A Rolling Horizon Auction Mechanism and Virtual Pricing of Shipping Capacity for Urban Consolidation Centers

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Abstract

A number of cities around the world have adopted urban consolidation centers (UCCs) to address some challenges of their last-mile deliveries. At the UCC, goods are consolidated based on their destinations prior to their deliveries into the city center. Typically, a UCC owns a fleet of eco-friendly vehicles to carry out deliveries. A shipper/carrier who buys the UCC's service hence no longer needs to enter the city center in which time-window and vehicle-type restrictions may apply. As a result, it becomes possible to retain the use of large trucks for the economies of scale outside the city center. Furthermore, time which would otherwise be spent in the city center can then be used to deliver more orders. With possibly tighter regulation and thinning profit margin in near future, requests for UCC's services shall become more and more common and there is a need for a mechanism to allocate UCC's resources to provide sustainable services for shippers/carriers. Handoko et al. (2014) proposed a profit-maximizing auction mechanism for the use of UCC's last-mile delivery service. In this paper, we extend that work with the idea of a rolling horizon to give bidders greater flexibility in competing for the UCC's resources in advance. In particular, it addresses the challenge that many shippers/carriers plan their deliveries many weeks ahead, and simultaneously allows last-minute bidders to compete for the UCC's resources. Under a rolling horizon framework, shipping capacity of the same truck is bid in several successive auctions. To allocation of truck capacities among these auctions, we also propose a virtual pricing method which makes use of Target-oriented Robust Optimization techniques to address future demand uncertainty.

1 Introduction

Last-mile deliveries in urban areas exert serious pressures on environmental, social, and economic well-being of a city. These three aspects are usually referred to as *planet*, *people*, and *profit* Quak and Tavasszy (2011). On the planet, the impacts are contributed by the use of unsustainable natural resources like the fossil fuel. On the people, the impacts are primarily due to air pollution and noise. On the profit, the impacts include economic losses because of traffic congestion and low utilization of transport vehicles. Addressing these issues, local authorities may then impose time-window or vehicle-type restriction. The earlier complicates the scheduling of the last-mile deliveries from the perspective of carriers/shippers. Quite-so-often, wait time becomes inevitably necessary. Efficiency of the deliveries has thus been compromised. The latter, on the other hand, forces the carriers/shippers to operate small eco-friendly trucks for deliveries into the city center. These trucks, however, are not efficient for long-distance inter-city transport. It is then clear that one aspect may be affected while addressing the others. Both the time-window and the vehicle-type restrictions affect the profit while trying to address the planet and the people. This prompts carriers/shippers to collaborate and consolidate shipments for greater efficiency.

The urban consolidation center (UCC) is an alliance concept where oders served by various participating carriers get consolidated at the UCC. First, they are sorted according to their destination addresses. Then, they are assigned to a sufficient number of vehicles for the actual last-mile deliveries. The cost savings obtained are finally shared among the relevant carriers. As a consequence, higher truck utilization is attained, fewer trucks are required, and lower delivery cost is incurred. This effectively addresses the potential inefficiency due to the time-window restriction. The possible wait time suffered by those carriers assigned to carry out the consolidated last-mile deliveries is compensated by the savings attained by those carriers that no longer need to enter the city center. A fair allocation of the total savings earned among participating carriers enhances the profit.

To-date, there have been a number of UCC establishments with their own transport vehicles that are in compliance with the rules and regulations set by local authorities. These UCCs provide last-mile delivery service at a charge. Occasionally, the UCCs may be governments' initiatives or pilot runs and provide last-mile delivery service free-of-charge. In essence, carriers/shippers can simply drop their loads off at the UCCs and pay the UCCs accordingly to get the loads delivered into the city center. Examples of these UCCs are La Petite Reine in Paris, France, Westfield Consolidation Center in London, and Binnenstadservice.nl in Nijmegen, the Netherlands. This addresses not only the time-window but also the vehicle-type restrictions. By using the UCCs' service, carriers/shippers no longer need to enter the city center. Retaining the use of large trucks for the economies of scale outside the city center thus becomes possible. Besides, the time which would otherwise be spent in the city center may then be used to deliver more orders. With these incentives, requests for using the UCCs' service would intuitively become more common. The UCCs could soon receive more demands than what they are capable of serving.

To our knowledge, most—if not all—UCCs operate with some fixed-rate mechanism on a first-come-first-serve basis. We found no literature discussing the automatic matching of orders to the available fleets of UCCs' transport vehicles for the efficient last-mile deliveries. Handoko et al. (2014) proposed an auction mechanism for the last-mile delivery via the UCC. Compared to the fixed-rate mechanism, the proposed auction is distinctively aimed at achieving both operational efficiency and economic viability—both of which are important for the sustainability of the UCC.

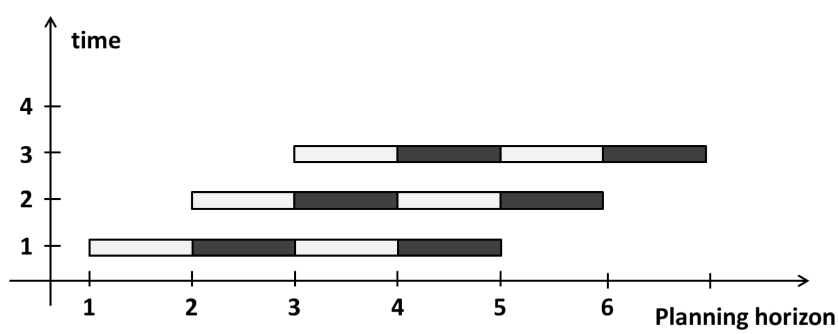


Figure 1: A rolling horizon framework.

