## ECON207 Session 8: Review Exercises

## AY2024/25 Term 1

In this review exercise, you are asked to elaborate on some of the derivations in Session 8 slides:

Question 1 (Slide 19) Show that

$$E(Y - \beta_0 - \beta_1 X) = 0$$
 and  $E((Y - \beta_0 - \beta_1 X)Z) = 0$ 

 $\text{implies } \beta_1 = \operatorname{\mathit{Cov}}(Z,Y)/\operatorname{\mathit{Cov}}(Z,X).$ 

Question 2 (Slide 44) Show that

$$\hat{\beta}_{mm} = (X^{\mathrm{T}} Z Z^{\mathrm{T}} X)^{-1} X^{\mathrm{T}} Z Z^{\mathrm{T}}$$

(You can skip the second order conditions.)

Question 3 (Slide 64) Show that solving the minimization problem

$$\hat{\beta}_{W}^{gmm} = \mathrm{argmin}_{\hat{\beta}} \; \underbrace{(Z^{\mathrm{T}}y - Z^{\mathrm{T}}X\hat{\beta})^{\mathrm{T}}W(Z^{\mathrm{T}}y - Z^{\mathrm{T}}X\hat{\beta})}_{"J(W)"}$$

produces

$$\hat{\beta}^{gmm}_W = (X^{\mathrm{T}} Z W Z^{\mathrm{T}} X)^{-1} X^{\mathrm{T}} Z W Z^{\mathrm{T}} y \,.$$

(You can skip the second-order conditions).

**Question 4** (Slide 65) Show that if M = G then  $\hat{\beta}_W^{gmm}$ ,  $\hat{\beta}^{mm}$  and  $\hat{\beta}^{2sls}$  all reduce to the IV estimator formula given on slide 28.