Inconsistency and Contradiction

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Inconsistency and contradiction are important concepts. Unfortunately, they are easily confused.

A proposition or belief which is inconsistent is one which is self-contradictory and vice-versa. Moreover two propositions or beliefs which are contradictories are inconsistent with each other. Nonetheless it is a mistake to suppose that inconsistency is the same as contradiction.

If a man believes that s1, believes that s2, and believes that \( \neg(s1 \& s2) \), then he does not hold contradictory beliefs, as he would were he to both believe that \( s1 \& s2 \) and believe that \( \neg(s1 \& s2) \). Nor does he hold a self-contradictory belief, as he would were he to believe that \( [s1 \& s2 \& \neg(s1 \& s2)] \).

He does not hold contradictory beliefs, since there are no two beliefs which contradict each other. Moreover, the subtraction of any one of his beliefs has the result that it is now possible for all his remaining beliefs to be correct. Neither does he hold any self-contradictory beliefs, since there is no one thing believed from which two contradictory things follow. Yet since the conjunction of what he believes is self-contradictory and hence not possibly true, the fact that he holds all the beliefs which he does entails that they are necessarily not all true. Therefore his beliefs are inconsistent.

Hence, while all believers who hold contradictory beliefs or hold a self-contradictory belief hold inconsistent beliefs, not all believers who hold inconsistent beliefs hold either contradictory ones or self-contradictory ones.

In the light of this, the only reason that one could have for supposing that inconsistency is the same as contradiction is the view, conscious or unconscious, that to hold a conjunction of beliefs is to believe a conjunction. For were this so, then one who inconsistently believed that s1, believed that s2 and believed that \( \neg(s1 \& s2) \) would thereby also believe that \( s1 \& s2 \) and hence hold contradictory beliefs.

This view is mistaken, however. To believe that p and to believe that q is not necessarily to believe that p and q.

Were this not the case then all cases of holding beliefs inconsistent with each other would be cases of holding self-contradictory beliefs. For on the disputed view, one who both believed \( \neg(s1 \& s2) \) and believed \( s1 \& s2 \) would thereby believe that \( [(s1 \& s2) \& \neg(s1 \& s2)] \). But it is surely implausible to suppose that holding inconsistent beliefs is always a matter of holding beliefs of what is self-contradictory. One can legitimately doubt whether self-contradictory beliefs are even possible, for normally, the falsity of such a belief is perspicuous to the believer. At the very least, lack of such perspicacity is uncommon. Yet notoriously, inconsistent sets of beliefs are not only possible, but difficult to avoid, especially when there are a large number of beliefs in the set, e.g. a belief that s1, a belief that s2, ... a belief that sn and a belief that \( \neg(s1 \& s2 \& \ldots \& sn) \).
A man who believes that p and believes that q, may have considered whether p and considered whether q, without having considered whether p and q, in which case he does not consciously believe that p and q. But there is neither any reason to hypothesise an unconscious belief that p and q, for the only reason one could have for doing so would be the very same view under dispute. Indeed, an extreme example of this latter case involves holding contradictory beliefs. A may present an argument containing many premises, including p and including ∼p, and being sincere, believe each premise separately. Although these premises may be perspicuously contradictory when considered together, A may have only considered them separately, which allows him to believe that p and believe that ∼p.

Suppose also, that A never believes what he knows to be false. This is surely possible. One surely does not believe what one does not understand. It would be ludicrous to credit someone with beliefs of things of which he had never heard or of which he had no understanding. But while A may believe and hence understand the proposition p, and also may believe and hence understand the proposition ∼p, he cannot believe the proposition p & ∼p even unconsciously. This would entail that he understood this proposition, which being perspicuously self-contradictory, he would know to be false, and given the supposition of this particular example he would not believe.

Moreover, a man who can ‘only hold a few ideas in his head’ might severally believe each of a large number of propositions without being able to understand or consider their conjunction, and therefore without being able to believe it. Such a man would be like a person in a supermarket with a shopping list who can remember to buy each item upon seeing it, but who lacks the memory to write out the list for himself.

The importance of the difference between inconsistency and contradiction arises from the fact that whereas the discovery of contradiction in a man’s beliefs is conclusive ground for censure, the discovery of inconsistency need not be. For if a man holds contradictory beliefs, e.g. believes that p and believes that ∼p, then clearly, he should give one of them up, for any evidence for the one will be evidence against the other and vice-versa, so that he will be unjustified in continuing to hold both beliefs. A self-contradictory belief, e.g. a belief that p & ∼p, is likewise irrational, since clearly there can be no evidence at all to justify it. On the other hand, an inconsistent set of beliefs may be epistemologically virtuous, for the evidence for each belief may be independent of that for any other, so that one is quite justified in holding each belief. For unlike a set of contradictory propositions, each of a set of inconsistent propositions may be logically independent of any one of the others. Where a man believes that s₁, believes that s₂ and believes that ∼(s₁ & s₂ & … & sₙ) his confidence in each of s₁ to sₙ may be justifiably high yet justifiably less than complete, so that the gaps in his confidence add up to a justifiable lack of confidence in their conjunction.

Substantial evidence for the belief that ∼(s₁ & s₂ & … & sₙ) may simply be that n is a very large number. In this case it is hard to see how it is evidence against any of the individual beliefs in s₁ to sₙ. Indeed
the very same evidence which both justifies a belief that $s_1$ and justifies a belief that $s_2$ may justify a belief that $\sim (s_1 \& s_2)$. Suppose the evidence is that the probability that $s_1$ is the same as the probability that $s_2$ and this is $\frac{3}{4}$, i.e. more probable than not. This justifies both a belief that $s_1$ and a belief that $s_2$. But it is also evidence that the probability that $s_1$ and $s_2$ is $\frac{3}{4} \times \frac{3}{4}$ i.e. $\frac{9}{16}$, i.e. less probable than not.

The moral of this is that one's attitudes to inconsistency should not be uncritically coloured by one's attitudes to contradiction,\(^1\) for inconsistent beliefs may nonetheless be rational.\(^2\)

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