The basic auction mechanism proposed in (Handoko et al., 2014) is however quite restrictive in that bidders are only allowed to compete for the UCC's resources in the immediate period following the winner determination. In that paper, a period of one week was observed. Indeed, one can argue that this is somewhat unrealistic as many shippers/carriers plan for their deliveries far in advance. To address this issue, not only the period needs to be lengthened but the auction needs to be conducted over a rolling horizon. This is as illustrated in Figure 1. Consider a planning horizon of 4 weeks and the UCC starts Auction #1 at the beginning of Week #0 and accept bids for deliveries in Week #1 to Week #4. Prior to the start of Week #1, the UCC determines the winning bids for Auction #1. The UCC then starts Auction #2 at the beginning of Week #1 and accept bids for deliveries in Week #2 to Week #5, and so on. This gives greater flexibility for the bidders in that the participating shippers/carriers can choose to bid far in advance or at the last minute. Unlike the basic UCC auction model, there is an overlap in the planning horizons of the UCC between two consecutive auctions. Intuitively, there may be only a few bids for deliveries in Week #4 in Auction #1. Profitable consolidation may thus be impossible at the time the winners of Auction #1 is determined. However, there should be more bids to come for deliveries in Week #4 in Auction #2 to Auction #4. Hence, profitable consolidation may in fact be possible after the upcoming auctions. This suggests that the UCC needs to be able to anticipate the potential revenue due to future bids in the upcoming auctions. For deliveries in Week #3, there may be enough bids to consolidate but some of the bids have low bid prices. Rather than accepting bids with low value to make profitable consolidation, it could be better for the UCC to accept only highly profitable bids in the current auction in the anticipation of other highly profitable bids in the upcoming auctions.

Our contribution in this paper is an auction mechanism with a rolling horizon that determines which demands are to be served in the anticipation of future demands. This is achieved by virtually pricing truck capacities so that only highly profitable bids are selected and the truck capacities are reserve for upcoming auctions. To our knowledge, this is the first auction with rolling horizon in the context of last-mile deliveries via the urban consolidation center. Our second contribution is to propose an approach determining the virtual prices of truck capacities in the face of uncertain future demand. The proposed approach makes use of Target-oriented Robust Optimization techniques, and can determine a solution that is robust again demand uncertainties. Note that this is not a trivial problem, since the price should not be too conservative nor too optimistic to maximize the profit of UCC. We then verify this through computational experiments.

The remaining of this paper is then organized as follows. Section 2 briefly reviews some related works on auction in the logistics. Section 3 elaborates the basic auction mechanism presented in (Handoko et al., 2014) and forms the basis of our extension described in this paper. Section 4 proposes the auction mechanism in elaborative manner. Mathematical formulation of the augmented winner determination problem is also presented therein. Section 5 describes a virtual price determination approach using robust auction techniques. Finally, Section 6 concludes the paper.

2 Related Works

Auction has been commonly used in the logistics context. Solving winner determination problems in logistics auctions is similar to solving scheduling problems in order to minimize certain transportation cost. Combining the services provided by different providers to fulfill some deliveries can be modeled as the set-partitioning (Song and Regan, 2003) or lane-covering problems (Agarwal and Ergun, 2010; Özener and Ergun, 2008). Typically, the model is an Mixed-Integer Program (MIP) with an objective of minimizing the cost subject to constraints on delivery time, capacity, and network structure. Such an MIP, when optimally solved, guarantees a least-cost solution preferred by decision makers. However, it is usually computationally expensive even for medium-sized problems. A linear relaxation may be used to come up with a feasible solution in polynomial time (Özener and Ergun, 2008). A greedy algorithm can also be used to provide an efficient way for a procurement schedule (Agarwal and Ergun, 2008). A greedy approach is first used to construct an initial sub-optimal solution to different scheduling components. The Benders- or column-generation-based algorithm is then used to optimize the combination of the lanes. A column-generation-based algorithm solves some form of a restricted problem with a set of selected columns, reducing the size of the original problem considerably. Benders-based algorithm, also known as the row-generation-based algorithm, solves optimization problems in two stages. In the first stage, the master problem is solved to formulate some constraints for sub-problems. In the second stage, scheduling solution is identified for each sub-problem. Note that despite the numerous literature on logistics auction, we found none pertaining to the use of the UCC. Furthermore, the concept of rolling horizon (Sethi and Sorger, 1991) has been extensively used for decision making(Mula et al., 2006; Chand et al., 2002; Ouelhadj and Petrovic, 2009) . Recently, the rolling horizon concept has also been adopted in transportation and logistics context (Berbeglia et al., 2010; Wang and Kopfer, 2013; Andersson et al., 2010). However as long as UCC is concerned, we believe no work has been done regarding an auction of shipping capacities with a rolling planning horizon.

In contrast to traditional auctions where each item for sale is typically one entity and all belongs to one winner once sold, the auction for truck capacities of UCC differs in some ways. First, the capacity of one truck load may be shared by several winning bidders. Second in the case of auction with a rolling planning horizon, bidders in different auctions may compete for the same truck capacity and the truck capacity is gradually assigned in several auctions. Such features make it necessary to develop a method for reserving truck capacity. In this paper, we consider a dynamic virtual pricing approach for this purpose. The literature of dynamic pricing is rich and expanding fast, especially in the area of logistics and inventory control (Elmaghraby and Keskinocak, 2003; Bitran and Caldentey, 2003). In the context of UCC, the pricing of truck capacity is primarily for the reservation of truck capacity and the "price" is never release to bidders. Therefore, this problem is different from most of the problems considered in the literature and that is why we call it virtual pricing. In this work, we determine the optimal price using robust optimization techniques. Adida and Perakis (2007) proposed a dynamic pricing approach using robust optimization techniques, but it is for a nonlinear continuous time inventory control problem. Our approach is also different in the sense that we propose a target value for the profit of UCC and maximize the uncertainty set that can be accommodated.

