ECON207 Session 11-12: Review Exercises

AY2024/25 Term 1

Question 1 (Regression with lagged dependent variable vs Regression with AR errors)

(a) Show that models A1 and A2 below describe the same process

$$\begin{array}{ll} \text{[A1]} & Y_t = \beta_0 + \beta_1 Y_{t-1} + \epsilon_t\,,\ |\beta_1| < 1\,,\ \epsilon_t \sim (0, \sigma^2) \\ \text{[A2]} & Y_t = c + u_t\,,\ u_t = \rho u_{t-1} + \epsilon_t\,,\ |\rho| < 1\,,\ \epsilon_t \sim (0, \sigma^2) \end{array}$$

(b) Show that models B1 and B2 below describe the same process

$$\begin{array}{ll} \text{[B1]} & Y_t = \beta_0 + \beta_1 t + \beta_2 Y_{t-1} + \epsilon_t\,,\ |\beta_2| < 1\,,\ \epsilon_t \sim (0, \sigma^2) \\ \text{[B2]} & Y_t = \alpha_0 + \alpha_1 t + u_t\,,\ u_t = \rho u_{t-1} + \epsilon_t\,,\ |\rho| < 1\,,\ \epsilon_t \sim (0, \sigma^2) \end{array}$$

(c) Show that models C1 and C2 below *do not* describe the same process

$$\begin{array}{ll} \text{[C1]} & Y_t = \beta_0 + \beta_1 X_t + \beta_2 Y_{t-1} + \epsilon_t\,,\ |\beta_2| < 1\,,\ \epsilon_t \sim (0, \sigma^2) \\ \text{[C2]} & Y_t = \alpha_0 + \alpha_1 X_t + u_t\,,\ u_t = \rho u_{t-1} + \epsilon_t\,,\ |\rho| < 1\,,\ \epsilon_t \sim (0, \sigma^2) \end{array}$$

Question 2 Suppose we are interested in the long-run cumulative dynamic multiplier $\beta_0 + \beta_1 + \beta_2 + \beta_3$ from the regression

$$Y_t = \alpha_0 + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \epsilon_t\,,$$

This is easily found by estimating the regression by OLS and then adding up

$$\hat{\beta}_0^{ols} + \hat{\beta}_1^{ols} + \hat{\beta}_2^{ols} + \hat{\beta}_3^{ols}\,.$$

We can also easily find the standard error for the long-run cumulative dynamic multiplier by calculating the square root of

$$c^{\mathrm{T}}\, Var(\hat{\beta}^{ols}) c$$

where $c = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}}$ and $Var(\hat{\beta}^{ols})$ is the variance-covariance matrix of the 5×1 vector

$$\hat{\beta}^{ols} = \begin{bmatrix} \hat{\alpha}_0^{ols} & \hat{\beta}_0^{ols} & \hat{\beta}_1^{ols} & \hat{\beta}_2^{ols} & \hat{\beta}_3^{ols} \end{bmatrix}^{\mathrm{T}}\,.$$

Yet another way of finding the standard error is to “reparameterize” the regression, by transforming it so that $\beta_0 + \beta_1 + \beta_2 + \beta_3$ appears as a coefficient on some regressor. Running that transformed regression then gives you both a direct estimate of the long-run cumulative dynamic multiplier and its standard error. Find one such transformation.