Session 11: Review Exercises

AY2025/26 Term 1

Question 1: If your sample $\{Y_i\}_{i=1}^{n}$ is such that $Y_i \overset{iid}{\sim} N(\mu, \sigma^2)$ for $i = 1, \ldots, n$, show that that the maximum likelihood estimators for μ and σ^2 are

$$\hat{\mu}_{ml} = \frac{1}{n}\sum_{i=1}^{n} Y_i = \overline{Y}$$
$$\widehat{\sigma^2}_{ml} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \overline{Y})^2 \,.$$

Show also that

$$-E\left(\frac{\partial^2 \ln L(\theta \mid Y_1, \ldots, Y_n)}{\partial\theta\partial\theta^{\mathrm{T}}}\right) = -E\begin{bmatrix} \frac{\partial^2 \ln L(\theta \mid Y_1, \ldots, Y_n)}{(\partial\mu)^2} & \frac{\partial^2 \ln L(\theta \mid Y_1, \ldots, Y_n)}{\partial\mu\partial\sigma^2} \\ \frac{\partial^2 \ln L(\theta \mid Y_1, \ldots, Y_n)}{\partial\mu\partial\sigma^2} & \frac{\partial^2 \ln L(\theta \mid Y_1, \ldots, Y_n)}{(\partial\sigma^2)^2} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{n}{\sigma^2} & 0 \\ 0 & \frac{n}{2\sigma^4} \end{bmatrix}$$

where θ is the 2×1 vector containing μ and σ^2.

Question 2: In your own words (and without using equations or math symbols), explain the conceptual differences between corner solution models and censored/truncated dependent variable models. You should make clear what distinguishes “corner” solution observations vs truncated observations vs censored observations, and why it matters for how we model and interpret the results.