3 Problem description and Auction mechanism for UCC

We consider a UCC operating her own storage resources and delivery trucks. By consolidating the customers' packages into truckloads to the city center, the UCC can achieve economics of scale to reduce the total delivery cost, which benefits herself and all the costumers. This consolidation effort also mitigates the negative impact on the city's environment by reducing traffic in the city center.

In the value chain of the UCC, the packages originate from a *shipper* (for example, a manufacturer). They are transported to the UCC by the shipper himself or by a *carrier* (for example, a logistics service provider). The UCC consolidates them with other packages and then delivers them to a *receiver* (for example, a retailer, a restaurant, or a hotel in the city center). This last segment of distribution is also known as *the last-mile delivery.* Since the UCC is not obliged to deliver the packages for all the shippers and carriers,Handoko et al. (2014) proposed an auction mechanism for the UCC to select the packages to deliver.

3.1 Auction Protocol

We assume there are Z zones indexed as $j = 1, \dots, Z$ in the city center. The UCC operates K trucks indexed as $k = 1, \dots, K$ to deliver packages to these zones. Assume there are T periods (for example, each period represents a day) in the planning horizon. Each truck k has volume capacity V_k^t in period t. To plan for the last-mile deliveries in its nearest upcoming planning horizon, at the start of the planning horizon each shipper or carrier is invited to submit a bid for his package to be delivered to the city center. Each bid i specifies the following information in a tuple:

$$[v_i, d_i, a_i, \ell_i, p_i]$$

where

- v_i is the volume of the package,
- the zone $d_i \in [1, Z]$ that the package is to be delivered to,
- the arrival period $a_i \in [1, T]$ of the package to the UCC,
- the deadline period $l_i \in [1, T]$ of delivery to its receiver,
- the bid price p_i of the package.

Let B denote the total number of bids when the auction is close (after that no more bids are accepted). Based on this information, the UCC selects the packages to serve subject to her trucks' capacity constraints. She then notifies the bidders about the result of the auction and arranges the deliveries accordingly.

3.2 Winner Determination

To determining which bids are to be served such that the profit of the UCC over its planning horizon is maximized, a winner determination problem is to solve. To model this problem and determine the winning bids mathematically, we define x_{ik}^t as a decision variable that equals 1 if bid i is delivered by truck k in period t, and equals 0 otherwise. We also define y_{jk}^t as decision variable that equals 1 if truck k delivers to zone j in period t, and equals 0 otherwise. For notational convenience, define $\mathcal{B} := \{1, 2, \ldots, B\}$, $\mathcal{Z} := \{1, 2, \ldots, Z\}$, $\mathcal{K} := \{1, 2, \ldots, K\}$, and $\mathcal{T} := \{1, 2, \ldots, T\}$. We have the following constraints.

Constraint (i): Each bid is served at most once and this can be represented as

$$\sum_{k \in \mathcal{K}, t \in \mathcal{T}} x_{ik}^t \leq 1, \qquad \forall i \in \mathcal{B}. \tag{1}$$

Constraint (ii): Each truck serves at most one zone in one period and this can be represented as

$$\sum_{j \in \mathcal{Z}} y_{jk}^t \leq 1, \qquad \forall t \in \mathcal{T}, \forall k \in \mathcal{K}. \tag{2}$$

Constraint (iii): The truck capacity constraint can be expressed as

$$\sum_{i \in \mathcal{B}} v_i x_{ik}^t \leq V_k^t, \qquad \forall t \in \mathcal{T}, \forall k \in \mathcal{K}. \tag{3}$$

Constraint (iv): A truck will visit zone d_i in period t if bid i is served by the truck in period t, which can be expressed as

$$x_{ik}^t \leq y_{jk}^t, \qquad \text{for } j = d_i, \forall t \in \mathcal{T}, \forall i \in \mathcal{B}, \forall k \in \mathcal{K}. \tag{4}$$

Constraint (v): Each bid should be served within its delivery time window, which can be expressed as

$$x_{ik}^t = 0, \qquad \forall t \notin [a_i, l_i], \forall i \in \mathcal{B}, \forall k \in \mathcal{K}. \tag{5}$$

Constraint (vi): The binary decision variables can be expressed as

$$x_{ik}^t, y_{jk}^t \in \{0, 1\}, \qquad \forall i \in \mathcal{B}, \forall j \in \mathcal{Z}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T}. \tag{6}$$

For notational convenience, we define the following sets of decision variables

$$\mathbf{X} := \left\{x_{ik}^t, i \in \mathcal{B}, k \in \mathcal{K}, t \in \mathcal{T}\right\}, \tag{7a}$$

$$\mathbf{Y} := \left\{y_{jk}^t, j \in \mathcal{Z}, k \in \mathcal{K}, t \in \mathcal{T}\right\}. \tag{7b}$$

We define the set of feasible solutions as

$$\mathbb{F} := \{(\mathbf{X}, \mathbf{Y}) | \mathbf{X}, \mathbf{Y} \text{ satisfy (1)-(6)}\}. \tag{8}$$

We assume the cost of the UCC consists of two major components: the warehousing cost and the delivery cost. If a package is stored in the UCC before its delivery, it incurs a holding cost per volume per period h for the UCC. For truck k to deliver to zone j in period t, a delivery cost c_{jk}^t is incurred. Given a solution $(\mathbf{X}, \mathbf{Y})$ and the delivery costs c_{jk}^t, the profit of UCC can be expressed as

$$r(\mathbf{X}, \mathbf{Y}) := \sum_{i \in \mathcal{B}, k \in \mathcal{K}, t \in \mathcal{T}} [p_i - h v_i (t - a_i)] \, x_{ik}^t - \sum_{j \in \mathcal{Z}, k \in \mathcal{K}, t \in \mathcal{T}} c_{jk}^t y_{jk}^t. \tag{9}$$

The basic *Winner Determination Problem* is to

$$\max r(\mathbf{X}, \mathbf{Y}) \tag{10a}$$

$$s.t. \ (\mathbf{X}, \mathbf{Y}) \in \mathbb{F}. \tag{10b}$$

In the rest of this paper, we propose an auction with rolling horizon based on the above formulation.

