Chaos and Convergence on Bucket Brigade Assembly Lines

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One way to coördinate the efforts of workers along an assembly line that has fewer workers than work stations is to form a bucket brigade. Each worker in a bucket brigade simultaneously assembles a single item (an instance of the product) along the line. The worker carries the item from work station to work station until either he hands off his item to a downstream co-worker or he completes the work for his item. The worker then walks back to get another item, either from his co-worker upstream or from a buffer at the beginning of the line. The most notable application of bucket brigades is to coördinate workers to pick products for customer orders in distribution centers, as reported in Bartholdi and Eisenstein (1996b) and Bartholdi et al. (2001). Bucket brigades have also been used in the production of garments, the packaging of cellular phones, and the assembly of tractors, large-screen televisions, and automotive electrical harnesses (see Bartholdi and Eisenstein (1996a,b, 2005), and Villalobos et al. (1999a,b)).

In the Normative Model of bucket brigades (Bartholdi and Eisenstein 1996a) the work content of the product is assumed to be deterministic and to be continuously and evenly distributed along the assembly line. Workers are not allowed to pass one another so that their sequence along the line is preserved. Each worker proceeds forward with a finite *work* velocity reflecting the worker's familiarity with the work content. Furthermore, the time for each worker to walk back to get more work is assumed to be negligible (that is, workers walk back with an infinite velocity). Bartholdi and Eisenstein (1996a) showed that when workers are sequenced from slowest to fastest (according to their work velocities) in the direction of production flow, they will eventually hand off items to their co-worker downstream at fixed locations. Every worker will eventually repeat their respective portion of work content on each item produced. The system is said to converge to a *fixed point* (Alligood et al. 1996). Furthermore, after converging to the fixed point the throughput (number of items produced per unit time) of the line attains the maximum possible. Convergence to the fixed point is desirable because it creates several positive effects, such as the skills of workers are reinforced by repetition, the effort of each worker contributes directly to the output, and the output is regular, which simplifies the coördination of downstream processes. All these effects are created without the intervention of management or engineering.

In this paper, we consider an extended model in which each worker i is characterized by a constant work velocity v_i and a constant walk-back velocity w_i. This generalization applies to some application contexts, such as McMaster-Carr, an industrial supply company that carries over 350,000 stockkeeping units (SKUs). A typical customer order at McMaster-Carr requests fewer than three SKUs. To ensure a high service level, orders must be picked within 30 minutes of receipt, and so workers typically pick few orders per trip. Therefore, workers spend significant time in walking during a trip. The time to work forward picking orders is comparable to the time required to walk back for the next batch of orders.

Consider a team of n workers indexed from 1 to n. Workers $1, \ldots, i-1$ are the *predecessors* of worker i and workers $i+1, \ldots, n$ are his *successors*. Each worker must

Table 1: Each team member independently follows the extended Bucket Brigade Rules.

Forward Rule: Work forward with your item until

1. your item is handed off to a successor; or
2. you complete your item;

then follow the Backward Rule.

Backward Rule: Walk back to get more work,

1. if you encounter a predecessor working forward then take over his item;
2. otherwise, begin a new item at the start of the line;

then follow the Forward Rule.

be able to distinguish his predecessors from his successors and must follow the extended Bucket Brigade Rules given in Table 1. Unlike the Normative Model, workers following these extended rules are not restricted in a fixed sequence along the line because of the following behaviors:

Overtaking, in which one worker catches up and passes another as both work forward or as both walk back.

Passing, in which a worker going back to get more work walks past a successor who is working forward. (They must pass because a worker may not take work from a successor.)

The main result of this paper is that when workers follow the extended rules the bucket brigade converges to a unique fixed point (emergence of a stable partition of work) if the following condition holds.

Convergence Condition: The workers on a bucket brigade should be indexed so that

$$\frac{1}{v_1} - \frac{1}{w_1} > \frac{1}{v_2} - \frac{1}{w_2} > \ldots > \frac{1}{v_n} - \frac{1}{w_n}; \tag{1}$$

or, in other words, from most-slowed to least-slowed.

