

## ECON207 Session 7: Review Exercises

### AY2024/25 Term 1

In this review exercise, you are asked to elaborate on some of the derivations in Session 7 slides:

**Question 1** On Slide 26, we have the equations

$$\begin{aligned} E((Y^o - \hat{Y}^o)^2 \mid X, X_1^o) &= E((\beta_0 - \hat{\beta}_0^{ols} + (\beta_1 - \hat{\beta}_1^{ols})X_1^o + \epsilon^o)^2 \mid X, X_1^o) \\ &= Var(\hat{\beta}_0^{ols} \mid X, X_1^o) + (X_1^o)^2 Var(\hat{\beta}_1^{ols} \mid X, X_1^o) + Var(\epsilon_o \mid X, X_1^o) \\ &\quad + 2X^o Cov(\hat{\beta}_0^{ols}, \hat{\beta}_1^{ols} \mid X, X_1^o) \\ &= \frac{\sigma^2}{n \sum_{i=1}^n (X_{i1} - \bar{X}_1)^2} \left( \sum_{i=1}^n X_{i1}^2 + n(X_1^o)^2 - 2n\bar{X}_1 X_1^o \right) + \sigma^2 \\ &= \frac{\sigma^2}{n \sum_{i=1}^n (X_{i1} - \bar{X}_1)^2} \left( \sum_{i=1}^n (X_{i1} - \bar{X}_1)^2 + n(X_1^o - \bar{X}_1)^2 \right) + \sigma^2 \end{aligned}$$

from which we get the expression for the  $MSPE(X_1^o)$  on Slide 25. Explain carefully how to get each of the four lines above. The correctness of the formula for  $MSPE(X_1^o)$  assumes (i) correct regression specification and (ii) homoskedasticity. Explain where these assumptions enter into its derivation.

**Question 2** (Slide 64) Show that selecting a regression model from a group of alternative regression models by maximizing  $\text{Adj-}R^2$  is equivalent to choosing a regression model by minimizing

$$\frac{n}{n-p} \tilde{\sigma}^2$$

where  $\tilde{\sigma}^2 = \frac{1}{n-p} \sum_{i=1}^n \epsilon_{i,ols}^2$  and where  $p$  is the number of coefficients, including intercept.