Table 1 summarizes the parameters and the notation used in this paper. The rest of the paper is organized as follows. Section 4 extends the basic Winner Determination Problem and proposes an auction with rolling horizon. Section 5 introduces two virtual pricing approaches for the choice of key parameters in the auction with rolling horizon. Section 6 concludes this paper.

Table 1: **Notation**

Sets of indices	$\mathcal{B} =$	$\{1, 2, \ldots, B\}$, where B is the number of bids
	$\mathcal{Z} =$	$\{1, 2, \ldots, Z\}$, where Z is the number of zones
	$\mathcal{K} =$	$\{1, 2, \ldots, K\}$, where K is the number of trucks
	$\mathcal{T} =$	$\{1, 2, \ldots, T\}$, where T is the number of planning time periods
Parameters	h:	holding cost per volume per period of the UCC
	V_k^t:	capacity of truck k in period t
	c_{jk}^t:	delivery cost for truck k to visit zone j in period t
	v_i:	volume of bid i
	d_i:	destination of bid i, $d_i \in \mathcal{Z}$
	a_i:	arrival time of bid i, $a_i \in \mathcal{T}$
	l_i:	delivery deadline of bid i, $l_i \in \mathcal{T}$
	p_i:	bid price of bid i
Decision variables	x_{ik}^t:	binary variable, which equals 1 iff bid i is served by truck k in time period t
	y_{jk}^t:	binary variable, which equals 1 iff truck k visits zone j in time period t
Sets of variables	$\mathbf{X} =$	$\{x_{ik}^t, i \in \mathcal{B}, k \in \mathcal{K}, t \in \mathcal{T}\}$
	$\mathbf{Y} =$	$\left\{y_{jk}^t, j \in \mathcal{Z}, k \in \mathcal{K}, t \in \mathcal{T}\right\}$

4 Proposed UCC Auction with Rolling Horizon

Winner determination problem for the basic UCC auction elaborated in Section 3 assumes that the planning horizons of two consecutive UCC auctions never overlap one another. As established in Section 1, this restricts shippers/carriers as the bidders to compete only for the UCC's delivery resources in the immediate period following the winner determination.

In practice, longer planning horizon is often desirable so as to allow shippers/carriers to bid for delivery resources not only in the immediate period but also in the subsequent few periods following the winner determination. Announcement of the results of each auction—and hence, determination of the winning and the losing bids—should remain as frequent so that the losing bidders could have a chance to arrange for some other means of delivery or to alter their bid prices and resubmit their bids in the subsequent auction. This gives rise to the UCC auction with a rolling horizon. The requirements of longer planning horizon and high-frequency update makes the implementation of rolling horizon in the UCC auction an interesting and significant topic.

As illustrated earlier in Figure 1, winners for the multiple consecutive delivery periods across the planning horizon are determined simultaneously at the end of each auction before the start of the next auction. Upon closing Auction #1, bids for potential deliveries at any days in Week #1 to Week

#4 could have been received and the corresponding winners are determined simultaneously. While the committed deliveries in Week #1 are carried out, Auction #2 is accepting bids for the potential deliveries at any days in Week #2 to Week #5. At the closure of Auction #2, the winners for deliveries in Week #2 to Week #5 are determined. Committed deliveries in Week #2 are then carried out while Auction #3 accepts bids for deliveries in Week #3 to Week #6. The cycle then continues. From this illustration, it is clear that the UCC's delivery resources in one period are considered in a number of auctions altogether. The delivery resources for Week #3, for instance, are considered in 3 consecutive auctions. When determining the winners of Auction #1, the auctioneer may intuitively wish to reserve some capacity for profitable bids yet to come in Auctions #2 and #3 for deliveries in Week #3. When determining the winners of Auction #3, however, it is intuitive to use as much remaining capacity as possible since capacity left unused will no longer have any potential value.

Motivated by this, we propose herein an augmentation to the profit expression of the winner determination problem (10) . This augmentation aims at pricing the unused capacity with its potential to be allocated to more profitable bids in upcoming auctions. This is equivalent to introducing virtual bids to the current auction, which could potentially be replaced by real bids of equal or higher values in the future auctions. Other than reserving capacity for highly profitable future bids, such augmentation additionally allows selection of few profitable bids in the current auction as the winners although there may not be enough bids to realize profitable consolidation at the moment.

