Session 3: Review Exercise

AY2025/26 Term 1

Question 1 (Econometrics Notes Exercise 4.3) Show that the F-statistic given in (4.17) in Econometrics Notes can be written as

$$F = \frac{(R_{ur}^2 - R_r^2)/J}{(1 - R_{ur}^2)/(n - K)}$$

where R_{ur} and R_r are the R^2 from the unrestricted and restricted regressions respectively, J is the number of restrictions being tested, n is the number of observations used in the regression, and K is the number of coefficient parameters in the unrestricted regression model (including intercept). What does this expression simplify to if the hypothesis being tested is that all the slope coefficients (excluding the intercept) are equal to zero?

Question 2 (Econometric Notes Exercise 4.10) Suppose *male* is a dummy variable indicating if an observation is male (=1) or not male (=0), and the variable $female$ is defined as $female = 1 - male$.

(a) Explain why you cannot run the regression

$$\text{(A)} \quad \ln earn = \beta_0 + \beta_1 male + \beta_2\, female + \epsilon$$

but you can run the regression

$$\text{(B)} \quad \ln earn = \beta_1 male + \beta_2\, female + \epsilon\,.$$

(b) In the regression

$$\text{(C)} \quad \ln earn = \alpha_0 + \alpha_1\, male + \epsilon\,,$$

show that the OLS estimator $\hat{\alpha}_0$ is equal to the sample mean of $\ln earn$ for all female individuals in the sample, and $\hat{\alpha}_0 + \hat{\alpha}_1$ is the sample mean of $\ln earn$ for all male individuals in the sample. By suitable re-parameterization of equation B, show that the OLS estimator $\hat{\beta}_1$ is equal to the sample mean of $\ln earn$ for all male individuals in the sample and $\hat{\beta}_2$ is equal to the sample mean of $\ln earn$ for all female individuals in the sample.