## **ECON207** Session 4: Review Exercise

## AY2024/25 Term 1

**Question 1** Let X be a  $n \times k$  data matrix where each column contains the n observations of a variable. Let  $i_n$  be a  $n \times 1$  vector of 1s, i.e.,  $i_n^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}$ . Let  $M_0$  be the matrix

$$M_0=I_n-i_n(i_n^{\rm T}i_n)^{-1}i_n^{\rm T}$$

- a. How does  $M_0$  transform the  $n \times 1$  vector y when y is pre-multiplied by  $M_0$ ?
- b. Describe how  $M_0$  transforms X when X is pre-multiplied by  $M_0$ .
- c. Show that  $M_0$  is symmetric and satisfies the property  $M_0M_0 = M_0$ .

d. Show that  $X^{\mathrm{T}} M_0^{\mathrm{T}} M_0 X = X^{\mathrm{T}} M_0 X$  and describe the contents of the matrix

$$\frac{1}{n-1}X^{\mathrm{T}}M_0X.$$

**Question 2** (a) Show that the inverse of a symmetric non-singular matrix is symmetric. (*Hint: Use the fact that*  $(A^{T})^{-1} = (A^{-1})^{T}$  which we proved in class.)

(b) Suppose X is  $n \times k$  with n > k, such that  $X^{T}X$  is non-singular. Show that  $(X^{T}X)^{-1}$  is symmetric.

**Question 3** Suppose x is a  $n \times 1$  vector of random variables such that E(x) and  $E(xx^{T})$  exists. Show that  $Var(x) = E(xx^{T}) - E(x)E(x)^{T}$ .