Precisely, we adjust the profit function as follows. Let q_{kz}^t denote the potential value of one unit of unused truck capacity if truck k delivers to zone z at day t and hereafter we refer to it as *virtual price*. Also Let V_k^t denote the remaining capacity of truck k at day t. If $y_{kz}^t = 1$, the potential value for the remaining capacity of the truck after the auction is $q_{kz}^t(V_k^t - \sum_i v_i x_{ik}^t)$, and 0 otherwise. Additionally if truck k does not deliver to any zone in period t, we assume the potential value is the average potential value of the full truckload $(\sum_z q_{kz}^t/Z)V_k^t$. Hence, the profit after adjustment can be expressed as

$$\begin{aligned}\hat{r}(\mathbf{X},\mathbf{Y}) := r(\mathbf{X},\mathbf{Y}) &+ \sum_{k,z,t} q_{kz}^t \min\{M \cdot y_{kz}^t,\ V_k^t - \sum_i v_i x_{ik}^t\} \\ &+ \sum_{k,t}\left(\left(\sum_z q_{kz}^t/Z\right) V_k^t \left(1 - \sum_z y_{kz}^t\right)\right), \end{aligned} \tag{11}$$

where M in (11) is a large constant. In the rolling horizon implementation, we update the value of V_k^t in each auction, replace $r(\mathbf{X},\mathbf{Y})$ by $\hat{r}(\mathbf{X},\mathbf{Y})$, then solve the optimization problem (10) for new winning bids. This process is repeated for every auction.

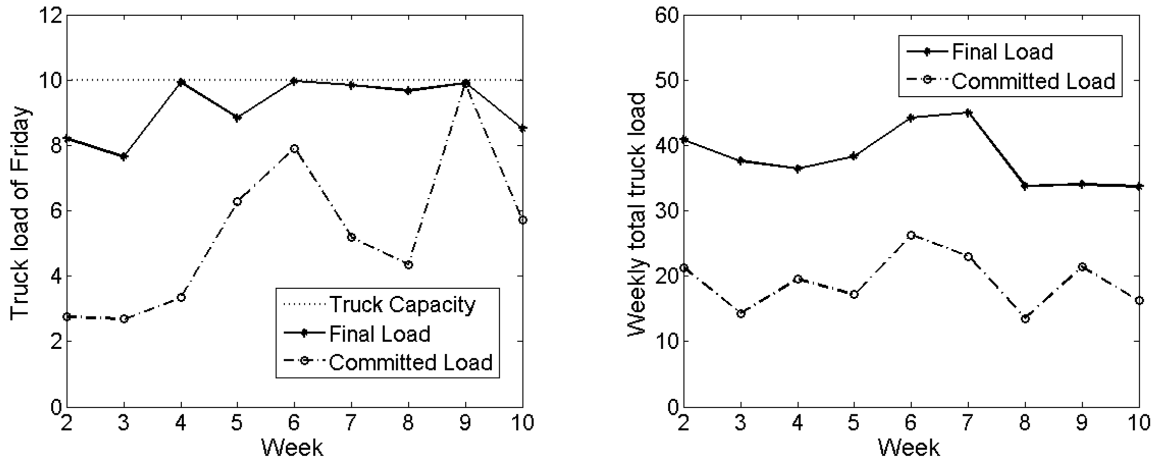
4.1 An example of auction with rolling horizon

The UCC: In this section, the efficacy of the proposed UCC auction with rolling horizon will be demonstrated via computational experiments. To understand the contribution of the rolling horizon framework more clearly and easily, we consider a problem with one zone and one truck with capacity 10. The planning horizon is 10 weekdays (2 weeks) and the auction result is released weekly. The delivery cost is deterministic and equals 10.

Bid Generation: For each auction, a total of 30 bids are generated in which 15 bids compete for the UCC's delivery resources on the first week of the planning horizon and another 15 for resources on the second week. The price-to-volume ratio of the bids is uniformly distributed between 0 and 3.

Efficacy of UCC Auction with Rolling Horizon In our first experiment, we set $q_{11}^t = 0$ for each $t = 1, 2, ..., 5$ and $q_{11}^t = 1$ for each $t = 6, 7, ..., 10$ and let the simulation runs for 10 weeks. The truck load on Friday of the Week #2 to Week #10 are shown in Figure 2(a). The result of Week #1 is not shown since it is only involved in one auction and no truck load is committed before. In the figure, the circles show the truck load committed one week before the delivery date and the stars show the final truck load. It can be observed that in most of the days, the final truck load contains a portion that is committed one week before the delivery date. For truck load on Monday, Tuesday, Wednesday and Thursday we can also observe the similar pattern, which also lead to a weekly total truck load as shown in Figure 2(b). From Figure 2(b), we can observe that about half of the weekly truck load are committed one week before and this is shown specifically by Figure 3. In the figure, the dash line shows the percentage of the weekly load that is committed one week before. It can be seen that around 50% of the total load is committed one week before. In Figure 3, the solid line also shows the percentage of the total profit that is due to such load. In most weeks, the solid line is above the dash line, suggesting that by pricing the unused capacity properly (i.e. setting q_{kz}^t appropriately) the winning bids chosen one week before are more profitable.

Effect of Various Pricing of Unused Capacity: To further show the importance of the potential value rate, we let q_{11}^t where $t = 6, 7, ..., 10$ change from 0 to 3.4 while keeping q_{11}^t for each $t = 1, 2, ..., 5$ to see how the total revenue (profit) changes with the pricing of the unused capacity. Since the delivery cost remains the same for all cases, we just compare revenue contributed by different types of bids. The result is shown in Figure 4 where the darker bars at the bottom shows the revenue contributed by the winning bids for deliveries on the second week of the planning horizon and the lighter bars on the top corresponds to the revenue attributable to the winning bids for deliveries on the first week of



(a) (b)

Figure 2: (a) Truck load of Friday. (b) Weekly total truck load.

