ECON207 Session 1: Review Exercise

AY2024/25 Term 1

Question 1. In Session 1 we defined a population made up of individuals from which we obtained a representative random sample. Another type of "population", better described as a "process", is one that generates a sequence of observations. Think of tossing a certain coin, giving you a sequence of heads and tails. The objective may be to find the probability of obtaining 'heads'. As another example, think of a machine that produces a certain good with some (hopefully small) probability of the good being defective. The objective may be to discover that probability of obtaining a defective good. A more complicated example of a "process" is an economy generating a sequence of macroeconomic observations. We consider the simple coin toss (or defective/non-defective goods) example in this question.

(a) Let Y be a random variable with possible values 1 and 0, such that the probability of obtaining 1 is $\Pr(Y = 1) = p$ (take $Y = 1$ to mean that the outcome of the coin toss is 'heads', or that a defective good was produced). The probability of obtaining 0 is then $\Pr(Y = 0) = 1 - p$. We say that Y is a Bernoulli random variable with parameter p. Complete the following:

i. Show that the probability distribution function of Y can be written as

$$f(y) = \Pr(Y = y) = p^y(1-p)^{1-y}\,,\ y = 0, 1.$$

ii. Show that $E(Y) = p$ and $Var(Y) = p(1-p)$.

(b) Suppose you have an iid sample $\{Y_1, \dots, Y_n\}$ of a Bernoulli(p) random variable Y. Since $E(Y) = p$, it makes sense, given the theory developed in class, to estimate p as

$$\hat{p} = \overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i\,.$$

Furthermore, since an unbiased estimate of $Var(Y)$ is

$$\widehat{Var(Y)} = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2$$

and the variance of the sample mean is $Var(Y)/n$, it makes sense to estimate $Var(\hat{p})$ as

$$\widehat{Var(\hat{p})} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(Y_i - \overline{Y})^2\,.$$

However, since we know that $Var(Y) = p(1-p)$, another possibility is to estimate $Var(Y)$ using $\widetilde{Var(Y)} = \hat{p}(1-\hat{p})$ and thereafter estimate $Var(\hat{p})$ using

$$\widetilde{Var(\hat{p})} = \frac{1}{n}\hat{p}(1-\hat{p})\,.$$

Show that this is equivalent to estimating $Var(Y)$ and $Var(\hat{p})$ as

$$\widetilde{Var(Y)} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \overline{Y})^2 \ \text{ and } \ \widetilde{Var(\hat{p})} = \frac{1}{n^2}\sum_{i=1}^{n}(Y_i - \overline{Y})^2$$

respectively.