Qn 1  Check if the following AR(2) processes are stationary

i. \( Y_t = 0.8 + 0.9Y_{t-1} - 0.2Y_{t-2} + \epsilon_t \), \( \epsilon_t \sim (0, \sigma^2) \)

ii. \( Y_t = 0.8 + 1.6Y_{t-1} - 0.73Y_{t-2} + \epsilon_t \), \( \epsilon_t \sim (0, \sigma^2) \)

iii. \( Y_t = 0.8 + 1.3Y_{t-1} - 0.3Y_{t-2} + \epsilon_t \), \( \epsilon_t \sim (0, \sigma^2) \)

Qn 2  (a) Consider the model

\[ Y_t = \beta_0 + \beta_4 Y_{t-4} + \epsilon_t \], \( \epsilon_t \sim (0, \sigma^2) \)

which is an AR(4) but with “zero-restrictions” on \( \beta_1, \beta_2, \) and \( \beta_3 \). Assume for this question that \( | \beta_4 | < 1 \).

i. Write \( Y_t \) as a sum of current and infinite lags of the error term \( \epsilon_t \).

ii. Derive the autocorrelation function of the process. Explain why this model describes quarterly seasonal behaviour.

iii. Show that the condition \( | \beta_4 | < 1 \) implies that the process is stationary. What are the inverse characteristic roots of this model if \( \beta_4 = 1 \)?

(b) The model above is sometimes described as a “seasonal AR” model. It is often used in conjunction with ‘regular’ AR models, such as in the following model

\[ (1 - \beta_1 L)(1 - \beta_4 L^4)Y_t = \beta_0 + \epsilon_t \]

which we denote as \( AR(1)(1)_4 \), where the integer in the second parentheses refers to the number of seasonal lags.

i. Expand the \( AR(1)(1)_4 \) into the usual AR form. Viewed as a “restricted” \( AR(p) \), what is the value of \( p \), and what are the restrictions?

ii. Assuming that \( | \beta_1 | < 1 \) and \( | \beta_4 | < 1 \), find the unconditional mean of \( Y_t \), and show that the model can be written as

\[ (1 - \beta_1 L)(1 - \beta_4 L^4)\bar{Y}_t = \epsilon_t \]

where \( \bar{Y}_t = Y_t - E[Y_t] \).

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(c) Run three simulations of the model in part (b) using

i. a specification with $0 < \beta_1 < 1$ and $0 < \beta_4 < 1$;

ii. a specification with $\beta_1 = 1$ and $0 < \beta_4 < 1$;

iii. a specification with $0 < \beta_1 < 1$ and $\beta_4 = 1$.

Analyze your simulated series.

You are encouraged to try many different specifications, but report only three. Be as concise as you can in your report. For replicability, use `set.seed(5)` to set the seed of your random number generator. Include your code.

Qn 3 “MA and ARIMA Models” Notes Question 2

Qn 4 “MA and ARIMA” Models Notes Question 4