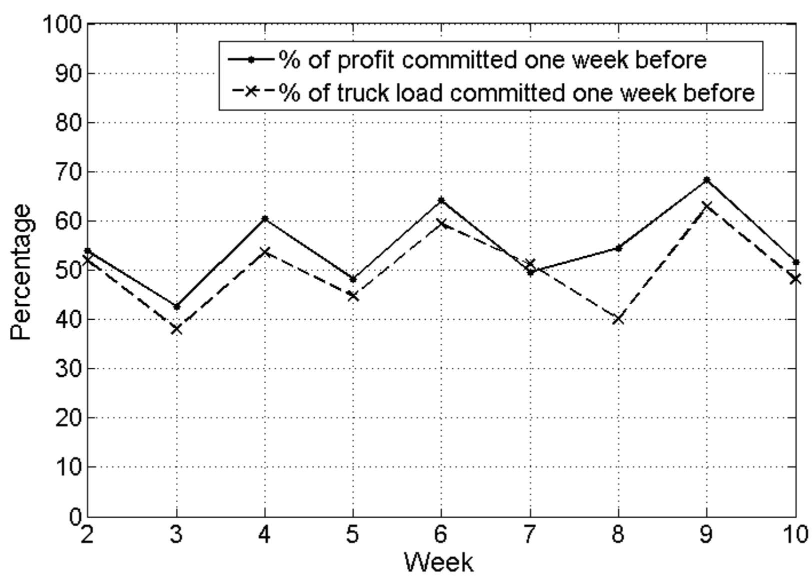


Figure 3: Percentage of profit and truck load committed in advance (i.e. one week ahead of the actual delivery week).

the planning horizon. Note that when $q_{11}^t = 0$ for all t, the winning bids are selected exactly according to Problem (10). As q increases, the revenue contributed by the winners of the second week's delivery resource first increases and then decreases. The total revenue follows the same trend. When $q > 3$, all the shipping capacity is reserved for the very last auction before the delivery date as no bids have the price-to-volume ratio larger than 3. Therefore, there is no winning bids for deliveries on the second week of the planning horizon in such cases, and it is equivalent to run the auction with one week planning horizon every week without rolling horizon. Figure 4 also verifies the idea of choosing value of q described in Section 4. As the expected total volume of the 15 bids is 22.88, the value of q should be $3 * 10/22.88 = 1.31$ in the ideal case. But due to non-splittable bid volumes and the small number of bids, the best value of q appears around 1 which is smaller than 1.31.

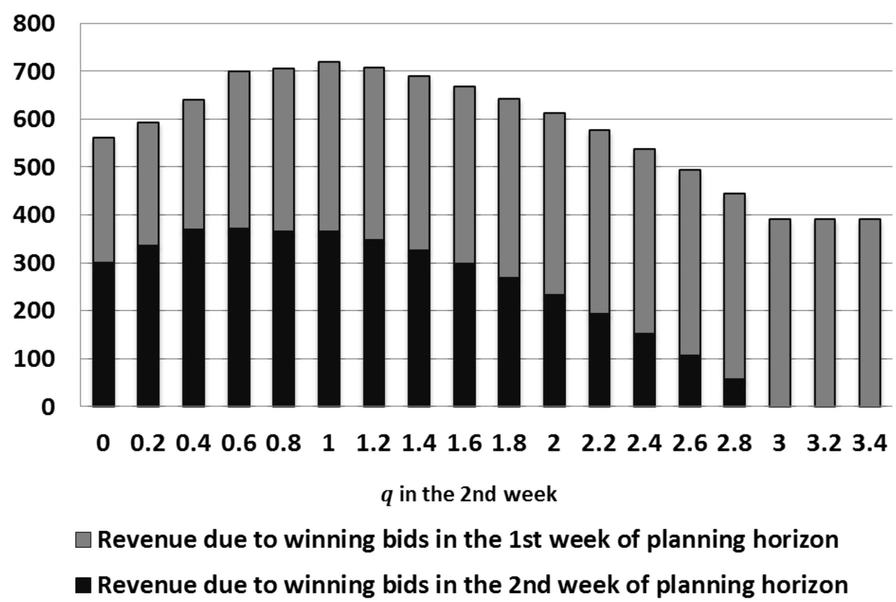


Figure 4: Revenue v.s. value of q_{11}^t for all $t = 6, 7, ..., 10$.

5 Virtual price determination

The virtual price q_{kz}^t is a critical parameter in rolling horizon implementation and has great impact on the performance. In this section, we introduce two approaches for the determination of its value, one for the case with known price/volume distribution and the other for the case with uncertainties.

5.1 The case with known price/volume distribution

For an ideal case, a reasonable value of q_{kz}^t can be roughly determined in the following way. Suppose the volume of each bid is relatively small compared with a truck load so that almost all the highly profitable bids can be consolidated in a truck load. We also assume the volume and price of each bid are independent and the distribution of the ratio of price to volume of a bid, namely p/v, is available or can be estimated from historical data. We let $F(\cdot)$ denote the cumulative distribution function of the ratio p/v. Then we want to fill the remaining truck capacity V_k with the bids with the highest value of price-volume ratio. If the total volume of the oncoming bids is V, then the best value of q is $F^{-1}(1 - V_k/V)$, as shown in Figure 5. This value is for the optimistic case where all bids with the high price-volume ratio can be consolidated into a truck load. However, this is almost not possible in reality due to non-splittable volume of bid and limited number of bids. Therefore, the value of q in practice is usually appropriately smaller than $F^{-1}(1 - V_k/V)$. This is already observed in the results of numerical example in Section 4.1. In the next section, we will discuss another scenario where the distribution of p/v is unknown and the total volume is uncertain.