Furthermore, if configured inappropriately, a bucket brigade described by our extended model can be capable of chaotic behavior. The top graphs of Figure 1 show the convergence of a two-worker line when the Convergence Condition (1) holds. Hand-off locations converge quickly to a single point (top left) and each successive item is produced in a

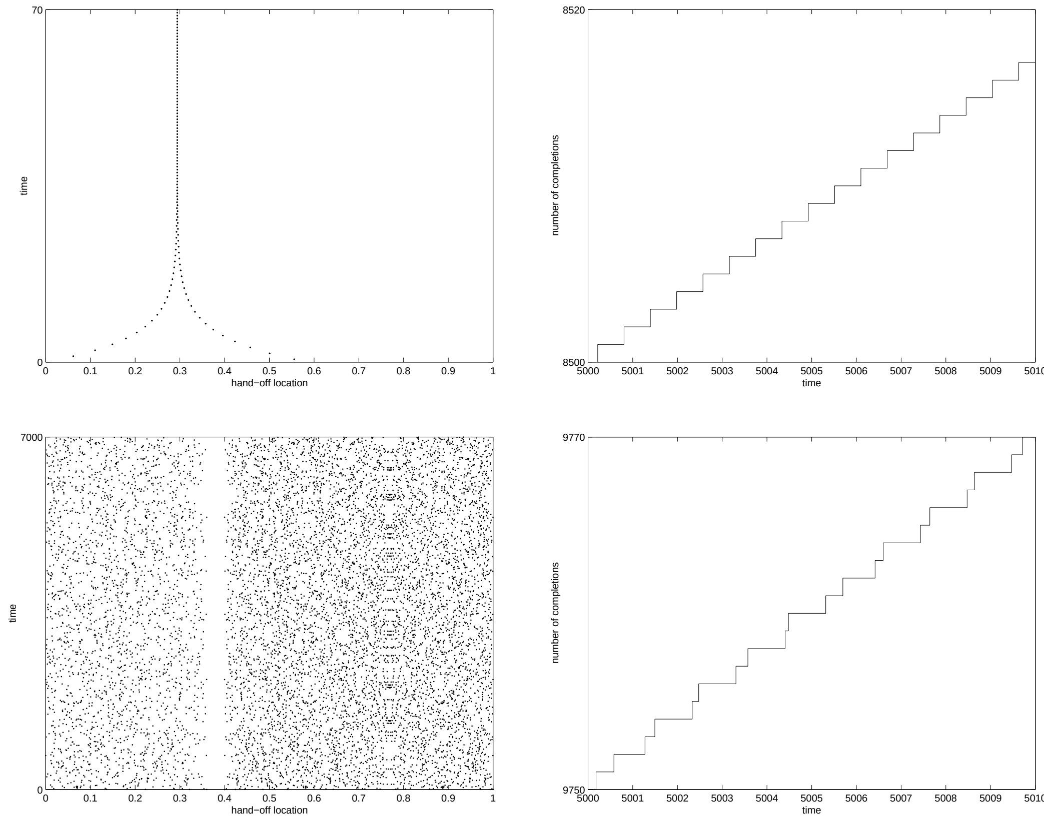

Figure 1: **Convergence and chaos:** The top graphs show the convergence of a two-worker line when the Convergence Condition (1) holds. The bottom graphs show the chaotic behavior of the line when the Convergence Condition is violated. In all graphs $w_1 = 1, v_2 = 3, w_2 = 2$, and $v_1 = 1$ and 3 in top and bottom graphs respectively.

fixed time interval (top right). The bottom graphs show the chaotic behavior of the line when the Convergence Condition is violated. Hand-offs occur erratically along the assembly line (bottom left) and completion times become irregular (bottom right). This demonstrates that significant variability can be induced by the dynamics of a purely deterministic system.

