} = \frac{1}{2}\operatorname{tr}\left[\frac{1}{1-\rho^2}(2\rho I_T + V_\rho^{-1}F_\rho)\otimes(W'+W)\right] = 0,$$

$$J_{44} = \frac{1}{2}\operatorname{tr}[I_T\otimes(W'+W)^2] = \frac{T}{2}\operatorname{tr}[\{(W'+W)\}^2] = Tb.$$

This yields the information matrix given in (3.23).

A.8. (C.6) LM test for $\mathrm{H}_0^m\colon \sigma_\mu^2=\rho=0$ allowing $\lambda\neq 0$

Under $\mathrm{H}_0^m\colon \sigma_\mu^2=\rho=0$ allowing $\lambda\neq 0$, the variance–covariance matrix in (2.7) reduces to $\Omega_0=\sigma_e^2 I_T\otimes(B'B)^{-1}$ and $\Omega_0^{-1}=(1/\sigma_e^2)I_T\otimes(B'B)$.

$$\left.\frac{\partial\Omega}{\partial\sigma_e^2}\right|_{\mathrm{H}_0^m}=I_T\otimes(B'B)^{-1},$$

$$\left.\frac{\partial\Omega}{\partial\sigma_\mu^2}\right|_{\mathrm{H}_0^m}=J_T\otimes I_N,$$

$$\left.\frac{\partial\Omega}{\partial\rho}\right|_{\mathrm{H}_0^m}=\sigma_e^2 G\otimes(B'B)^{-1},$$

$$\left.\frac{\partial\Omega}{\partial\lambda}\right|_{\mathrm{H}_0^m}=I_T\otimes(B'B)^{-1}(W'B+B'W)(B'B)^{-1}$$

with

$$\left.\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_e^2}\right|_{\mathrm{H}_0^m}=\left(\frac{1}{\sigma_e^2}I_T\otimes I_N\right),$$

$$\left.\mathrm{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_e^2}\right]\right|_{\mathrm{H}_0^m}=\frac{NT}{\sigma_e^2},$$

$$\left.\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_e^2}\Omega^{-1}\right|_{\mathrm{H}_0^m}=\frac{1}{\sigma_e^4}I_T\otimes(B'B).$$

Using (A.2), we get

$$\left.\frac{\partial L}{\partial\sigma_e^2}\right|_{\mathrm{H}_0^m}=-\frac{NT}{2\sigma_e^2}+\frac{1}{2\sigma_e^4}\hat{u}'[I_T\otimes(B'B)]\hat{u}=0.$$

Similarly,

$$\left.\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_\mu^2}\right|_{\mathrm{H}_0^m}=\frac{1}{\sigma_e^2}J_T\otimes(B'B),$$

$$\left.\mathrm{tr}\left[\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_\mu^2}\right]\right|_{\mathrm{H}_0^m}=\frac{T}{\sigma_e^2}\mathrm{tr}[B'B],$$

$$\left.\Omega^{-1}\frac{\partial\Omega}{\partial\sigma_\mu^2}\Omega^{-1}\right|_{\mathrm{H}_0^m}=\frac{1}{\sigma_e^4}J_T\otimes(B'B)^2.$$

Using (A.2), we get

$$\left.\frac{\partial L}{\partial \sigma_\mu^2}\right|_{\mathrm{H}_0^m} = -\frac{T}{2\sigma_e^2}\,\mathrm{tr}[B'B] + \frac{1}{2\sigma_e^4}\hat{u}'[J_T \otimes (B'B)^2]\hat{u}$$

which is (3.26).

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \rho}\right|_{\mathrm{H}_0^m} = G \otimes I_N,$$

$$\left.\mathrm{tr}\left[\Omega^{-1}\frac{\partial \Omega}{\partial \rho}\right]\right|_{\mathrm{H}_0^m} = 0,$$

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \rho}\Omega^{-1}\right|_{\mathrm{H}_0^m} = \frac{1}{\sigma_e^2}G \otimes (B'B).$$

Using (A.2), we get

$$\left.\frac{\partial L}{\partial \rho}\right|_{\mathrm{H}_0^m} = \frac{1}{2\sigma_e^2}\hat{u}'[G \otimes (B'B)]\hat{u}$$

which is (3.27). Finally,

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \lambda}\right|_{\mathrm{H}_0^m} = \frac{1}{\sigma_e^2}I_T \otimes (W'B + B'W)(B'B)^{-1},$$

$$\left.\mathrm{tr}\left[\Omega^{-1}\frac{\partial \Omega}{\partial \lambda}\right]\right|_{\mathrm{H}_0^m} = \frac{T}{\sigma_e^2}\mathrm{tr}[(W'B + B'W)(B'B)^{-1}],$$

$$\left.\Omega^{-1}\frac{\partial \Omega}{\partial \lambda}\Omega^{-1}\right|_{\mathrm{H}_0^m} = \frac{1}{\sigma_e^4}I_T \otimes (W'B + B'W).$$

Using (A.2), we get

$$\left.\frac{\partial L}{\partial \lambda}\right|_{\mathrm{H}_0^m} = \frac{T}{2\sigma_e^2}\mathrm{tr}[(W'B + B'W)(B'B)^{-1}] + \frac{1}{2\sigma_e^4}\hat{u}'[I_T \otimes (W'B + B'W)]\hat{u} = 0.$$

Using (A.9), the elements of the information matrix under H_0^m are given by

$$J_{11} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^4}I_T \otimes I_N\right] = \frac{NT}{2\sigma_e^4},$$

$$J_{12} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^4}J_T \otimes (B'B)\right] = \frac{T}{2\sigma_e^4}\,\mathrm{tr}[B'B],$$

$$J_{13} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^2}G \otimes I_N\right] = 0,$$

$$J_{14} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^4} I_T \otimes (W'B + B'W)(B'B)^{-1}\right] = \frac{T}{2\sigma_e^4}\,\mathrm{tr}[(W'B + B'W)(B'B)^{-1}],$$

$$J_{22} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^4} J_T^2 \otimes (B'B)^2\right] = \frac{T^2}{2\sigma_e^4}\,\mathrm{tr}[(B'B)^2],$$

$$J_{23} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^2}(J_T \otimes (B'B))(G \otimes I_N)\right] = \frac{T-1}{\sigma_e^2}\,\mathrm{tr}[B'B],$$

$$J_{24} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^4}(J_T \otimes (B'B))(I_T \otimes (W'B + B'W)(B'B)^{-1})\right] = \frac{T}{2\sigma_e^4}\,\mathrm{tr}[W'B + B'W],$$

$$J_{33} = \frac{1}{2}\,\mathrm{tr}[G^2 \otimes I_N] = N(T-1),$$

$$J_{34} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^2}(G \otimes I_N)(I_T \otimes (W'B + B'W)(B'B)^{-1})\right] = 0,$$

$$J_{44} = \frac{1}{2}\,\mathrm{tr}\left[\frac{1}{\sigma_e^4} I_T \otimes \{(W'B + B'W)(B'B)^{-1}\}^2\right] = \frac{T}{2\sigma_e^4}\,\mathrm{tr}[\{(W'B + B'W)(B'B)^{-1}\}^2].$$

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