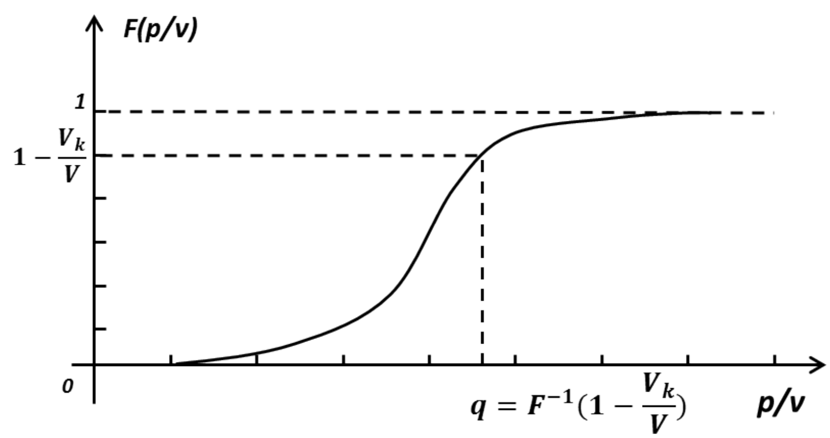


Figure 5: Determination of value of potential value rate.

5.2 Approach using robust optimization

This section consider a scenario that is more realistic. Let n denote the index of future auctions and s_j^n denote the total volume that is to be allocated for zone j in the future auction n. Also for the clear presentation, we assume the trucks are homogeneous and q_{kj}^t remains the same for all $k \in \mathcal{K}$. We let q_j^n denote the value of q_{kj}^t that is set for the next N auctions and Z zones. Then typically, s_j^n is a decreasing function of q_j^n in general and may also be subject to uncertainties. In this paper, we assume s_j^n, as a function of q_j^n, takes the following form,

$$s_j^n(q_j^n, \delta_j^n) = (a_j^n - b_j^n q_j^n)(1 + \delta_j^n) \tag{12}$$

where a_j^n and b_j^n are known parameters, δ_j^n is an uncertain factor round 0 and lies in the range $[\underline{\delta}_j^n,\ \bar{\delta}_j^n]$. Then an lower bound of the revenue due to the winning bids to zone j in auction n is

$$\Upsilon(q_j^n, \delta_j^n) := s_j^n q_j^n = p_j^n(a_j^n - b_j^n q_j^n)(1 + \delta_j^n). \tag{13}$$

It has been pointed out (Chen and Sim, 2009) that the primary target of decision makers is to meet certain pre-specified target on profit instead of maximizing simply maximizing the profit. This motives us to propose a Target-oriented Robust Optimization approach to determine the value of q_j^n. Let V_n denote the total remaining capacity of UCC's truck fleet before auction n and let τ denote the total revenue that is expected from the future N auctions. We aim to determine the vale of q_j^n so that the revenue target is met for an uncertainty set that is as much as possible, so the optimization problem takes the following form.

$$\max_{0\le\gamma\le 1} \ \gamma \tag{14a}$$

$$s.t. \ \sum_{j\in\mathcal{Z}} s_j^n(q_j^n, \delta_j^n) \le V_n, \qquad \forall \delta_j^n \in \left[\gamma\underline{\delta}_j^n, \gamma\bar{\delta}_j^n\right], \ \forall n \in \mathcal{N}; \tag{14b}$$

$$\sum_{n\in\mathcal{N}} \sum_{j\in\mathcal{Z}} \Upsilon(q_j^n, \delta_j^n) \ge \tau, \qquad \forall \delta_j^n \in \left[\gamma\underline{\delta}_j^n, \gamma\bar{\delta}_j^n\right]. \tag{14c}$$

As the worst case scenarios of each constraint in Problem (14) can be determined by observation, Problem (14) is equivalent to

$$\max_{0\le\gamma\le 1} \ \gamma \tag{15a}$$

$$s.t. \ \sum_{j\in\mathcal{Z}} (a_j^n - b_j^n q_j^n)(1 + \gamma\bar{\delta}_j^n) \le V_n, \qquad \forall t \in \mathcal{N}, \tag{15b}$$

$$\sum_{n\in\mathcal{N}} \sum_{j\in\mathcal{Z}} p_j^n(a_j^n - b_j^n q_j^n)(1 + \gamma\underline{\delta}_j^n) \ge \tau. \tag{15c}$$

Problem (15) can be solved by performing a binary search over γ and solving the following problem a few times.

$$r^*(\gamma) := \max \ \sum_{t\in\mathcal{N}} \sum_{j\in\mathcal{Z}} q_j^n(a_j^n - b_j^n q_j^n)(1 + \gamma\underline{\delta}_j^n) \tag{16a}$$

$$s.t. \ \sum_{j\in\mathcal{Z}} (a_j^n - b_j^n q_j^n)(1 + \gamma\bar{\delta}_j^n) \le V_n, \qquad \forall t \in \mathcal{N}. \tag{16b}$$

The specific procedure is summarized in Algorithm 1.

Algorithm 1 Binary search for solution of Problem (15)

procedure $TRO_{BS}(\tau, i_{max})$
  $LB = 0$, $UB = 1$, $i = 1$
  if $r^*(LB)$ of Problem (16) is less than τ **then** Return infeasible
  end if
  if $r^*(UB)$ of Problem (16) is no less than τ **then** Return 1
  end if
  while $i \leq i_{max}$ **do**
    if $r^*((UB + LB)/2)$ of Problem (16) is no less than τ **then** LB =(UB+LB)/2
    else UB =(UB+LB)/2
    end if
    $i = i + 1$
  end while
  Return LB
end procedure

5.3 An example of virtual pricing

UCC: In this section, we demonstrate the pricing approach proposed in Section 5.2 using an example. The UCC considered in this section is the same as the one in Section 4.1 with one truck, one zone and a planning horizon of 10 days.

Bid Pattern: We assume the mean of the daily total bid volume increases evenly from10 on Monday to 30 on Friday, and remains the same for every week. The actual daily total volume is uniformly distributed within ±10% around its mean value. The distribution of the ratio p/v is the same for all bids, and is uniformly distributed between 3 and 5. Therefore for a pricing problem with a planning horizon of 2 weeks, the parameters is summarized in Table 2. When we generate detailed bid information, we draw the total daily volume first, and then determine the number of bids such that the average bid volume is 0.5.