Figure 2 summarizes the asymptotic dynamics of two-worker lines. The Convergence Condition (1) holds in the unshaded area above the diagonal line. All bucket brigades lying in this area converge to a fixed point. On the other hand, all bucket brigades lying in the shaded area below the diagonal line fail the Convergence Condition and behave chaotically. In the unshaded area, all overtaking and all passing are transient. In Region 1b, forward overtaking can persist. In Region 3b, backward overtaking can persist. In

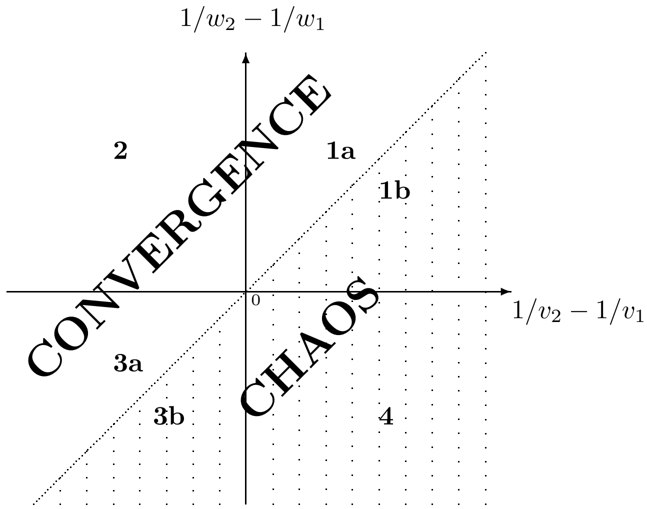

Figure 2: The Convergence Condition (1) is violated in the shaded area below the diagonal line where the system behaves chaotically.

Region 4, both forward overtaking and backward overtaking can persist.

Figure 3 shows the transition of a two-worker line from stability to chaos. We set $w_1 = 1, v_2 = 3$, and $w_2 = 2$, and recorded the hand-off locations with different values of v_1. For each $v_1 \in [1, 10]$ we started with both workers located at point 0, computing the hand-off locations through 10,000 iterations (presumably long enough for transients to fade away), and then plotted the next 1,000 hand-off locations vertically above the corresponding v_1. For $v_1 < 6/5$ the Convergence Condition holds (this is within Region 3a of Figure 2), and all hand-offs occur at a fixed point as expected. The value of the fixed point increases with v_1. At the threshold of chaos, $v_1 = 6/5$, the Convergence Condition fails to hold, and the formerly attracting fixed point becomes explosively repelling as the system moves into the shaded Region 3b of Figure 2. The asymptotic sets corresponding to each v_1 in this region appear Cantor-like (Alligood et al. 1996). Another regime of behavior occurs as $v_1 > 3$ and the system moves into Region 4. Lim (2005) provides explanations of the fine structure.

We will explain the observed behaviors analytically. We will also discuss the impact of chaos on simulation models of manufacturing systems and the potential errors that

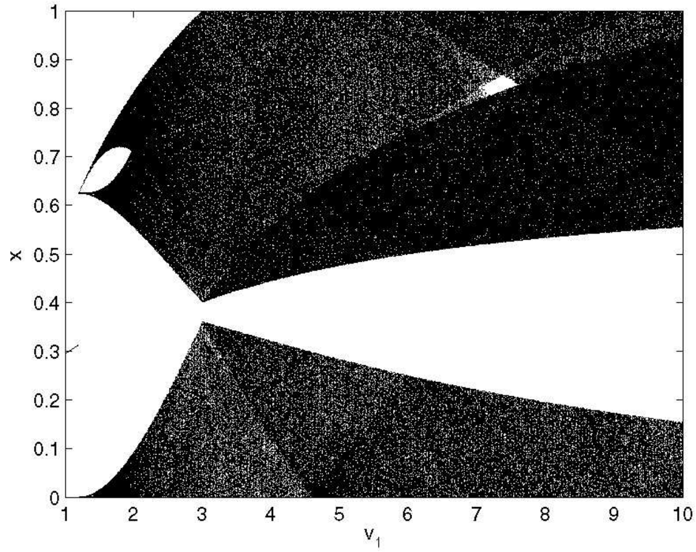

Figure 3: The hand-off locations between the 10,000th and 11,000th hand-offs are plotted with different values of v_1 while other velocities are fixed ($w_1 = 1, v_2 = 3$, and $w_2 = 2$).

the chaotic behavior may result in numerical computation.

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