ECON207 Session 6: Review Exercises

AY2024/25 Term 1

Question 1 Consider the multiple linear regression

$$Y_{i} = \beta_{0} + \beta_{1}X_{i1} + \dots + \beta_{k}X_{ik} + \epsilon_{i}, \ i = 1, 2, \dots, n.$$

F-tests are used for testing multiple hypotheses, but it can also be used for testing a single hypothesis. Show that the F-statistic for the single hypothesis $H_0: r^T\beta = r_0 \text{ vs } H_A: r^T\beta \neq r_0$ is the square of the t-statistic for this same hypothesis. (Hint: Use the version of the F-statistic presented in Session 6 Slide 19, and compare it with the square of the version of the t-statistic presented in Session 6 Slide 14, after replacing R with r^T and setting J = 1).

Question 2 Consider the problem of deciding whether or not the variable X_{ik} should be included or excluded. That is, you are trying to choose between the models

$$\begin{split} & [\mathbf{A}] \ \ Y_i = \beta_0 + \beta_1 X_{i1} + \dots + \beta_{k-1} X_{i,k-1} + \epsilon_i \\ & [\mathbf{B}] \ \ Y_i = \beta_0 + \beta_1 X_{i1} + \dots + \beta_{k-1} X_{i,k-1} + \beta_k X_{ik} + \epsilon_i \,, \ i = 1, 2, \dots, n. \end{split}$$

Suppose you used the following rule: include X_{ik} if doing so increases the Adjusted- R^2 . Show that this is the same as include X_{ik} if the F-statistic (and therefore the t-statistic) for the hypothesis $H_0: \beta_k = 0$ vs $H_A: \beta_k \neq 0$ is greater than one. Explain why the proposed Adjusted- R^2 rule is far more liberal than the usual t-test or F-test for deciding whether or not to include a variable.