Virtual pricing using robust optimization: We choose $\tau = 200$ for Problem (14) which is about 60% of the theoretical maximum mean of revenue in two weeks' time. With this target and the parameters in Table 2, Algorithm 1 determines an optimal $\gamma^* = 0.53$ and Figure 6 shows the trajectories of the

Table 2: **Parameters in pricing problem**

	Monday	Tuesday	Wednesday	Thursday	Friday
a	50	75	100	125	150
b	10	15	20	25	30
$\bar{\delta}$	10%	10%	10%	10%	10%
$\underline{\delta}$	-10%	-10%	-10%	-10%	-10%

upper bound and lower bound of γ in Algorithm 1. The resultant optimal virtual prices for one unit of

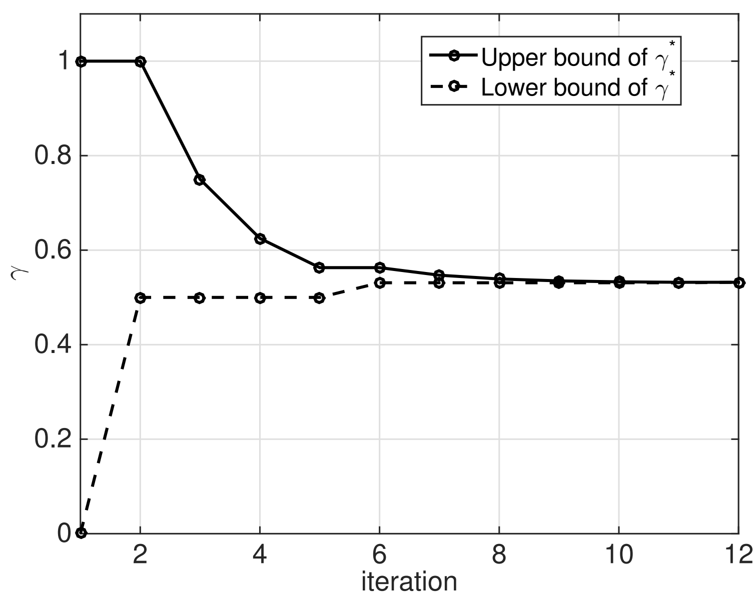


Figure 6: Trajectories of upper and lower bounds of γ in Algorithm 1.

shipping capacity on weekdays are given in Table 3.

Daily revenue: We also conduct simulation to run the UCC for 10 weeks, and therefore 10 auctions are closed. Assuming that each truck load captures the highest profitable portion of the total bid volume, we can calculate the upper and lower bounds of the theoretically highest revenue of each weekday, and they are shown in Figure 7 together with the average revenue of each weekday over the 10 weeks in our simulation. It can be observed that the average daily revenue in our simulation is actually very close to its theoretically highest value. This verifies the validity of our virtual pricing approach.

Table 3: **Optimal virtual prices for weekdays**

	Monday	Tuesday	Wednesday	Thursday	Friday
optimal q	4.05	4.37	4.53	4.62	4.68

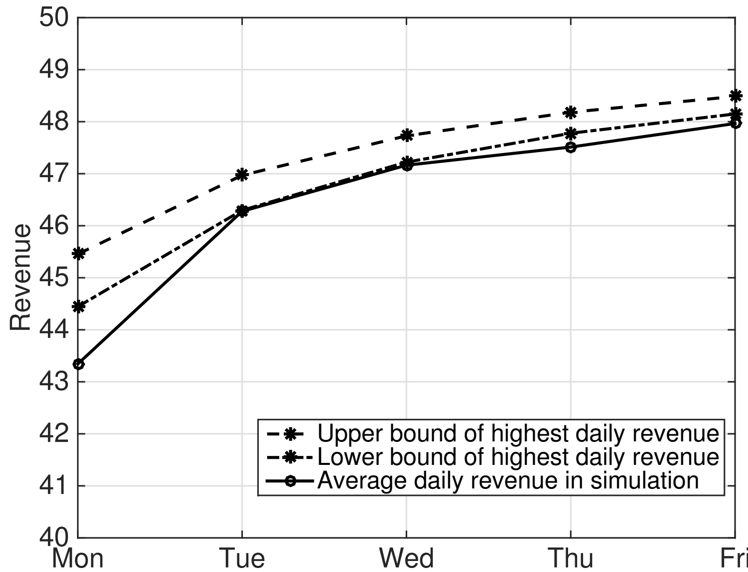


Figure 7: Average daily revenue of each weekday.

6 Conclusion

In this paper, we have presented auction mechanism with rolling horizon for the consolidation of last-mile deliveries into the city center via an urban consolidation center (UCC). To anticipate profitable bids in future auctions, we augment the profit function of the basic winner determination problem with additional terms to allow pricing of the unused capacity. This is essentially equivalent to introducing some virtual bids to the current auction, which may potentially be replaced by the real bids in the upcoming auctions. Simulation results suggest that by setting the virtual price appropriately, more revenue can be made out of the auctions of UCC.

For the determination of appropriate virtual price, we also proposed approaches for two different scenarios. One is for the ideal case where the distribution of price/volume ratio of upcoming bids and the total bid volume are known. In this case, the optimal virtual price can be determined using the distribution function. The other pricing approach is for a more realistic scenario where the distribution of price/volume and the total volume are unknown and subject to uncertainties. For this case, we proposed an approach which use Target-oriented Robust Optimization to determine virtual prices that makes the resultant revenue robust against uncertain factors. The validity of the proposed approach is verified by numerical experiments.

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